

»STATISTICAL INQUIRIES«

Nr. 14

Issued by »The Statistical Department«, Denmark

Income-Expenditure Relations of Danish Wage and Salary Earners

By Erling Jørgensen

Published by »The Statistical Department«
in Collaboration with »The Institute of Statistics«
and »The Institute of Economics«
of the University of Copenhagen

KØBENHAVN

1965



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PREFACE

The present analysis of the expenditure behaviour of Danish households of wage and salary earners is essentially based on the model used by Prais and Houthakker in their analysis of a similar English material (*The Analysis of Family Budgets*, Cambridge, 1955).

The scope of the analysis was determined by the Statistical Department, the Institute of Statistics of the University of Copenhagen and the Institute of Economics of the University of Copenhagen in concert. Mr. Kjeld Bjerke, lecturer, chief of division, the Statistical Department, and the heads of the two institutes, Professor Anders Hald, dr. phil., and Professor P. Nørregaard Rasmussen, dr. polit., having met regularly to discuss problems arising in the course of the work; this committee has also gone through the final report.

The day-to-day work was directed by Mr. Erling Jørgensen, cand. polit., The Statistical Department, assisted for a brief period by Mr. Åbling Thomsen, cand. polit., the Statistical Department. Mr. Erling Jørgensen has also prepared the manuscript of the present report. Computations have been carried out by The Danish Institute of Computing Machinery, the staff of which has rendered valuable assistance in the programming process. Mrs. Lis Taxøe Jensen, the Statistical Department, has displayed great skill and care in preparing the many tabulations and examples to be used in the analysis.

The Translator of the Statistical Department, Mr. Vagn K. Sandberg, has translated the draft manuscript into English, and Mr. Niels Thygesen, cand. polit., has carried out a terminological revision of this manuscript.

The expenditure in connection with the computations carried out by The Danish Institute of Computing Machinery has been covered by a grant from the Danish State Research Foundation, and a grant has been received from the Rask-Ørsted Foundation to cover the cost of the terminological revision of the manuscript. The expenditure incidental to the publishing of the book has been defrayed by the Statistical Department.

The analysis was started in the summer of 1959 and was concluded with a provisional report at the end of 1962.

Copenhagen 1964.

Chapter I.

BACKGROUND OF THE STUDY AND THE MAIN RESULTS

I a. Background of the study.

The survey of income, consumption and saving patterns in 1955 of households of Danish wage and salary earners which was carried through at the beginning of 1956 is the most comprehensive and the most detailed of the consumption surveys undertaken by the Danish Statistical Department since it started making this kind of surveys in 1897¹⁾. The primary object of the surveys was originally to procure information of the "Conditions of life in the different classes of society, including nutrition and consumption"²⁾, but after the system of adjusting salaries, wages, benefits and other payments in accordance with a price index became generally adopted, the consumption surveys were primarily undertaken in order to provide the basic material for constructing a system of weights to be used in the calculation of price indices. During recent years, however, the generally descriptive purpose, which was the primary one in the first consumption surveys, seems to be gaining ground again. One of the reasons for this development is the fact that it has been realized that the basic material which is obtained by means of a consumption survey carefully planned and carried out—in this connection the substantial advances in survey techniques of the last decades should be borne in mind—provides information on essential economic relationships, particularly in connection with spending in relation to income, which cannot be illustrated so completely in any other way³⁾.

The Danish consumption survey for 1955 has, in fact, been subject to a more detailed processing than any of the previous surveys. Thanks to the scope and quality of the 1955 survey it has been possible, through this detailed processing, to arrive at results which are of direct interest to private institutions and persons as well as to public authorities.

A general outline of the 1955 survey, its planning and main results, was given in *Statistiske Efterretninger* in 1957⁴⁾. Food consumption was dealt with separately in an article

¹⁾ A complete list of publications on Danish consumption surveys will be found p. 118.

²⁾ Act concerning the Central Statistical Bureau 1895.

³⁾ Cf. I.L.O. (11).

⁴⁾ *Statistiske Efterretninger* 1957, No. 83.

in *Statistiske Efterretninger* in 1958⁵). The data collected concerning the saving and personal wealth of the households of wage and salary earners were subject to a special analysis, the results of which were given in a volume of the series of *Statistiske Undersøgelser* in 1960⁶). Two volumes in the same series dealt with the data collected on the distribution and composition of the wage and salary incomes⁷).

The greater part of the information obtained from the households of wage and salary earners concerned their consumption expenditure in the year 1955, and it was decided to subject the consumption behaviour of these households to a more detailed analysis. It is the results of this study which are contained in the present publication.

I b. Main results of the analysis.

The analysis aimed at giving a precise description of the relationship between the disposable income of the Danish households of wage and salary earners and their expenditure on some essential items in the year 1955. This relationship between disposable income and the expenditure on given items is undoubtedly of considerable importance in helping to explain differences in consumption behaviour from one household to another, although, of course, many other factors must be included if all such differences are to be explained, such as type of household, residential and social classification, etc. However, the income-expenditure relationship is of great importance in another connection, namely for forecasting the development of consumption in response to given changes in income, for one household or group of households or for all households as a whole⁸).

The analytical work thus consisted mainly in deriving the best possible description of the income-expenditure relationships. Such income-expenditure relationships often go by the name of Engel curves after the German economist and statistician, Ernst Engel⁹). More specifically, the work in connection with the analysis has consisted in calculating estimates of the parameters of five types of functions selected in advance and then comparing these functions by means of a number of tests for goodness of fit in order to find the Engel function most suitable for each expenditure item.

This is on the whole the same type of analysis as was adopted by J. S. Prais and H. S. Houthakker in their study of British family budgets from 1955¹⁰). In fact, in many respects the present inquiry may be considered an application to Danish data of the analytical tools which Prais and Houthakker present and discuss in their work.

⁵) *Statistiske Efterretninger* 1958, No. 46.

⁶) *Opsparing i lønmodtagerhusstandene 1955, Statistiske Undersøgelser*, No. 3, Copenhagen 1960. (Summary in English).

⁷) *Lønmodtagerindkomster, Fordeling og sammensætning, Statistiske Undersøgelser*, No. 6, Copenhagen 1962 and *An Analysis of the Personal Income Distribution for Wage and Salary Earners in 1955. Statistical Inquiries*, 1964.

⁸) Cf. chapter III, p. 28, and Erling Jørgensen (10), pp. 54–61.

⁹) Cf. Ernst Engel (10)

¹⁰) Prais J. S. and Houthakker H. S. (10)

Prais and Houthakker's study contains a discussion of alternative approaches to analyses of budget data as well as a list of literature dealing with family budget studies¹¹). These background problems, therefore, will not be elaborated here.

As regards the main lines of the present study a few points deserve mention in this summary of methods and results.

To eliminate the most disturbing influences deriving from differences in the size of the households interviewed, all expenditure and income amounts were converted into amounts per person for each of the 3100 households for which data were obtained.

The income concept used, which is the independent variable of the Engel curve, was then determined as disposable income (all cash receipts less paid personal taxes) per person, and Engel functions were derived for 13 main expenditure items. In the appendix will be found a detailed classification of these expenditures which together amount to 85 per cent of total consumption expenditures for all households of wage and salary earners. The 13 main items were the following:

1. Dwelling
2. Fuel and lighting
3. Food (incl. regular eating out, beer, wine, and liquor within the usual household consumption)
4. Tobacco
5. Clothing
6. Footwear
7. Washing and cleaning
8. Durable goods (excl. motor vehicles)
9. Personal hygiene
10. Books, newspapers, etc.
11. Sports, holidays, hobbies, etc. (incl. visits to restaurants, theatres, cinemas, and beer, wine, and liquor outside the usual household consumption)
12. Transport (incl. motor vehicles)
13. Subscriptions, union fees, insurance premiums, etc. (excl. life and pension insurance)

The calculations were carried out separately for 12 groups of wage and salary earners, namely four social groups within each of the three regions of the country.

The three regions were the following

1. The capital incl. suburbs
2. The provincial towns incl. suburbs
3. Urban districts in the rural municipalities

In regions 1 and 2 the following social grouping was used

1. Higher public servants and salaried employees
2. Lower public servants and salaried employees
3. Skilled workers
4. Unskilled workers

¹¹) Prais J. S. and Houthakker H. S. (10) p. 169.

In region 3, the urban districts in the rural municipalities, separate calculations were also carried out for the social group of *Agricultural workers*. In this region no calculations were carried out for the group of *Higher public servants and salaried employees*.

The five types of functions used were the following, x denoting disposable income per person and y the expenditure per person on the item in question:

$$\begin{aligned} \text{(I,1)} \quad & \log \eta = \alpha + \beta (\log v - \overline{\log v}), \\ \text{(I,2)} \quad & \log \eta = \alpha + \beta (v^{-1} - \overline{v^{-1}}), \\ \text{(I,3)} \quad & \eta = \alpha + \beta (\log v - \overline{\log v}), \\ \text{(I,4)} \quad & \eta = \alpha + \beta (v^{-1} - \overline{v^{-1}}), \\ \text{(I,5)} \quad & \log \eta = \log x + \log [\Phi (\alpha + \beta \log v)] \end{aligned}$$

a bar denoting average value and $\Phi (t)$ denoting the cumulative distribution function

$$\text{of the normal distribution } \varphi (t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

Considerable parts of the report on the analysis are devoted to a discussion of the estimation procedure so that it may be said that an evaluation of the *methods of analysis* was another main objective of the analytical work besides the calculation of *the results of the analysis*.

The tests applied showed almost consistently that the double-logarithmic function (I,1) gave the best description of the Engel curve for all 13 expenditure items as a whole. This result is in a way surprising because it implies that the income elasticity¹²⁾ of the households in their demand for each of the 13 commodity groups is constant over the income range (for given social group, since the calculations have, as mentioned, been carried out separately for 12 groups of wage and salary earners). It might have been expected that commodity groups which are considered necessities in the higher income groups would be regarded as luxuries in the lower income groups. This, however, is not confirmed by the estimates of the income elasticities.

It might then be thought that this constancy of the income elasticity would hold good only in the case of the individual groups of wage and salary earners, which do not each of them cover any wide income interval, but that the matter would be different if all households were grouped together, in other words, that the estimates of β should be different for the different groups. However, there proves¹³⁾ to be a remarkable stability as regards the mentioned estimate of β , when we move from one group of wage and salary earners to another. In the case of six expenditure items a hypothesis of constant income elasticity through all 12 household groups can be maintained, and in the case of the remaining 7 items the deviations, though statistically significant, are not very great. The demonstration of this stability in the income elasticity of the households in

¹²⁾ Note that the income elasticity is identical with the parameter β in the double-logarithmic Engel function (I, 1).

¹³⁾ Cf. chapter V p. 83.

their expenditure on the most important items is one of the most conspicuous results of the analysis¹⁴⁾

This stability renders it justifiable to calculate the average income elasticity for the 12 groups of wage and salary earners for each of the 13 expenditure items. These average elasticities are shown in table I,1, where the expenditure items have been arranged by size of the average income elasticity.

Table I,1.
Income elasticities for 13 expenditure items; average values for
12 groups of wage and salary earners.

Item	Average income elasticity
Fuel and lighting.....	0.51
Footwear.....	0.56
Food.....	0.61
Subscriptions, union fees, etc.....	0.82
Personal hygiene.....	0.86
Washing and cleaning.....	0.86
Dwelling.....	0.89
Books, newspapers, etc.....	0.98
Tobacco.....	0.98
Durable goods (excl. motor vehicles).....	0.99
Clothing.....	1.04
Transport (incl. motor vehicles).....	1.39
Sports, holidays, hobbies, etc.....	1.50

It will be seen from this table that the expenditure items fall into three clearly defined groups:

1. A group, which might be called necessities, in which the elasticity is just over 0.5, consisting of three items, *food, footwear, fuel and lighting*.

2. A second group, which might be called neutral commodities, with an elasticity close to unity. This group includes 8 items, among which the two important items of *dwelling and clothing*.

3. Finally, there is the third group, which might be called luxuries, in which the elasticity is significantly higher than unity; this group consists of the two items of *transport* (incl. motor vehicles) on the one hand and *sports, holidays, hobbies, etc.* on the other

As already mentioned, the main objective of the analysis has been to give a description of the relationship between the income of the households of wage and salary earners and their expenditures on important items. The analytical method employed, which consists chiefly in linear regression analysis, seems to yield satisfactory results in the case

¹⁴⁾ This result invites the postulate that the income elasticities found for the population of wage and salary earners have general validity for all population groups. Concerning the consequences of this postulate, see Erling Jørgensen (12).

Expenditure per person. Danish Kroner

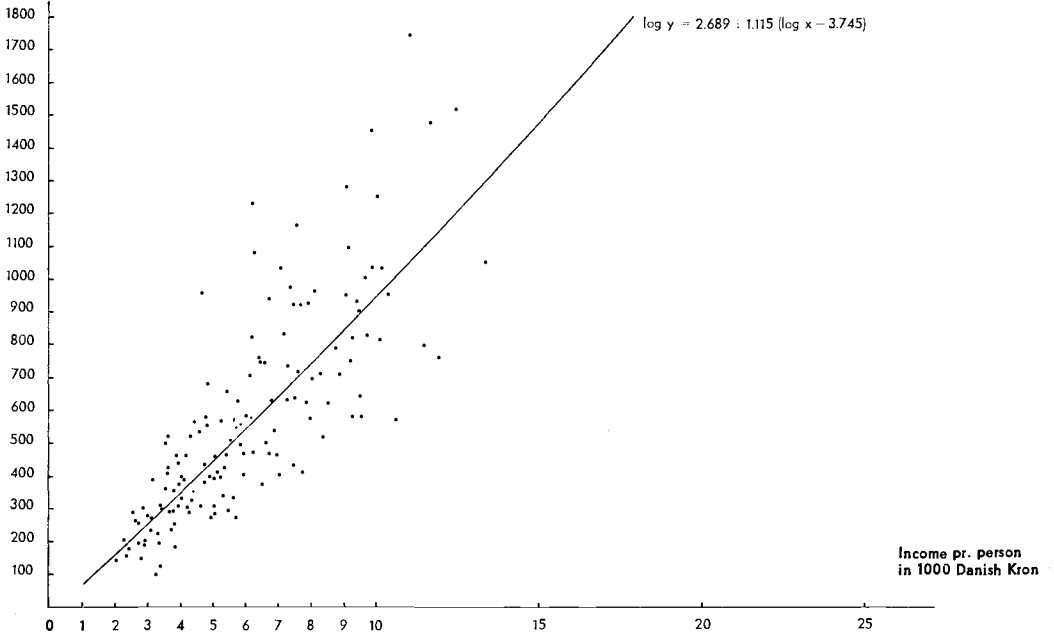


Fig. I, 1. Income and expenditure on *clothing*. Average values of 154 groups of 3 observations among lower public servants and salaried employees in the capital.

Expenditure per person. Danish Kroner

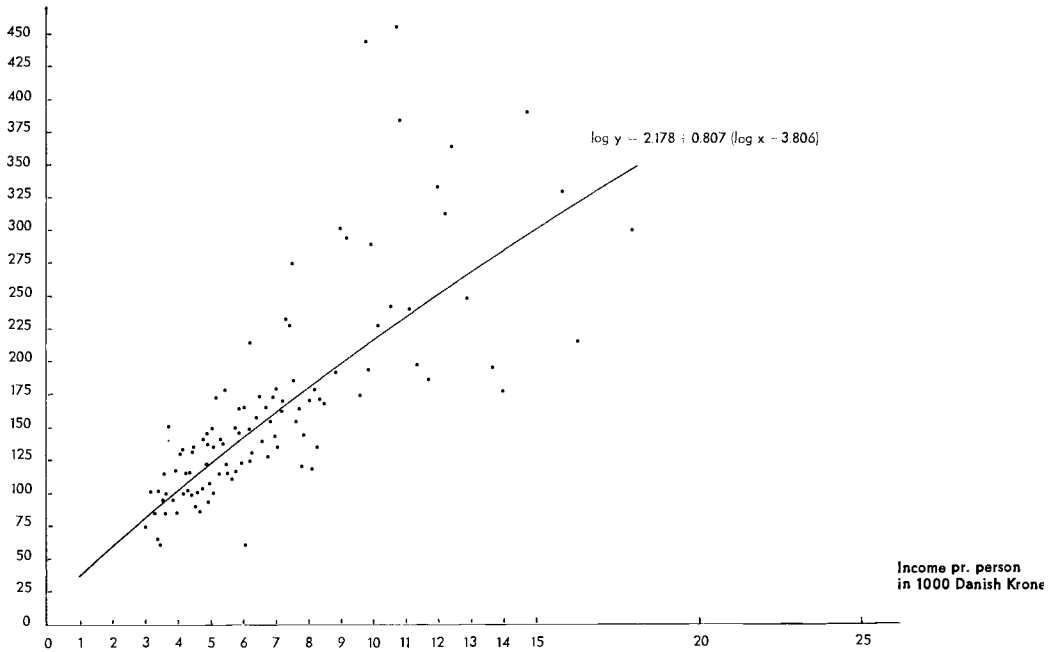


Fig. I, 2. Income and expenditure on *personal hygiene*. Average values of 112 groups of 3 observations among higher public servants and salaried employees in the capital.

Expenditure per person. Danish Kroner

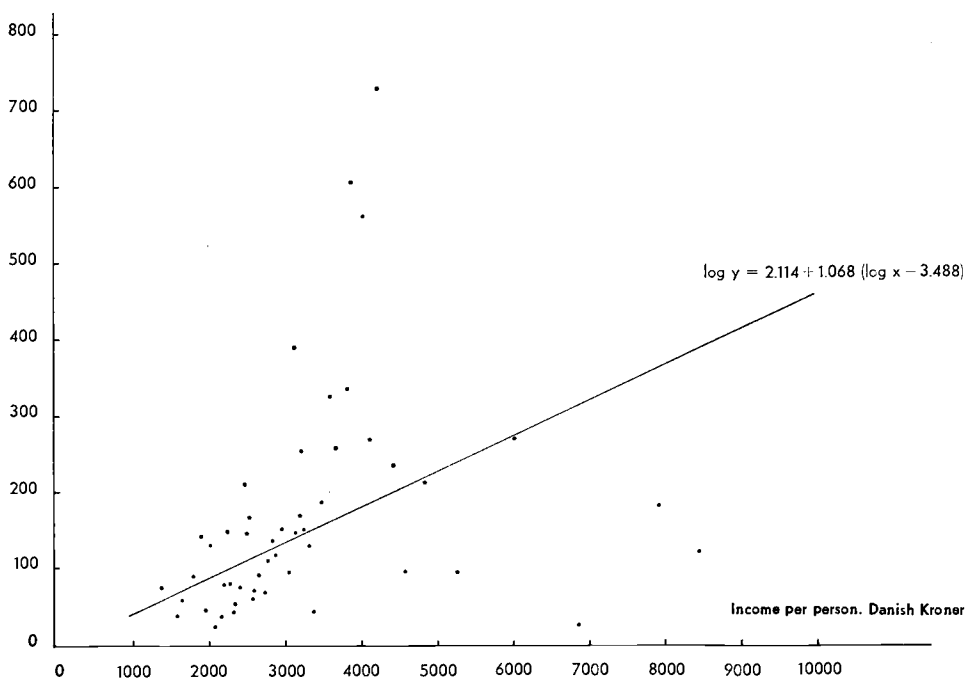


Fig. I, 3. Income and expenditure on *durable goods* (excl. motorcars). Average values of 51 groups of 3 observations among skilled workers in the rural districts.

of most expenditure items, cf. fig. I, 1 and fig. I, 2. For a few items, however, particularly for *durable goods* and *transport incl. motor vehicles*, the residual variation in expenditure from household to household is very high; the introduction of the disposable income of the households as explanatory variable has not reduced the variation appreciably. Fig. I, 3 demonstrates the high residual variation as regards durable goods in the group of skilled workers in the rural municipalities.

It may probably be concluded that the analysis of the expenditures of the households on these items will have to be tackled differently, by including information on type of household and other environmental factors and particularly on income changes and the consumption behaviour in earlier periods. Such a dynamic analysis has, however, been outside the scope of the present study, but it must be admitted that in the case of durables and transport the results presented here are rather unsatisfactory.

In a few respects the report goes beyond the objective of the analysis as set out above. In a concluding chapter it is examined to what extent the 13 expenditure items are correlated, i. e., whether households which spend much or little on one item display a characteristic expenditure behaviour as regards one or more of the other items. It was attempted to discover, e.g. whether households with a high consumption of tobacco have a lower consumption of food than households with a low consumption of tobacco. It was also tried to outline the importance of differences in type of households (size and

composition of household) to the consumption behaviour of households for given income classes.

As regards the first problem—the interrelationships of the 13 expenditure items—the calculations show only a slight correlation. Only in the case of the two items of *dwelling* and *fuel and lighting* was there a significant (positive) correlation. This result is a consequence of the design of the analysis, since the grouping of the many goods and services for which information was obtained into a moderate number of main expenditure items aimed precisely at a grouping with only a slight positive or negative correlation between the individual groups. This attempt to arrive at stable relationships between income and a few groups of expenditures at the same time ruled out a description of the consumption behaviour of the households towards individual goods and services; if such a description were to be attempted, the expenditure on other closely related goods and services would undoubtedly have to be taken into account.

As regards the importance of type and size of the household to the consumption behaviour of the households, the examinations show that the size (i.e. number of persons) of the household was the dominant factor, and that the conversion into amounts per person from amounts per household eliminated the greater part of this “disturbing” influence. In the case of certain expenditure items, among them *dwelling* and *tobacco*, other influences made themselves felt; a general influence, as was to be expected, was the “economies of scale” effect, i.e. the expenditure per person falls as the number of persons per household rises.

I c. The report.

After this introductory survey of the background and plan of the analysis and of some of its main results, chapter II will present a *review of the basic material*. This review consists of a description of the practical work of carrying through the survey of consumption and saving, i.e. the collection and processing of the basic material, and also a description of the inaccuracy attaching to the figures derived from the basic material. Chapter II also contains a brief summary of average expenditure per household on the main expenditure items. In chapter III the *aim of the analysis* will be defined, various models for an analysis of the expenditure behaviour of the households being discussed, a discussion which concludes in a statement of the reasons for choosing the Engel curve approach as the main subject of the analysis. Chapter IV contains a detailed discussion of the *methods of analysis*. What types of functions are to be chosen as a basis for deriving Engel curves for the different expenditure items? How are the variables to be specified? How is the suitability of the functions employed in the description of the income-expenditure relationship to be tested?

In chapter V *the results of the analysis* are presented. The double-logarithmic Engel curve was, according to the test made, found to be the “best” of the 5 types of function tested.

Finally, chapter VI suggests *examples of some further calculations* which should make it possible to achieve a more exhaustive description of the consumption behaviour of the

households than has been possible with the main tool of the present analysis, the Engel curve. In order to explain the variations observed in the expenditures of the households on a given item, differences in the size and composition of the households will be discussed as well as the expenditures of the households on one or more other items.

An appendix to the report contains partly the basic material and a detailed description of the expenditure items comprised by each of the 13 main items and partly tables showing the results of the computations. These tables fall into two parts, the results of the main analysis, cf. chapter V and the results of the further calculations, cf. chapter VI.

A list of the literature used will be found on pages 117–118.

Chapter II.

REVIEW OF THE SURVEY MATERIAL

IIa. Introductory remarks.

The present analysis of the consumption patterns of Danish wage and salary earners in 1955 is based on the family budget survey of households of Danish wage and salary earners undertaken in 1956¹).

This survey comprised a total of 3100 households, selected by stratified sampling among all households of wage and salary earners; the sampling procedure is described below.

The questionnaire used in the survey was very detailed as it was desirable to collect information on household expenditures for a large number of consumer goods cf. the detailed list in the appendix p. 240. For the purpose of the present analysis, however, only main expenditure items are of interest as the emphasis of the analysis is on the consumption pattern as a whole rather than on consumption of individual commodities.

IIb. Concepts and methods of the survey.

1. Collecting the information.

The consumer survey has been carried through by personal interviews. This method has been chosen in preference to the far cheaper one of mailing questionnaires to the households for two reasons. Firstly because some of the questions were so complicated that the interpretation of an interviewer was considered necessary in order to ensure that the households would understand them, and secondly to reduce the non-response rate to a minimum. Both as regards the quality of the information collected and as regards the response rate, gratifying results were achieved. Only 61 questionnaires out of a total of 3161 had to be rejected owing to unsatisfactory completion, and only 473 households, or less than 12 per cent of all households approached, refused to cooperate. (Besides, 345 other households could not be contacted because of illness, change of address, etc.).

The survey comprised income and assets of the household as well as its expenditures and savings during 1955, the expenditure being broken down into various items, and saving being distributed by the various forms of saving.

¹) Cf. references p. 118.

Total saving, defined as net change in assets, was calculated on the basis of the figures for changes in debts payable and receivable in the course of the year. The interviewer checked the figures against the difference between income and total consumption. Where appreciable discrepancies were found, the household was contacted again, and substantial errors in the figures for saving as well as in the various consumption items were eliminated. On the whole, it may perhaps be concluded that both the interview method and the fact that the total budget of the household was included in the survey have helped in keeping what might be called errors of measurement at a minimum in the data collected for consumption, saving and personal wealth.

2. Income and expenditure concepts ; unit of analysis.

The purpose of the 1955 consumer survey was to illustrate expenditures and savings in households of wage and salary earners. Hence it follows directly that it is the *household* which is the relevant unit of analysis both as regards consumption and saving. This gives rise to the problem of defining the household concept on which the survey was to be based.

In drawing up such a definition there are two considerations to keep in mind. Firstly, the household should be defined in such a way that it contains those—and only those—persons who behave as a unit both in relation to the earning of income (income unit) and to the spending of income (spending unit). Secondly, the household unit adopted should be practical for the purpose of selecting, collecting and processing the survey material.

Without going into detailed definitional problems it should be emphasized that these two considerations may in fact be irreconcilable. The consideration that the persons included in the household should act as one income and spending unit might lead to the selection of a household concept which will prove to be impractical in the selection of the sample or in the collection and processing of the material. Moreover, even if we insist only on the point that the household should act as one income and spending unit we are not assured of an unambiguous definition. Thus with regard to board and lodging, domestic servants take part in the consumption of the household, but their incomes are not included in the joint income of the household. On the contrary, they are paid out of this income; domestic servants in some respects form part of the spending unit, but not of the income unit. If it is desired, e.g., to inquire into the relationship between the income of the household and its food consumption, information supplied by the households in which there are domestic servants will give misleading results.

Further, it may be mentioned that the household concept which would be most relevant in an analysis of consumption, will not necessarily be the one that is most relevant in an analysis of saving, since it may very well be imagined that persons who act as one unit as regards consumption will not make their saving decisions in common; examples are: households in which there are boarders and/or older children living at home who pay a certain amount towards the joint consumption of the household, but otherwise dispose independently of the rest of their income.

The household concept actually used was the following: those persons (and only those) who take part in the joint consumption, i.e., husband, wife, and children without an income of their own are included; also included were children living at home who had incomes of their own and others who stayed permanently with the household, provided that these persons did not spend more than 50 per cent of their incomes outside the households.

As regards income, consumption and saving the following concepts were used:

Income :

Cash wages and salaries.—Contributions to pension schemes withheld out of the salaries of public servants and salaried employees.—Interest and Dividends.—Pension, incl. old-age pension.—Disablement pension.—Contributions from separated or divorced spouse.—Unemployment relief.—Contributions to housekeeping made by children and relatives.—Payment by lodgers for board and lodging.—Amounts received under insurance policies.—Gifts.—Inheritance, scholarships.—Sales of motor car, moped, bicycle, furniture, clothing, etc.²⁾—Savings certificates received³⁾.

Consumption expenditure :

Expenditures on purchases of all consumer goods, including all expenditures in connection with purchases of durable consumer goods (motor cars, motor cycles, furniture, household appliances, radio and television sets²⁾, etc.), i.e. both initial payments on durable consumer goods acquired in the course of the year and instalments on hire-purchase debt relating to acquisitions in this or previous years; taxes, subscriptions, etc.—Also cash contributions to relatives and gifts.

Saving :

Amounts spent on increasing, or received by reducing, the below-mentioned items:

Cash-in-hand.—Bank and savings bank deposits.—Bonds and shares.—Premium bonds.—Private mortgage deeds.—Compulsory saving and savings certificates.—Value of real property.—Business assets.—Other assets.—Life and deferred annuity insurance (incl. contributions of public servants to pension funds).

Amounts spent on reducing, or received by increasing, the below-mentioned items:

Debt to bank and savings bank not secured by mortgage in real property.—Mortgage debt in real property.—Other debt apart from hire-purchase debt, etc.

Only a few comments are necessary in connection with these definitions. As mentioned above, saving was calculated also as the difference between income and consumption in the course of the year. Since this method of calculation must, of course, give the same result as the calculation according to the above definition⁴⁾—if the figures are correct

²⁾ In the case of purchases of motor vehicles, the value of any motor vehicle traded in has been set off against the value of the new vehicle.

³⁾ In connection with the imposing of new indirect taxes in 1955 saving bonds were issued to all persons with assessed income of kr. 4000 or more. The face value of the bonds was increasing with increasing income of the persons concerned.

⁴⁾ Cf. Statistiske Undersøgelser, No. 3, Opsparing i Lønmodtagerhusstandene. 1955, Copenhagen 1960, pp. 11–16.

—the interviewers were able to get a good check on the data collected by comparing the amounts of saving resulting from the two definitions.

Besides, it should be emphasized that the definitions used are based on a “cash point of view”. Income comprises all cash payments to the household, incl. gifts and amounts received under insurance policies. On the other hand, consumption contains, as a general rule, all amounts actually paid by the household; this involved, for instance, that in the case of purchases of durable goods, only the initial cash payment and any instalments paid during the survey period were included.

3. *Period of the survey.*

In the choice of *survey period* two conflicting considerations have to be taken into account. Firstly, it is desirable that the households interviewed should be able to remember, at the time of the interview, the size of their income during the survey period and, in particular, how they have spent this income. For this reason, it would be desirable to have as brief a survey period as possible. On the other hand, however, it is desirable that accidental fluctuations should not be allowed to have too much influence on the results, neither as regards the income earned nor as regards the spending of it. If both the earning of the income and the consumption took place at a regular rate, this consideration would not give rise to any problems, but since particularly some consumption expenditures occur irregularly, it would be reasonable to make the survey period so long that these irregularities will be smoothed out. Since seasonal factors must be presumed to play a dominant part in these fluctuations, it was found reasonable to use the year as the survey period.

Especially as regards income earned experience shows that most households will have a precise idea of it only for a period of one year and only once a year, namely when they fill in their income tax returns. Therefore the survey was carried out immediately after the date for delivering of the income tax returns, viz. the 1st of February.

4. *Method of selection.*

The selection of a sample of basic sampling units on the basis of probability theory (i.e., in such a way that it becomes possible to calculate the standard error of the results) requires, firstly, a specification of the population from which the sample is to be drawn (setting up a *frame* for the selection), and secondly, the choice of a *sampling design* based on random selection (i.e., a selection by which all the elements of the population have a specified probability of being selected).

As regards the setting up of a *frame* for the consumer survey, the population census on the 1st October, 1955, provided a complete “list” of all households in Denmark. In view of the main object of the survey, which was an analysis of the consumption patterns of households of wage and salary earners, it was decided to exclude from the frame all rural municipalities without urban areas because there are very few wage and salary earners in those municipalities. The few wage and salary earners who were to be found there were considered to be represented by the households of wage and salary earners

selected in the rural municipalities with urban areas. The frame was accordingly those households in the whole of Denmark, except in the “purely” rural municipalities, which were recorded in the population census schedules as having a wage or salary earner as head of household.

The choice of *sampling design* was influenced by a number of factors, the most important of which will now be briefly discussed.

The guiding principle in the considerations which preceded the choice of sampling design was that the standard error of the estimates calculated on the basis of the sample drawn should be below a certain limit, and that the costs of the survey should be held at a minimum given this maximum standard error⁵).

However, the sampling design which gives the lowest standard error for *one* of the estimates, e.g. for total food consumption expenditure per household, will not always at the same time give the lowest standard error for all the other estimates. As soon as a survey is to form the basis of a calculation of several estimates, it is therefore necessary to specify one of the quantities which it is desired to estimate on the basis of the sample as the decisive one in the choice of sampling design. One may then hope that this design will also be favourable as regards the other quantities to be estimated. Alternatively, all the quantities to be estimated must be arranged by order of priority and an overall evaluation must be made for the purpose of arriving at a design which minimizes the sum of the standard errors for all the quantities estimated, the individual standard errors being assigned weights corresponding to their order of priority.

One of the objects of the 1955 survey was to provide the basis for calculating a system of weights for the Danish price index. Therefore the estimation of average expenditure on the main items of goods and services which are covered by the price index were assigned a high priority. As estimates made on the basis of preceding consumer survey (1948) showed that there was a high correlation between the total expenditure of a household and expenditures on certain main items, the desired end was assumed to be attained by fixing certain limits of the standard error for the total expenditures per household for each of twelve groups of wage and salary earners⁶).

In a following section an account will be given of the calculation of these standard errors.

With the mentioned point of departure (that the survey should be planned with a view to minimizing the standard error for the total consumption expenditure), the sampling design was otherwise determined by a number of practical and theoretical considerations.

Firstly, already the choice of method of enumeration places certain restrictions on the sampling procedure. The decision to carry through the survey by means of interviewers who are to call on each sample household up to six times, makes it natural to assign to each interviewer as many households as he is able to call on within the period of the survey. This procedure ensures that interviewers gain a maximum of experience in taking interviews. It may also be mentioned that the possibilities of supervision for the central authorities will be considerably reduced if there are too many interviewers.

⁵) See E. Lykke-Jensen: (13), pp. 16–18.

⁶) Viz. four social status groups separately within three district categories; cf. below p. 34.

Consequently, it was desirable that the households should be selected in clusters within geographical areas, whereby the transport costs of the interviewers would be considerably reduced. Each cluster corresponds to the capacity of one interviewer, in this survey approximately twenty households.

Besides, the very form of the frame will play a part in the considerations concerning the method of selection. In this case, as already mentioned, the schedules from the 1955 population census provided the frame from which the sample was drawn, and as these schedules are arranged by municipalities (in Copenhagen by "roder" (tax collection districts), in Frederiksberg and Gentofte by parishes), it seemed natural to base the sampling on whole municipalities (parishes or "roder"). As it was possible to group these municipalities in accordance with the criteria which were considered relevant to this inquiry, viz. distribution by industry and degree of urbanization, it was found reasonable to use stratified sampling. Finally, the desirability of illustrating the consumption patterns of the individual social status groups separately within each of the three district categories, (the capital, provincial towns with suburbs, and rural municipalities with urban areas) made it natural to conduct the survey in such a way that it would be possible to calculate separate estimates for each status group within these three district categories.

The result of these considerations was accordingly that the sampling was made in two stages within each of the three mentioned district categories. At the first stage municipalities ("roder", parishes) were drawn by random selection from strata of uniform municipalities already formed, the probability of selection of each municipality ("rode", parish) being proportionate to the number of households in the municipality. Actually, the selection ought to have been made in proportion to the number of households of wage and salary earners, but this number was unknown. As the households of wage and salary earners constituted a more or less constant share of the total number of households within each stratum of municipalities this procedure seems permissible. At the second stage households of wage and salary earners (basic sampling units) were drawn from each municipality in the first stage sample of municipalities, households belonging to different status groups⁷⁾ drawn with different probability.

In the capital 16 first stage units were selected, comprising about 36000 households of wage and salary earners, from which were drawn 1262 second stage or final units, i.e. individual households. In the provincial towns the numbers of first and second stage units were 17 and 920 respectively, the sample of first stage units comprising about 85000 households. In the rural districts the numbers were 26, 918 and about 4000 respectively. Whereas the final sample of 3100 basic sampling units comprised only about 0.45 per cent of all households of wage and salary earners, the number of such households in the first stage sample of municipalities comprised about 18 per cent of the total number⁸⁾.

Finally, it should be mentioned that the definition adopted of the basic unit of analysis—all members of the expenditure unit—did not quite correspond to the units

⁷⁾ Higher salaried, lower salaried, skilled and unskilled, cf. p. 5.

⁸⁾ A similar approach was used in the Danish labor force surveys in 1951 and 1952, cf. The Danish Labor Force Surveys. *Statistical Review*, New-Series vol. 2, No. 7, pp. 259-267.

selected at the second stage (the sampling units), as these units had been determined by the choice of the frame of the survey, namely the schedules from the 1955 population census. Since, according to the definition used in the population census, the household comprises all persons staying permanently in the household, with the exception of lodgers providing their own food, whereas in the consumer survey the household comprises only the persons who contribute at least fifty per cent of their income towards the consumption of the household⁹⁾, the population census household will in some cases comprise more persons than the basic sampling unit of the consumer survey. This fact leads to certain complications in estimating averages for the whole country and also in estimating the true standard errors of these averages, but in the following this has not been taken into account as we have assumed that the inaccuracy introduced hereby is insignificant compared with the inaccuracies which arises in the course of the collection and processing of the questionnaires.

IIc. Estimating mean values and their standard errors.

1. Accuracy of the results of the survey.

The estimates based on the 1955 consumer survey are subject to a certain degree of inaccuracy. This inaccuracy consists of two components. The first originates in the collection and processing of the material, i.e., wrong or inadequate information, errors in coding and punching, etc. The errors of this type are often called systematic errors (bias), cp. the following section. The other component is called sampling error, and it occurs because only a sample of households and not the entire population is observed.

As the sample of households of wage and salary earners has been selected by stratified two-stage sampling, the sampling error of the estimates for each of the twelve groups of wage and salary earners will consist of two elements; firstly, the error due to the variation, within strata, among the sampling units at the first stage, municipalities, and secondly the error which is due to the variation among the sampling units at the second stage, i.e., among the individual households within municipalities.

While it is impossible to arrive at more precise estimates of the systematic errors, the sampling method adopted makes it possible to form estimates of the two elements of the sampling error¹⁰⁾.

The calculation have shown that the error element due to variation among individual households within municipalities is dominant.

Table II,1 shows estimates of average expenditure per household for 14 expenditure groups; the table shows also the average saving and cash income per household for each of the twelve groups of wage and salary earners. The standard sampling error of the estimated total expenditure per household is estimated at kr. 140 or approximately 1 per cent of the total expenditure; 70 per cent of the standard error is due to variation between households within first-stage sampling units.

⁹⁾ Cfr. the exact definition, p. 14 above.

¹⁰⁾ Cf. Statistiske Undersøgelser No. 3, Opsparing i lønmodtagerhusstandene 1955, Copenhagen 1960, pp. 3-4.

In the regression analyses which form the greater part of the present inquiry the observations for each of the twelve groups of wage and salary earners into which the 3100 households observed have been divided, have been treated as deriving from a simple random selection. The estimates of the standard errors of the parameter estimates will therefore become a little too high since the stratification effect is ignored, and besides, some bias may be expected to occur in the estimation of the parameters because the deviation of the observations from the regression line is evaluated on an assumption of simple random selection, whereas the actual procedure is two-stage stratified sampling, cf. chapter 3, page 23. However, this bias must be considered insignificant in relation to the total variance in the distribution of the deviations from the regression line of the observations.

2. Processing of the material.

The inaccuracy of the estimates, discussed above, refers only to the sampling error, i.e., the error which will inevitably occur when estimates for the whole population are to be made on the basis of a sample of the population. With a given standard deviation in the distribution of the elements of the population the sampling error depends on the size of the sample and the sampling methods; it has been attempted, within the given cost framework, to make this sampling error as small as possible.

However, the estimates can also be subject to another type of error, which also occurs in complete enumerations, namely the so-called systematic errors, i.e., errors caused by wrong or inadequate completion of questionnaires and from the processing of the material, that is, errors in the scrutiny, coding and punching of the material received.

In the paragraph above on the enumeration method it was mentioned that the survey was conducted through interviews, partly to induce the sample households to cooperate, and partly to reduce the number of wrong answers. The 160 interviewers had received thorough instruction concerning the survey through letters and at special lessons at which officers from the Statistical Department went through the problems in connection with the completion of the questionnaires. A provisional scrutiny of the answers could therefore be made by the interviewers themselves at the time of the interview, the interviewers making a rough comparison of incomes and expenditures. In cases of discrepancy the interviewer was to take care that the household interviewed provided, wherever possible, the necessary supplementary information. There is reason to believe that thereby more correct figures have been obtained for the size of income and for items of expenditure which people might otherwise fail to state correctly.

It is obviously extremely difficult to indicate, even with rough approximation, the magnitude of errors which have arisen owing to people giving wrong answers to the interviewer. Experience from similar surveys abroad supports a belief that such incorrect statements are particularly frequent within the field which is often designated conspicuous consumption, i.e., such items as tobacco, liquor, consumption in restaurants, etc¹¹).

¹¹) Cf. Prais S. J. and Houthakker H. S. (10) p. 42.

Table II,1. Average income, saving and expenditures in 1955 in kroner per household.

	Capital and suburbs				Provincial towns	
	Higher public servants and salaried employees	Lower public servants and salaried employees	Skilled workers	Unskilled workers	Higher public servants and salaried employees	Lower public servants and salaried employees
Number of households in the sample.....	336	469	206	251	212	341
Average number of persons.....	2,9	2,3	2,8	2,6	3,0	2,6
of whom						
{men.....	1,5	1,0	1,5	1,3	1,5	1,2
{women.....	1,4	1,3	1,3	1,3	1,5	1,4
Expenditure on:						
1. Dwelling.....	1789	1183	1160	995	1514	1044
2. Fuel and lighting.....	867	552	618	570	967	679
3. Food.....	4318	3235	4160	3761	3822	2978
4. Tobacco.....	739	575	862	792	645	506
5. Clothing.....	1682	1194	1246	1044	1681	1120
6. Footwear.....	353	267	295	255	338	257
7. Washing and cleaning.....	363	251	296	253	264	191
8. Durable goods (excl. motor vehicles).....	1011	705	803	642	1111	791
9. Personal hygiene.....	451	354	373	332	383	289
10. Books, newspapers, etc.....	727	493	449	368	660	320
11. Sports, holidays, hobbies (incl. cinemas, theatres, restaurants, etc.).....	1975	1112	1210	925	1461	812
12. Transport, (incl. motor vehicles).....	1489	726	902	639	1444	519
13. Union fees, subscriptions ¹⁾	244	225	413	394	248	227
14. Other expenditure ²⁾	1744	1044	1085	859	1502	868
15. Saving.....	1384	554	475	405	1433	617
16. Total cash income.....	22606	13921	16111	13437	20866	12530

¹⁾ Excluding life insurance, deferred, annuity insurance, etc. which have been included in saving.

²⁾ Personal taxes not included.

A detailed comparison with figures for total consumption per household for the whole country obtained from production statistics and import—export figures for all expenditure items showed an over-all agreement, which confirms our impression that deliberately incorrect answers occurred only in few cases.

After the material was received at the Statistical Department it was subjected to a thorough scrutiny, in the course of which particularly the information concerning personal assets and liabilities as well as the changes in these items were critically examined as it turned out that it was these items in the questionnaire which had caused the greatest difficulty. The questionnaires which were found to be inadequately completed were returned with a request for supplementary information. A few questionnaires (61 in all) had to be rejected altogether, i.e., because the quality of the information provided was on the whole found to be too poor.

and their suburbs		Rural districts with urban areas				All households (weighted average)	Capital total (weighted average)	Prov. towns total (weighted average)	Rural districts with urban areas total (weighted average)
Skilled workers	Unskilled workers	Lower public servants and salaried employees	Skilled workers	Unskilled workers	Agri-cultural workers				
154	213	322	155	281	160	3100	1262	920	918
3,0	2,9	2,7	3,3	3,3	3,3	2,8	2,6	2,9	3,2
1,5	1,4	1,3	1,7	1,7	1,8	1,4	1,3	1,4	1,7
1,5	1,5	1,4	1,6	1,6	1,5	1,4	1,3	1,5	1,5
993	824	792	884	695	503	1027	1217	997	771
747	647	708	745	706	603	683	629	719	715
3409	3196	2746	3181	3208	2879	3467	3838	3288	3139
659	517	478	511	477	349	616	751	567	472
1012	981	936	883	873	624	1108	1240	1103	904
232	225	217	205	195	144	251	285	247	202
218	173	175	189	163	150	226	282	199	175
749	533	665	555	482	359	690	761	711	546
287	249	230	228	200	153	300	368	283	215
262	213	263	224	201	148	357	479	302	242
756	610	655	580	453	302	903	1222	793	556
492	404	697	829	567	304	712	873	578	650
387	425	201	356	395	337	344	334	354	341
764	648	667	597	551	366	887	1111	828	619
799	468	581	352	562	294	635	626	703	555
12780	10876	10967	11192	10253	7743	13569	15810	12934	10928

After this scrutiny the information in the questionnaires was transferred to punch cards. Both the punching operation and the subsequent mechanical processing were checked; in the case of the punching operation the check consisted in a complete verification of all punch cards, and in the case of the tabulation in check runs on the sums; the risk of error at these two stages is very small.

The figures which have been worked out on the basis of the punch card material must be presumed to be subject to rather few systematic errors compared with earlier Danish surveys. The errors of this type which might exist originate from the first two stages of the process: the interviewer's collection of information and the scrutiny of this information. As mentioned, special efforts have been made in this survey to limit the possibilities of error at these two stages, cf. the section concerning the enumeration method.

Chapter III.

OBJECTIVES OF THE ANALYSIS. ENGEL FUNCTIONS

IIIa. Introductory remarks.

The basic material available from the 1955 consumer survey is, as mentioned above, very comprehensive. For each of the 3100 households included in the survey approximately fifty punch cards (80 columns) were prepared. A complete description of this material, including an analysis of the relationships among the many quantities of which it is made up, is naturally out of the question. In order to keep the analytical work within reasonable limits, it is necessary to concentrate on some essential, well-defined problems. More precisely: among the many possible models which could be tested by means of this material, a few are to be selected which are of substantial interest from the points of view of economic theory, social policy, etc. The analysis then consists in confronting these models with the information collected.

From the point of view of economic theory, interest would focus on a model capable of explaining the consumption expenditures of households as a function of quantities familiar from economic theory as determinants of consumer behaviour. Hereby it might be possible to evaluate consumption, once information on those quantities to which it is functionally related becomes available. If the quantities in question may be more confidently predicted than consumption itself, such functional relationships will be useful in predicting consumption.

From the point of view of statistical theory the greatest interest will attach to estimation procedures; how are the best estimates of the parameters in the chosen models to be computed? What tests are applicable for purposes of comparing the estimates?

The computational work involved in the analysis has been carried out on an electronic computer. Accordingly it has been possible to choose more labour-consuming models and methods of calculation than if only the traditional calculating facilities had been available.

IIIb. Choice of model.

1. Determinants of expenditure.

According to traditional economic theory the expenditure of a household on a given commodity is determined primarily by the income of the household and by the price of the item in question. Prices of other commodities, expenditure of the household on other commodities as well as expenditure of other households on this and other commodi-

ties may also appear as important arguments. Other factors are of course, the composition of the household, its geographical location and social status. Also previous income and income change of the household as well as its assets might play an important role in determining the consumption behaviour.

As the present analysis is based upon a consumer survey relating to a given point of time and a given market, prices may be considered constant, independent of other variables as e.g. income and expenditure. All other variables mentioned above, however, can be found in the basic material of this inquiry, and if it was possible to set up a simple model of the relationships of these variables the parameters of such a model might be estimated.

However, there is no presumption that the relationships between the variables are simple at all. If the analysis is to be practicable, a relatively simple type of function must be chosen and the number of variables must be further reduced. Of the quantities mentioned above there is a strong presumption that household income is dominant in the determination of the expenditure pattern, while the household expenditure on other commodities plays a less prominent part, cf. chapter VI, p. 110. Therefore, if we further disregard income changes and assets as well as household expenditure on other commodities (and the consumption pattern of other households), a relationship remains within given social groups of households containing solely the two variables expenditure of the household and its income.

In a discussion of the relationship between these two quantities it is natural to start by emphasizing that expenditure is in the nature of a dependent variable to income, while income may reasonably be considered an independent or a determining variable cf. Prais & Houthakker (10) p. 80. It is quite obvious, however, that very often there is an influence the other way round, planned or incurred expenditures determining to some extent the income-earning behaviour of the household. On the whole this influence may be considered weak as compared to the influence of income on expenditure and especially as regards households of wage and salary earners as their possibilities for increasing income in the short run are rather limited.

Assuming that all households (household size and composition, social and geographical group held constant) show identical income-expenditure relationships except for random variation, a description of the "average" household of wage and salary earners will be of the form

$$(III,1) \quad y = f(x) + \epsilon$$

where y denotes household expenditure on a given item, x the household income and ϵ the effects on y from omitted determining variables plus random effects.

2. Engel curves and household survey data.

Formula (III,1) is the general expression of the Engel curve for a given expenditure item indicating the relationship between a household's income and its expenditure on that item. It was decided to place the Engel curve in the centre of the analysis, and the greater part of this and the following chapter is therefore devoted to a discussion of

methods of determining parameters in Engel functions by means of a household budget survey material.

Before discussing the question of the type of Engel functions a few remarks should be made in connection with the general approach of the analysis, which is indicated by the choice of the Engel curve as the main object of investigation.

Is it at all possible conceptually to estimate Engel curves on the basis of household budget surveys? Or stated more precisely: Assuming that all information in a survey provides reliable measurements of the incomes and expenditures of a number of individual households in the population group investigated, is it then possible to estimate true Engel curves based on that survey? Obviously, the degree of interest which would attach to the analysis from the point of view of economic theory depends very much on the answer to this question.

It is important to realize from the outset that our basic material does not allow of any direct testing of an Engel function relating to an individual household. For a given household only one set of income and expenditures is known, whereas several different sets of such observations would be necessary to enable us to test any hypothesis concerning the income-expenditure relationships of this household.

However, it may be possible to make up for this defect by inserting observations of the expenditures of other households on the commodity, these other households being selected in such a way that the relevant values of the income scale will be represented. Thus, instead of studying each household's expenditure reaction to various income levels, the relationship between the expenditures and incomes of many different households for one period is studied and it is postulated that by doing so the Engel curve of the "average" household in 1955 as written in (III,1) above will be obtained.

On the face of it, this postulated Engel curve is merely a description of the incomes and expenditures of various households. Such a description is, of course, valuable in itself since it enables us to make a statement of the following form: in the Danish population of wage and salary earners in 1955, households with an income of x_1 kr. spent an average of $f_1(x_1)$ kr. on the i 'th commodity, and households with an income of x_2 kr. spent $f_1(x_2)$ kr. apart from random deviations. Obviously the significance of the analysis as seen from the viewpoint of economic theory will be higher if this description of the income and the expenditure on certain commodities or groups of commodities of 3100 households will be a useful approximation to the Engel curves for the Danish population of wage and salary earners in 1955.

The Engel function, as defined in (III,1), is static, i.e., it gives an expression for the expenditure behaviour which the "typical" household will display, *ceteris paribus*, at alternative levels of income after any initial adjustment processes have been completed. This function, accordingly, entirely disregards the time factor and also the process whereby the households passes from one income level to the other. More concretely this process might be exemplified as follows: a household whose income rises will not adjust its expenditure behaviour to the new income level until some time has passed; hereby the saving of this household may temporarily be higher than the average for households whose incomes are permanently on this higher level. Conversely, a household which passes from a higher to a lower income will try to maintain consumption—

thereby reducing saving—than the average for households whose incomes are permanently on this lower level. Furthermore, the related ‘more general’ question arises whether the reaction of an individual household to changes in income will depend on income and consumption changes in neighbouring households¹).

These and other dynamic elements in the consumption behaviour of the households have been left out of account in the Engel functions of the type shown in (III,1)—but they are included in the estimates of the income-expenditure curve which can be made from the observations of the 3100 households in the basic material, and probably in such a way that the estimates are influenced systematically. It is thus highly probable that among the high-income households in the survey there will be relatively many who have experienced an appreciable increase in income since the immediately preceding period, while, conversely, there will be relatively many households with declining incomes at the lower end of the scale. The consumption expenditures observed for the high income groups will therefore tend to understate the “true” (static) propensity to consume, while among the low income groups the “true” propensity to consume will be lower than the observed expenditures, i.e., the Engel curve which is estimated will rise more slowly than a “true”, static Engel curve.

The postulate: that the observed relationships between income and expenditure for the 3100 households in 1955 are identical with the Engel curves as defined by (III,1) has, however, other weaknesses.

It does not allow for the dependence of the individual household on the consumption behaviour of other households. That such interdependence among the consumption expenditures of the individual households exists has long been recognized in demand theory²). The Engel curve is based on a *ceteris paribus* assumption and answers questions of the type: what amount would a household spend on the *i*'th commodity if its income rises by kr. 1000, kr. 2000, etc., *assuming that the other factors in the household situation are unchanged?* The most important factors here are: household size, residence, social status and the relative income position of the household in relation to its “neighbours”. The curves we can estimate from the available observation material, however, refer to households with, frequently, highly deviating environmental factors, and in observations of expenditures for households at different income levels it is therefore impossible to maintain the mentioned *ceteris paribus* assumption. We may assume that expenditures on durable goods are highly susceptible to the influence of environmental factors, whereas, e.g., the expenditure on typical necessities is less dependent on the consumption behaviour of other households.

The so-called layer effect³), may also lead to a wrong evaluation of the “true” Engel curve. If, e.g., we imagine that wage and salary earners in the rural districts, who are on an average at a lower income level than wage and salary earners in the towns, have a considerably lower expenditure on theatre and cinema than urban wage and salary

¹) Cf. Duesenberry, J. (3), Friedman, M. (7), Modigliani, F. (15) for a theoretical discussion of this aspect of the consumption pattern; Danish empirical studies on the subject are found in *Opsparing i l nmodtagerhusstandene 1955*, Copenhagen 1960.

²) Cf. Duesenberry, J. (3), Friedman, M. (7), Stone, R. (17).

³) Cf. Wold (19) p. 68.

earners on a corresponding income level, the observed income-expenditure relationships may come out as illustrated in fig. III,1.

The income-expenditure curve, 1 + 2, which is drawn as representing the households in both groups, exaggerates the income elasticity of the "average household of wage and salary earners" in the demand for theatre and cinema, because there is a marked shift in expenditure level from rural households to urban households. This shift may be due to the fact that these goods are not equally accessible to the two household types.

Now, the consumption survey of wage and salary earners in 1955 was so comprehensive that it was possible to make separate calculations for twelve different groups of wage and salary earners defined by residence and social status; to this should be added that adjustments were made also for observed differences in the size of the households. It may, perhaps, therefore be permissible to conclude that the shifting effects are smaller in this analysis than in most other similar analyses, in which the number of observations is most frequently so small as to render impracticable a breakdown into homogeneous subgroups. It should be emphasized however that this effect may still disturb the estimated Engel curves, cf. chap. V pg. 86.

In conclusion it must be underlined, therefore, that one cannot accept, without qualifications, Engel curves calculated on the basis of household surveys as representing the Engel curves as defined by (III,1).

If estimated Engel curves, based on household surveys, are to be used, e.g. for prediction of expenditure on certain commodities, income being known or guessed at, great care must be shown. Comparison should always be made to income-expenditure relationships calculated on the basis of other types of data primarily time-series data⁴).

On the other hand if time series data alone are used we are precluded from drawing conclusions as regards the situation at a specified time; we have instead to refer the calculated values to the whole of the period covered by the time series. Hereby the risk of introducing disturbing influences from other factors has grown—changed price relations, income level and distribution, etc.—so that the calculated values will, for that reason, become unreliable.

As will be shown in the last section of this chapter some of the biases mentioned above should not be excluded if the Engel-curve estimates are to be used on the macro-level; what are considered biases in one conception of the Engel curves are in other interpretations of the Engel curve rightly considered as true elements of the relationships.

In addition to these conceptual difficulties, which may cause serious biases in the estimates of the relationships between y and x , the estimated parameters in the functions of type (III,1) p. 23, are attended with errors from other sources. One important source of error is inaccuracy in the measurement of the independent variable, the household income. These errors of measurement are partly systematic and seem on the whole to lead to an understatement of income, a phenomenon which is well known from tax income statistics and which it has hardly been possible to avoid entirely in this survey

⁴) Cf. Wold (19) p. 50.

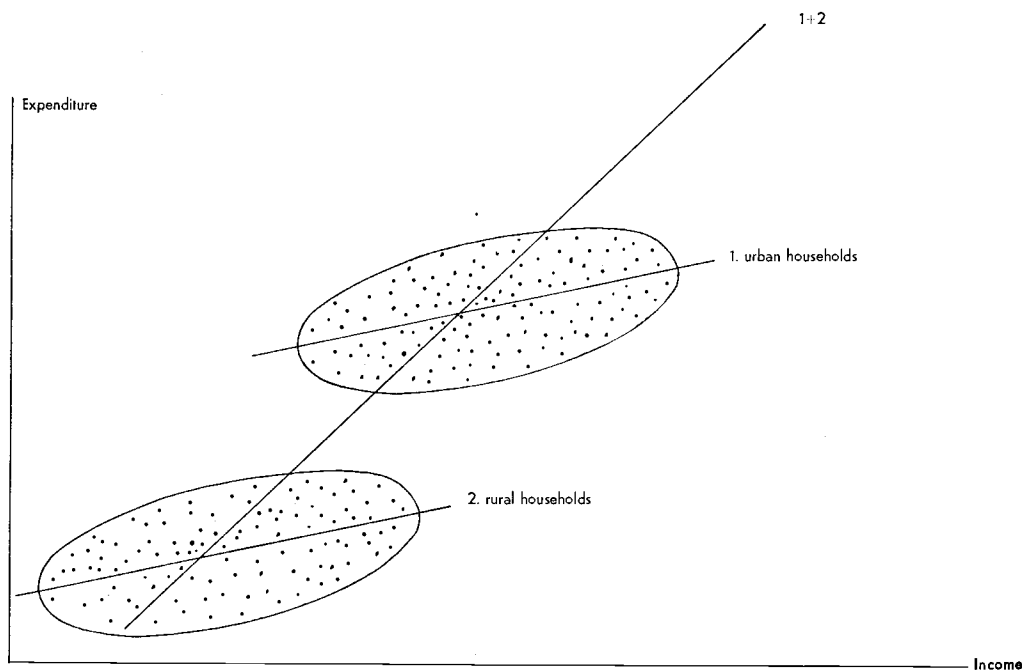


Fig. III, 1. Expenditure on theatre and cinema.

either⁵). The occurrence of inaccuracy in the independent variable even if there is no systematic error of measurement, leads to a systematic error in the evaluation of the slope of the regression lines. If the amount of inaccuracy can be estimated, it will be possible to adjust for it in the evaluation of the slope⁶), but this is not possible in our case, and the mentioned adjustment therefore cannot be made.

One of the requirements for determining unbiased estimates of the parameters of an Engel function of the form (III,1) by means of the regression analysis—which will be the main tool in the following—is that this function is specified in such a way that ε is independent of x . This involves either (1) that x is a quantity given in advance and accordingly not subject to variation in our experimental set-up, or (2) that any variation in x is independent of ε , which is an expression of the unexplained variation in y .

The observed x values do not fulfil the requirement mentioned under (1), already because x is subject to a considerable error of measurement, cf. above. On the other hand, it is not quite clear whether the variations in x are of such a nature that not even the requirement under (2) is fulfilled. The variation in x due to errors of measurement may perhaps to a great extent be presumed to be independent of ε , but it is possible

⁵) It turns out, indeed, that on an average for all households observed the sum of expenditures and savings exceeds the recorded incomes by kr. 145, or slightly over one per cent of the average recorded income.

⁶) Cf. Hald, A. (8) p. 615 and Stone, R. (17) p. 296.

that there are some causes of income variations which also affect y —or which have their origin in y . If, for instance, a household's purchases of a motor car or other durables influence the "income-earning" behaviour of this household, there will be a risk of bias⁷⁾.

In an evaluation of the conclusions which can safely be drawn from the estimated Engel curves, it is important to take the above-mentioned considerations into account. The fact is that it is not the "true" Engel curves we arrive at, and therefore care must be shown if the results are to be utilized in drawing further conclusions. Or, in other words, the validity of the analysis depends on the interpretation of the estimates.

3. *What, then, can the results be used for?*

Firstly, the estimated Engel curves give a more precise *description* of the income-expenditure relationships of the households of wage and salary earners in the year 1955 than would be possible by the mere presentation of summary averages of expenditures at different income levels. As the computations are made separately for twelve residential and social status groups, this description will give, in addition, useful illustration of existing differences in expenditure behaviour among these twelve groups.

Such a description of the expenditure pattern of the households of wage and salary earners is obviously of interest in many respects; questions concerning the marketing conditions of certain commodities or groups of commodities in the different parts of the country, and questions in connection with differences of the consumption patterns of the different social status groups and income groups seen in relation to existing or contemplated excise duties are two important fields. More generally, it may be mentioned that the official Danish statistics concerning the disposal of national income are considerably less developed than statistics concerning the formation of national income, for which reason any supplementing description of the kind mentioned here will be very useful. However, it is true of all the fields where the results could be applied that they would be substantially more valuable if they covered the whole population, whereas this survey, as already mentioned, covers only households of wage and salary earners.

From the point of view of economic theory, however, it is quite as interesting to ascertain whether the estimates are of any value except from a purely descriptive point of view. Are they of value in the analysis of demand at the microlevel? And can they be used as a basis for forecasts of total consumption at the macro level?

The conclusion of the considerations stated above concerning this problem is that the estimates shown in the following chapters—and estimates from other similar family budget surveys—form a very valuable supplement to the existing empirical basis of demand theory. The results are primarily applicable at the micro level, i.e., in an analysis where the point of view is that of the individual household or group of households, whereas it would be more questionable to draw inferences for the analysis of the demand of the total population for the different groups of commodities.

By definition, Engel curves are expressions of the behaviour of individual households at alternative income levels and, subject to the above reservations as regards the inter-

⁷⁾ Cf. Wallander, Jan (18) p. 52.

pretation of the estimates of Engel curves calculated here, the results can therefore directly only say something about the conditions of an individual household, or group of households, under alternative assumptions as regards the income of the household or group of households. These reservations, as will be remembered, involved especially four problems:

1. The estimates of the Engel curves were to a certain extent influenced by dynamic adjustment processes, whereas the "true" Engel curves are static.
2. The estimates were influenced by the fact that for some groups of commodities consumer behaviour was to a considerable degree determined by environmental factors (the interdependence effect), whereas the "true" Engel curves are based on the usual *ceteris paribus* assumption.
3. The layer effect.
4. The inaccuracy introduced by any errors of measurement.

With these reservations in mind the estimates should, however, be useful as a basis for conclusions concerning the consumption behaviour of individual households. However, it should be emphasized that the estimates refer to the year 1955 so that statements concerning consumption in any subsequent period will be attended with a further unknown error. The applicability of the Engel curve estimates on the macro level will be discussed in the final section of this chapter.

IIIc. Use of estimated Engel curves on the macro level.

When it is attempted—on the basis of the estimates of Engel curves concerning the consumption behaviour of individual households—to draw conclusions as regards the consumption of all households, i.e., total national consumption expenditure, the problem of the environmental influence on consumption is again brought to the fore. In estimating Engel curves of the type of formula (III,1) above, one of the main problems is how to avoid too much disturbing influence from the behaviour of other households. The object is to estimate independent income-expenditure relationships for individual households. But the relationships between total income and total expenditure, i.e. a function or curve illustrating alternative values of income and expenditure of *all* households, must necessarily take into account the effects on expenditure of the *j*'th household brought about by a change in the income of other households. Or in other words, on the macro-level a possible interdependence effect must be taken into account. A curve expressing the relationship between the whole population's total consumption of certain commodities and its total income will therefore be biased if it is formed by simple aggregation of the Engel curves of individual households.

The occurrence of interdependence among the consumption behaviour of the individual households naturally makes it difficult to say anything about the development of total consumption under alternative assumptions as to the development in total income on the basis of knowledge of Engel curves for individual households. For distinct necessities, where the interdependence effect is probably moderate, this drawback may not be of decisive importance.

An expression of the magnitude of the interdependence effect can be estimated under very simplified assumptions⁸⁾, and in such cases we might be able to arrive at a better estimate of the “macro Engel curve”. No such estimate of the interdependence effect has been made in this survey.

Assuming that a satisfactory estimate had been calculated of the “true” relation between the population’s total income and its total expenditure on various commodities, this estimate would still be subject to the limitation that it would be valid only for the survey period, viz. the year 1955. Such an estimate would not be directly applicable as a basis for a calculation of a consumption forecast, because it does not, of course, contain any elements of shifts in the trend of consumption owing to changes in fashion, taste, etc. These trend factors, which are often of great importance, are, on the other hand, contained in the time series covering the development, over long periods, in total consumption, total income, etc., which are the usual basis of forecasts. However, it must be realized that such trend factors can be extremely unstable, and as they appear only implicitly in the relationships an adjustment for structural change is impossible.

If time series data concerning total income and total consumption of certain groups of commodities were supplemented with Engel curves for all groups of the population, based on household survey material and estimated with due regard to the above-mentioned reservations—and preferably estimated for several consecutive survey periods—then forecasts on consumption expenditures could be improved considerably.

⁸⁾ Cf. Prais J. S. and Houthaker, H. S. (10) p. 18.

METHODS OF ANALYSIS

IVa. Introductory remarks.

The income-expenditure relation $\eta = f(v)$, is the main object of the present analysis; η denoting household expenditure on a given expenditure item and v household income. The question is now how this relation is to be estimated on the basis of the available observations.

In this situation it might be imagined that the form of the Engel function of a given commodity was given in advance or had been arrived at on the basis of, e.g., studies of the "expenditure process". The task would then "merely" consist in determining the parameters of this function, and the results of the analysis would then be of the following type: in the Danish population of wage and salary earners in the year 1955 the parameters of the Engel function for the expenditure on the i 'th commodity assumed the following values

However, no such "true" Engel function is given in advance. This is so because a more general theory on the basis of which a specific function could be derived does not exist. The first step of the present analysis therefore consists in selecting a functional form. Next comes the comparison of the different functions selected by means of suitable tests for goodness of fit.

In short it may be said that the analysis consists of three stages: 1) the selection of a number of functions, 2) the estimation of parameters of the selected functions, and 3) a comparison of these functions and the data by means of various tests for goodness of fit.

IVb. Choice of Engel functions and specification of the variables.

1. Criteria for selecting Engel functions.

In selecting Engel functions one may adopt two different points of view. Firstly, on the basis of the existing theory of consumer behaviour, try to set up, for each expenditure item, a model which fulfils the theoretical requirements to the greatest possible extent. Or secondly, on the basis of the available observations, select one or more functions showing a satisfactory goodness of fit, whether or not these functions can be justified by the theory of consumer behaviour.

Of course, it would be most satisfactory to choose the former approach, but it must be acknowledged that the theory of consumer behaviour does not at present offer sufficient guidance for the selection of "true" Engel functions. But economic theory can tell that the "true" Engel curve for a given commodity has certain characteristics. This

information can then be utilized as a supplementary criterion for selecting among alternative functions. This criterion being a supplement to the selection by means of different tests for goodness of fit.

If a given type of function deviates from the characteristics of the true Engel curve for extreme values of income, whereas the function otherwise "behaves" satisfactorily, this, however, should not exclude the use of the function in question.

Moreover, it is to be observed that computational problems in connection with the determination of the parameters of the function should not be too complicated, and this requirement naturally limits the types of functions which can be used.

2. Description of the functions selected.

In the present survey the following five functions were selected, in which ν is the household income, η the expenditure on a given expenditure item, and α , β and κ are parameters:

$$\begin{aligned} \text{(IV, 1)} \quad \log \eta &= \alpha + \beta (\log \nu - \overline{\log \nu}) \\ \text{(IV, 2)} \quad \log \eta &= \alpha + \beta \left(\frac{1}{\nu} - \frac{1}{\bar{\nu}} \right) \\ \text{(IV, 3)} \quad \eta &= \alpha + \beta (\log \nu - \overline{\log \nu}) \\ \text{(IV, 4)} \quad \eta &= \alpha + \beta \left(\frac{1}{\nu} - \frac{1}{\bar{\nu}} \right) \\ \text{(IV, 5)} \quad \log \eta &= \log \kappa + \log [\Phi (\alpha + \beta \log \nu)] \end{aligned}$$

Functions (IV, 1) to (IV, 4) find little justification in economic theory, whereas function (IV, 5) to a somewhat greater extent can be justified on the basis of studies of the "consumption process".

Functions (IV, 1) to (IV, 4) are two-parameter functions, which are linear in the two variables or in simple transformations of these variables. This means that we can use the computationally very convenient techniques of linear regression analysis. These functions represent to some extent alternative hypotheses as regards the income elasticity of the expenditure, $e = \frac{d\eta}{d\nu} \cdot \frac{\nu}{\eta}$ or the marginal propensity to consume $m = \frac{d\eta}{d\nu}$ and can thus be used for testing those hypotheses concerning the characteristics of the "true" Engel curves which are related to e and m as suggested by economic theory.

Table IV, 1 show the values of e and m for the five functions.

According to function (IV, 1) the income elasticity is a constant, being identical with the parameter β . According to function (IV, 2), e is inversely proportional to income, whereas according to function (IV, 3), e is inversely proportional to expenditure itself. If one considers the marginal propensity to consume, m , it will be found that according to function (IV, 3) m is inversely proportional to income and according to function (IV, 4) inversely proportional to the square of income. Among other characteristics

Table IV, 1.

Values of income elasticity, e , and marginal propensity to consume, m , for five Engel functions.

Function	e	m
$\log \eta = \alpha + \beta (\log v - \overline{\log v})$	β	$\beta \frac{\eta}{v}$
$\log \eta = \alpha + \beta \left(\frac{1}{v} - \overline{\frac{1}{v}} \right)$	$-\frac{\beta}{v}$	$-\beta \frac{\eta}{v^2}$
$\eta = \alpha + \beta (\log v - \overline{\log v})$	$\frac{\beta}{\eta}$	$\frac{\beta}{v}$
$\eta = \alpha + \beta \left(\frac{1}{v} - \overline{\frac{1}{v}} \right)$	$-\frac{\beta}{v\eta}$	$-\frac{\beta}{v^2}$
$\log \eta = \log \kappa + \log [\Phi (\alpha + \log v)]$	$\frac{\varphi (\alpha + \log v)}{\Phi (\alpha + \log v)}$	$\frac{\eta [\varphi (\alpha + \log v)]}{v [\Phi (\alpha + \log v)]}$

of functions (IV, 1) to (IV, 4) which are interesting from the point of view of economic theory may be mentioned that functions (IV, 1) and (IV, 2) reflect one feature of true Engel curves: that expenditure can never be negative, while functions (IV, 2), (IV, 4) and (IV, 5) reflect a theoretically desirable property of Engel curves of certain commodities, viz. that expenditure asymptotically tends towards a saturation expenditure. Concerning function (IV, 5), it should be mentioned firstly that it contains many of the qualities which can be said to be characteristic of the "true" Engel curves. Expenditure can never be negative; the income elasticity is falling with rising income, and the marginal propensity to consume is first rising and then falling. Secondly, the use of function (IV, 5) as model for the consumption behaviour can be justified by analogy to certain biological experiments²⁾.

In the actual estimation procedure it was decided to fix the parameter β at a given value *inter alia* because the 3-parameter estimation met with serious difficulties, cfr. chap. IV, p. 55.

3. Specification of the variables.

After the functions have been chosen, the observations must be put into a form suitable for computation, and here a number of problems arise. The following deserve special attention: 1) the precise specification of the two variables, v and η , the household income and the expenditure on a given commodity or group of commodities, and 2) problems concerning the grouping of commodities and households. In the foregoing the interpretation of the Engel functions and the selection of certain functions have been

²⁾ Aitchison and Brown (1), page 128, and the same authors in *The Review of Economic Studies*, No. 57, 1955.

discussed on the assumption that all variables other than income and expenditure, were "under control" (p. 23). Accordingly it has been assumed that the parameters of the functions selected were valid only for households in a certain area with that particular social status, of given size, etc.

Now, it is obvious that in real life one cannot estimate the parameters under such restricted assumptions. In this field it is impossible to make laboratory experiments in which all variables other than those examined are kept under control. It is therefore only with rough approximation that one can isolate and measure the influences due to the factors which are of interest in any given inquiry.

In the present survey of the income and expenditure in 1955 of households of wage and salary earners the factors which may be expected to influence the expenditure behaviour of households apart from the dynamic factors discussed above (p. 25) will especially be residential differences (whether the household lives in a rural district or in a provincial town or in the capital), differences as to social status (whether the household belongs to, for instance, the group of higher salaried employees or the group of unskilled workers), and differences in size and types of households.

The available basic material is so comprehensive that it is possible to make separate calculations for several subgroups of wage and salary earners, and by using domicile and social status as criteria in this subgrouping the greater part of the variation in expenditure attributable to differences in these two respects will be eliminated. The following subdivision was used in the survey:

1. Higher public servants and salaried employees in the Capital.
2. Lower public servants and salaried employees in the Capital.
3. Skilled workers in the Capital.
4. Unskilled workers in the Capital.
5. Higher public servants and salaried employees in the Provincial towns.
6. Lower public servants and salaried employees in the Provincial towns.
7. Skilled workers in the Provincial towns.
8. Unskilled workers in the Provincial towns.
9. Lower public servants and salaried employees in the Rural districts.
10. Skilled workers in the Rural districts.
11. Unskilled workers in the Rural districts.
12. Farm workers in the Rural districts.

By a further subdivision into subgroups by e.g. the size and composition of households, the number of observations would be so small in many subgroups that in spite of the subgroups being more homogeneous it would not be possible to calculate the parameters with reasonable accuracy³⁾.

The size of household may, however, be taken into account, if for all households ν and η represent *income per person* and *expenditure per person*, respectively⁴⁾. Differences in

³⁾ This in fact is the same thing as to say that no criteria for further subdividing are of "significant" importance for the stability of the relations in question.

⁴⁾ Cf. S. J. Prais and H. S. Houthakker (10) pp. 88-93.

type of household will probably still make themselves felt, but this influence can now, with good approximation, be considered as being of a random nature.

In a following part of this chapter the stochastic element of the model, will be taken up for discussion in greater detail (cf. p. 40), and the discussion at this point can therefore be finished with a few further remarks concerning the definition of η and ν , the dependent and the independent variable of the Engel functions.

The dependent variable, η , the expenditure per person on a given commodity group, is defined as the value of the goods (and services) in this commodity group which the household has *bought* during the survey period; there is one important exception to this rule, viz. with regard to goods bought on the hire purchase system. In the case of these goods (particularly certain *durable goods*, furniture, radios, refrigerators and not least *own means of transport*, motor-cars, motor-cycles, etc.) η is defined as the amount *spent* by the household during the survey period in connection with the purchase of these goods, i.e., down-payment plus any instalments.

The independent variable, ν , is defined as household disposable income per person, i.e., income earned less personal taxes paid.

The use of disposable income as the independent variable rather than total income is of varying importance for the different household types. An examination of all groups of wage and salary earners as regards the taxes paid as a percentage of total income in certain income groups, separately for 5 household types, showed that this percentage falls for a given total income with growing size of household (allowances for dependents and children).

For a given household type the tax percentage naturally increases with rising income owing to the taxation system.

These facts must be borne in mind when reading the discussion of the income-expenditure relationship in the following chapter.

Disposable income and total income are thus strongly correlated, but differences in household type, and income level and changes in income give rise to systematic deviations between the two income concepts.

In the present survey well-founded estimates of the incomes of the individual households in the survey period have been obtained by checking information on income with information on expenditure + savings, see chapter II; it was therefore decided to use this estimate of household income (less personal taxes paid) as the independent variable, cf. what has been said above concerning the importance of errors in the measurement of the independent variable (p. 27). In the corresponding British inquiry, household income could not be used as the independent variable because information about the size of this income had been collected only for part of the material; instead the sum of all recorded expenditures was used as the independent variable⁵).

In their arguments to justify this procedure, however, Prais and Houthakker tend to conclude that the sum of expenditures actually is a "better" independent variable than the income concept defined above because it enables them to arrive at more stable relationships:

⁵) S. J. Prais and H. S. Houthakker, (10) p. 80.

“The true determinants of the expenditure pattern of a household in a dynamic situation are a complicated function of past, present and expected incomes, and though this function can analytically be formulated in a precise way it is of little help here. The success of an empirical analysis must depend on the choice of some simple, readily obtainable, measure which substantially represents the facts. ... The use of total expenditure as the determining variable in the Engel curve can be justified on the assumption that while total expenditure may depend in a complicated way on income expectations and the like, the distribution of expenditures among the various commodities depends only on the level of total expenditure”⁶).

A priori the greatest interest seems, however, to attach to an elucidation of the relationship between income and expenditure rather than the relationship between the sum of all expenditures and components of this sum (cf. the discussion of the uses of the Engel curve functions on page 22).

If in many cases the use of total expenditure as the independent variable leads to better goodness of fit than the use of income, an important reason, of course, is the difference in saving behaviour of the households. It is obvious that differences in saving behaviour “disturb” the functional relationship between income and expenditure, but this does not mean that income is a bad independent variable. It means that one should explain both saving and consumption in the same model.

Any satisfactory model illustrating the relationship between income, expenditures on the different commodity groups, and saving would naturally also have to take *income changes* into account. This has not been done in the present survey, and therefore the relationship between ν and η will to a certain extent be less stable than if the available information on the influence of income changes had been utilized⁷). In Appendix D, which presents some of the basic material, information is given on saving as well as on income changes of households from 1953 to 1954 and from 1954 to 1955.

4. Zero-observations.

Three of the functions selected (IV,1), (IV,2) and (IV,5) namely the functions in which the dependent variable is the logarithmic transformation of expenditure, η , are only defined for $\eta > 0$. Therefore a problem arises if zero observations of η are found in the basic material (assuming that negative values cannot occur). The occurrence of zero observations not only creates a purely computational problem, but also raises the fundamental question of whether the functions selected can be used at all in the description of the observed relationship between income and expenditures on various commodity groups.

Assuming that zero observations occur with the same frequency in all income intervals, one may get a picture as shown in figure IV, 1.

⁶) S. J. Prais and H. S. Houthakker, (10) p. 81.

⁷) Cf. *Opsparing i lenmodtagerhusstandene*, Det Statistiske Departement, København, 1960, p. 31, where it is shown that households with increasing incomes save significantly more than households with falling incomes.

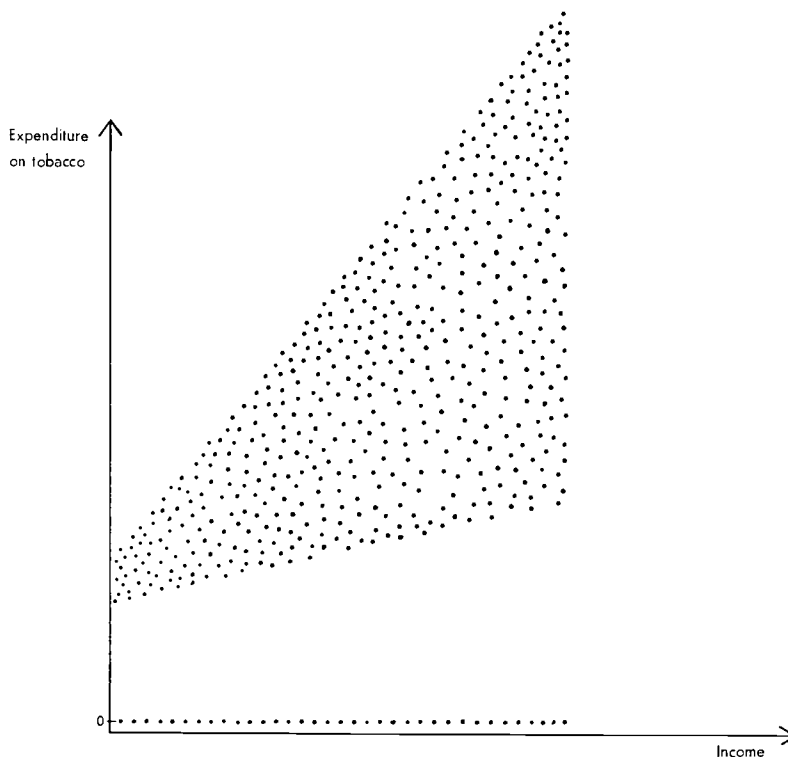


Fig. IV, 1. Two groups of Expenditures on tobacco.

Here it is evident that “true” zero observations exist and that it is therefore necessary to split the observation material into two groups before it is possible to give a satisfactory description of the relationship between ν and η .

In one group, which contains all non-smokers (or rather all who do not *buy* tobacco) the function $\eta = 0$ applies and for the remainder group one can then try to use the functions selected.

However, it turned out that a hypothesis concerning the occurrence of “true” zero observations can only be confirmed in exceptional cases. For most commodity groups the occurrence of zero observations is limited to the lowest income intervals, and it is perhaps then permissible to assume that the zero observations may be due to random deviations from the true values. If this is the case, it will not be justified to split up the material; instead it is necessary to work out a computational technique which permits the occurrence of zero observations. Here several possibilities seem open.

Firstly, one can assign an arbitrarily low value to the zero observation households, e.g., as suggested by Prais and Houthakker⁸⁾ $\eta = 0.25 m$, m being the unit of measure-

⁸⁾ S. J. Prais and H. S. Houthakker (10) p. 50.

ment. Assuming that all observations of $\eta < 0.5$ have been recorded as 0, and assuming a rectangular distribution of these observations their mean value will then be 0.25 m. This method leads to biases in the estimates of the parameters; especially $a \approx \overline{\log y}$ will be too big (compared with the corresponding uncorrected estimates in the other 3 Engel functions).

Even if, by means of suitable reductions of all $\eta > 0.5$, one might be able to avoid a systematic bias of the expenditure average, this method would nevertheless introduce a considerable element of arbitrariness into the calculation of the parameters of the functions and would therefore not be very satisfactory.

Another way out would be to try to estimate parameters direct from the functions, of which (IV, 1) and (IV, 2) have been formed by logarithmic transformation, i.e., in the functions

$$\begin{array}{ll} \text{(IV, 1*)} & \eta = a^* v^\beta \\ \text{and} & \\ \text{(IV, 2*)} & \eta = a^* e^{\frac{\beta}{v}} \end{array}$$

The parameters a^* and β can be estimated by an iterative process, where each stage of the iteration is a linear regression.

An examination of several examples showed that in the successive stages of iteration the estimates of a^* and β did not converge; the result of changes in one parameter seemed exactly to offset the result of the changes in the other parameter so that the results of the computations showed a continued oscillation. This method was therefore abandoned and instead it was decided not to use individual observations but to follow the method adopted in the British inquiry: to carry out the calculations on the basis of a grouped material. This, however, raises the problem of how to group the observations⁹⁾.

5. Grouping problems.

In the grouping of the households the zero expenditure observations will in most cases be grouped together with positive η values, and the group averages will therefore, except in very few cases, be higher than zero.

In the present inquiry the observations have been grouped in the following way:

Within a given social group (see the list of social groups above p. 34) the households are arranged according to size of income per person. The households are then grouped in threes so that the one or two excess households (if the number of households is not divisible by three) are rejected "from the middle" of the income range as it must be considered valuable to fully utilize the relatively few observations at the outer limits of the field of observation. The values of v and η which are accordingly included in the calculations are always the arithmetical average of the three household values observed

⁹⁾ Cf. S. J. Prais and H. S. Houthakker (10) pp. 50-51, and concerning the computational consequences of grouping, pp. 59-62.

for each group. Hereby it is achieved that transformations into logarithms or reciprocal values can be confined to the group averages.

In the very few cases where a group value of η becomes equal to zero, it is rejected.

Also rejected are a few individual households where the expenditure on certain necessities (food and dwelling) was extraordinarily low, namely households with an observed expenditure on food of zero or households whose expenditure on food was below kr. 300 at the same time as their expenditure on dwelling was kr. 0. In the case of these households (most often households of single persons who receive board and lodging as part of their remuneration), there were so severe errors of measurement that their exclusion from the observation material was deemed unavoidable.

However, the material is also grouped in another way: the several hundred individual commodities and services are grouped into main commodities or commodity groups, so that only expenditure on these commodity groups are considered. Unlike the above-mentioned grouping of the individual households, this grouping of commodities is indispensable, if an overall description of the expenditure pattern is aimed at. The problem in this connection is not, therefore, *whether* a grouping is to be undertaken, but *how* the material is to be grouped and *how far* this grouping is to be carried.

Here, there are several, more or less conflicting, points of view to be considered. A detailed classification of commodities will be desirable if the principal interest attaches to the marketing possibilities of the *individual commodity*. If the main interest—as in this analysis—attaches to an overall picture of the relationship between income and consumption expenditures, rather few groups should be considered. Another point has to be made; in order to arrive at a stable functional relationship between ν and η it would be desirable to group the material into groups which are felt by the households to be “natural”, i.e., that in spending their income the households think in terms of and actually distinguish among these categories of consumption expenditures. The breakdown into “natural” budget items which should contribute to stability in the consumption functions is, at the same time well in line with the aim of obtaining an overall description of the consumption behaviour. On the other hand, it must be borne in mind that this procedure may group together commodities with different income elasticities, although from other points of view a grouping which leads to a higher degree of homogeneity within the individual expenditure groups might be desirable.

In the present analysis the following grouping has been used:

1. Dwelling.
2. Fuel and lighting.
3. Food.
4. Tobacco.
5. Clothing.
6. Footwear.
7. Washing and cleaning.
8. Durable goods (excl. motor vehicles).
9. Personal hygiene.
10. Books, newspapers, etc.

11. Sports, holidays, hobbies, etc.
12. Transport (incl. motor vehicle).
13. Subscriptions, union fees, etc.

For all households together these groups comprise close to 90 per cent. of total consumption expenditures. The items which have been excluded are, *inter alia*, expenditures on education, domestic servants, gifts and charities. A detailed description of the 13 expenditure items will be found in the appendix.

IVc. Variance assumptions.

1. General remarks.

Now the actual calculation of the estimates of the parameters of the Engel curves is in sight. The alternative Engel functions have been set up; the dependent and the independent variables of these functions have been defined, and finally the problems relating to the grouping of commodities and households have been dealt with (whereby one also arrived at a workable procedure as regards the treatment of zero observations).

Before the calculation of parameter estimates of the Engel functions can be made it must be specified how the stochastic element enters. As mentioned above, the Engel functions which were chosen are of the form $\eta = f(v)$ where $f(v)$ is characterized by means of the parameters α and β (and further of κ for the log-normal distribution function); cf. p. 32 above. However, inserting in the model the actual income and expenditure observations x and y for v and η , $f(x)$ does not exhaustively describe a given household's expenditure on a given commodity group; each expenditure observation contains a stochastic element and it is necessary to specify the properties of this stochastic element, ε .

The simplest approach is to assume ε to be independent of x and normally distributed with mean value 0 and variance $V \{ \varepsilon \} = \sigma^2 = V \{ y|x \}$. If these assumptions are accepted, efficient estimates of the parameters of the five models will appear from a simple least-squares regression analysis¹⁰.

If, on the other hand, these simple assumptions are not fulfilled—and this they are not always in this analysis—then estimation of parameters carried out on these erroneous assumptions as to the distribution of the stochastic element will involve a loss in the efficiency of the parameter estimates; these estimates will accordingly have an unnecessarily high standard error. Moreover, an estimation on erroneous assumptions as regards the variance of the distribution of ε will make it difficult to apply the proposed tests for goodness of fit. If one accepts the—inefficient—parameter estimates obtained in this way, one may all the same be able to use the different tests for goodness of fit if certain corrections in the variance estimates are made¹¹. Prais and Houthakker, in their analysis of the material of the British family budget surveys, have disregarded these complications and have everywhere estimated on the variance assumption mentioned above—also in cases where this assumption obviously does not hold good.

¹⁰ The homoscedastic case of J. Aitchison and J. A. C. Brown (1), p. 46 and S. J. Prais and H. S. Houthakker (10), p. 78.

¹¹ Cf. S. J. Prais and H. S. Houthakker (10) p. 57 and p. 96.

However, it seems to be a more satisfactory alternative to try to specify the model in such a way that the least-squares regression estimator becomes the efficient estimator; thereby a correction to the testing procedure is, at the same time, avoided.

The efficient, least-squares regression estimate of the parameters will be achieved by weighting all y for given x values by the reciprocal value of their variance $V\{y|x\}$. Observations to which a high degree of variability attaches will then be included in the calculations with less weight than observations where $V\{y|x\}$ is small. If, therefore, we know the true value of $V\{y|x\}$ for all x , such a weighted calculation of the estimates will give the desired result. Now, this true value is unknown, and the problem then becomes to form suitable estimates of $V\{y|x\}$ for all x .

When plotting estimates of $V\{y|x\}$ against y^2 it was found in a number of cases that there seemed to be reason to assume that $V\{y|x\} = \sigma^2\eta^2$, i.e., that the variance of y for given x increases proportionally to the square of the dependent variable, cf. fig. IV,2.

If this assumption could be maintained, it would mean that the residual variance in the functions in which the logarithm of y is used as a dependent variable would become constant. This can be shown in the following way. The variance assumption $V\{y|x\} = \sigma^2\eta^2$ means that ε is included multiplicatively in the Engel function, i.e., that a given sample of expenditure observations can be described by a function of the form $y = f(x)(1 + \varepsilon)$. If, now, both sides of the equation are transformed logarithmi-

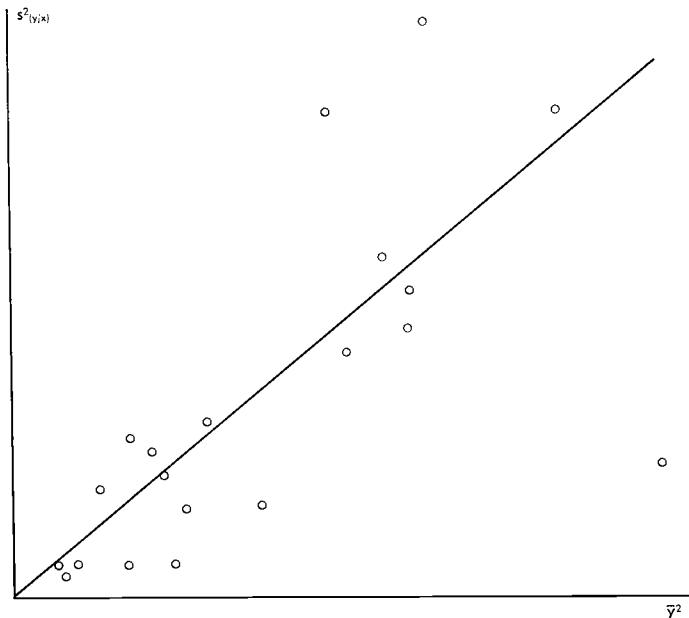


Fig. IV,2.—The variance of the expenditure on clothing (per person) within groups of ten households, arranged by the size of income per person, plotted against the square of this expenditure.

Higher salaried employees and civil servants in provincial towns.

cally, the result will be a function of the form $\log y = \log f(x) + \log(1 + \varepsilon)$. The stochastic element of this quantity, $\log(1 + \varepsilon)$, will then be independent of y and x and the simple least squares regression estimator is efficient and unbiased. As will be shown later rather simple efficient and unbiased estimators can be devised for the remaining Engel functions in this case, too.

It is evident that this convenient property of the hypothesis concerning the distribution of the stochastic element will further increase the interest in having the hypothesis tested, and it was therefore decided to examine this problem in greater detail.

2. Testing the hypothesis $V\{y|x\} = \sigma^2 \eta^2$.

Assuming $V\{y|x\} = \sigma^2 \eta^2$, it follows that the coefficient of variation, γ , in the distribution of y for given x is constant since

$$(IV, 6) \quad \gamma^2 = \frac{V\{y|x\}}{\eta^2} = \sigma^2.$$

In other words, the hypothesis can be tested by means of a test for the constancy of the coefficient of variation. If a test for such constancy does not show significant results for too many expenditure items, it would seem justified to maintain the hypothesis.

As mentioned above, the observations have everywhere been grouped in threes. For each group an estimate, c , of γ has been calculated, the c -value for group number m being calculated by the formula

$$(IV, 7) \quad c_m = \frac{S_{y_m}}{\bar{y}_m}.$$

A test for the constancy of γ can be developed if one can construct a theoretical c -distribution derived from groups of three observations which are known beforehand to follow the variance hypothesis being tested and compare the observed c -values with this theoretical distribution.

In an article Hendricks¹²⁾ has derived an approximation formula for the distribution of c , assuming y to be normally distributed and the number of observations per group = n .

Assuming $n = 3$, Hendricks formula can be developed into the following expression¹³⁾

$$(IV, 8) \quad p\{c\} dc \simeq \frac{\frac{3}{2} c}{\sqrt{1 + \frac{3}{2} c^2}} \left(1 + \frac{3 \eta^2}{\sigma^2 (1 + \frac{3}{2} c^2)}\right) \exp\left\{-\frac{\eta^2 \cdot 3 c^2}{\sigma^2 (3 + 2 c^2)}\right\} dc$$

and by integrating one obtains

$$(IV, 9) \quad P\{c\} \simeq 1 - \frac{\sqrt{3}}{\sqrt{3 + \frac{3}{2} c^2}} \exp\left\{-\frac{\eta^2 \cdot 3 c^2}{\sigma^2 (3 + 2 c^2)}\right\}$$

$P\{c\}$ and $p\{c\} dc$ denote the cumulative distribution function and the distribution function respectively for c , η denote the mean value of y and σ^2 the mean value of s^2 .

¹²⁾ Hendricks (9), cf. also Mc. Kay (14).

¹³⁾ Karl Vind, cand. polit., the Statistical Institute of the University of Copenhagen, has derived (IV, 8) and (IV, 9) and has taken part in the preparation of section IV, b, 2.

(IV, 8) and (IV, 9) are approximative, a good approximation depending on $\Phi\left(-\sqrt{3}\frac{\eta}{\sigma}\right)$ and $\Phi\left(-\frac{3\eta}{\sigma\sqrt{3+2c^2}}\right)$ being small, where $\Phi(u)$ denotes the cumulative normal distribution function. Not least the second of these assumptions is critical, since it implies that γ , i.e. the true c -value in a given expenditure group, must not be higher than about 0.5. For $\gamma = \frac{\sigma}{\xi} = 0.5$, $\Phi(u)$ will fluctuate around $\Phi\left(-\frac{6}{\sqrt{3+2\cdot 0.5^2}}\right) \approx 0.0006$, where $u = \frac{3}{\gamma\sqrt{3+2c^2}}$. For $\gamma = 0.6$, $\Phi(u)$ will fluctuate around 0.005. For γ higher than 0.6, $\Phi(u)$ will grow steeply.

If now γ is assumed equal to the observed average of the c -values from all groups of three, the test hypothesis that the observed c -values are distributed around the true value as indicated by the distribution function (IV, 8) above can be tested. By grouping the observed c -values in suitable intervals and calculating the expected number in the same intervals according to the theoretical distribution function shown above (formula (IV, 8)), the hypothesis can be tested by means of a χ^2 -test.

Before starting these calculations it is necessary to ascertain whether the assumptions under which the distribution function (IV, 8) was derived can be considered fulfilled in the present case. As mentioned, the assumptions of formula (IV, 8) and (IV,9) implied that γ , the "true" value of c should not be higher than approx. 0.5. A glance at the observed average c -values, cf. table IV,2, page 45, will show that this assumption in several cases *cannot* be considered fulfilled. What then? Is the approximation in formula (IV, 8) above nevertheless satisfactory or must the attempt to test the hypothesis be abandoned in those cases where $\bar{c} > 0.5$? This problem has been investigated experimentally. By means of random sampling numbers were formed distributions of c -values with given γ and then it was tested whether the theoretical c -distribution (IV, 8) differs systematically from the experimental c -distributions.

In the present case c -distributions were constructed from 100 groups of normally distributed random sampling numbers each group consisting of three numbers. For each group an estimate of σ was calculated by means of two of the three numbers; the third one is then taken as an independent estimate of η . By choosing a suitable mean of the random sampling numbers a series of c -quantities with given γ was produced. In the present case c -distributions were formed with $\gamma = 0.25, 0.33, 0.50, 0.67$ and 1.0. These c -distributions with known γ -values were then compared with the distributions calculated on the basis of the theoretical distribution (IV, 9) to ascertain the degree of approximation.

It turned out that the distribution formulas (IV, 8) and (IV, 9) produced c -distributions which did not differ significantly from the empirically derived "true" c -distributions even for $\gamma = 0.67$. However, it should be mentioned that this result is based on only one series of 100 groups, so that it cannot without hesitation be considered generally valid.

When γ tends towards 1.0 (IV, 8) and (IV, 9) is clearly useless.

Table IV,2. Average coefficient of variation, separately for 13

	Income	Dwelling	Fuel & ight	Food	Tobacco	Clothing
The capital:						
Higher salaried employees.....	0.0100558	0.4118768	0.3580842	0.1916989	0.6973217	0.4142853
Lower salaried employees.....	0.0099754	0.4138525	0.5104255	0.2391395	0.8289576	0.4674581
Skilled workers.....	0.0105403	0.4640940	0.3941377	0.1949627	0.5843378	0.4060097
Unskilled workers.....	0.0118760	0.4802885	0.4158627	0.2140957	0.6281060	0.5241140
Provincial towns:						
Higher salaried employees.....	0.0229228	0.4123087	0.4247304	0.2288049	0.6666245	0.4061404
Lower salaried employees.....	0.0105639	0.4859840	0.5399429	0.2534217	0.7225977	0.4221963
Skilled workers.....	0.0148521	0.4074703	0.4089206	0.1767807	0.6510627	0.4185785
Unskilled workers.....	0.0115769	0.4168361	0.3582310	0.2310122	0.6795800	0.4349383
Rural districts:						
Lower salaried employees.....	0.0085254	0.5753355	0.5261609	0.3213999	0.7005913	0.5041757
Skilled workees.....	0.0149660	0.4184816	0.3557065	0.1971511	0.6123814	0.4990967
Unskilled workers.....	0.0086862	0.4513609	0.3595431	0.2094953	0.6084371	0.4006761
Agricultural workers.....	0.0138114	0.5993484	0.3452973	0.2153365	0.6443781	0.4770866

The validity of the theoretical c-distribution formula for practically all items having thus been substantiated, the χ^2 -test for the postulated variance assumption can now be carried out in the way mentioned above. These test calculations were carried out for all expenditure items separately for each of the 12 social groups into which the materail has been divided.

Table (IV,3) shows the result of these calculations. The table indicates the calculated χ^2 -values; in brackets after each χ^2 -value has been given the number of degrees of freedom. All values which are outside the interval

$$\chi^2_{.025} < \chi^2 < \chi^2_{.975}$$

have been italicized. It will be seen that most of χ^2 -values fall within this interval, but the table shows that the items of durable goods and transport display many significant χ^2 -values, which seems to indicate that the c-values for these items cannot be considered distributed at random with a constant true value.

Also among the other expenditure items are there some significant χ^2 -values (tobacco, sports, holidays and hobbies), especially in the group of lower salaried employees.

In this group special factors make themselves felt as regards the distribution by household type which causes a higher variation in the amounts of expenditure on the various commodity groups. The many households consisting of single, often relatively young, employees, have in many cases special arrangements as regards their consumption of food and dwelling, and this again leads to an anomalous behaviour as regards their expenditure on other items.

expenditure items within each of 12 groups of wage and salary earners.

Footwear	Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.4112018	0.4458214	0.7144320	0.3626326	0.4883780	0.4814214	0.7731124	0.3964075
0.4372635	0.4506974	0.8814987	0.3819749	0.5080054	0.5579195	0.6946542	0.4351082
0.3462304	0.4120135	0.7127083	0.3624174	0.4516408	0.4951017	0.7926875	0.3205971
0.4440553	0.4946532	0.8143076	0.4150490	0.5411231	0.6219904	0.8325174	0.4192661
0.3602831	0.4769174	0.8262957	0.3476057	0.5117822	0.5397854	0.9513031	0.2363746
0.4233208	0.5308037	0.8837792	0.3897127	0.5188425	0.5658513	0.8252526	0.3484479
0.3699763	0.4177406	0.7241689	0.3894987	0.4751481	0.5430097	0.8953278	0.3066853
0.3959437	0.4723487	0.8089104	0.3510550	0.4778457	0.5694717	0.8622860	0.3109550
0.4252238	0.5500041	0.8916780	0.4078307	0.5945628	0.6114558	0.9503679	0.3585170
0.4358140	0.4724081	0.7867449	0.3860195	0.4983964	0.6142276	0.8669539	0.3510981
0.4227025	0.4618598	0.7643399	0.3981474	0.5260356	0.631437	0.8663685	0.2987965
0.4169857	0.4591317	0.7869866	0.3807334	0.5176211	0.6789559	0.7583866	0.4023374

As mentioned above, page 39, it was deemed necessary, before starting the main computation programme, to reject the households in which the expenditures on food and dwelling were near zero. 29 households out of a total of 39 households rejected belonged to the group of "lower employees and public servants", and of these 29 households, 25 belonged to the above-mentioned household type of one person. The χ^2 -values in the table were calculated before the 39 households were rejected.

IVd. Calculation of Estimates of Parameters.

1. Four linear functions.

Maximum-likelihood estimates of the parameters of the four linear relations can now be calculated in a simple manner, assuming that the above-mentioned variance assumptions are valid. As regards the type of function $\log y = f(x) + \epsilon$, in which $V\{y|x\}$ is assumed constant, maximum-likelihood estimates of the parameters can be obtained by means of a simple, unweighted least squares estimation¹⁴). For the other type of function $y = f(x)(1 + \epsilon)$ the maximum-likelihood estimate can be obtained by means of an iterative calculation. As mentioned above page 41, it is a prerequisite for obtaining the efficient estimate of the parameters that the observations (x, y) should be weighted by the reciprocal value of the variance $V\{y|x\}$.

¹⁴) Cf., e.g., A. Hald, (8) § 18,3, p. 528.

Table IV,3. χ^2 -test for constancy

Groups of wage and salary earners	Expenditure				
	Dwelling	Fuel & light	Food	Tobacco	Clothing
The capital:					
Higher salaried employees.....	5,5 (6)	2.4 (5)	3.7 (2)	20.2 (10)	6.9 (6)
Lower salaried employees.....	10.2 (7)	17.6 (9)	7.6 (3)	74.3 (12)	14.8 (9)
Skilled workers.....	6.2 (6)	9.4 (4)	3.4 (2)	11.5 (7)	9.1 (4)
Unskilled workers.....	5.0 (6)	13.7 (5)	2.8 (2)	8.3 (8)	3.8 (7)
Provincial towns:					
Higher salaried employees.....	5.0 (4)	2.9 (4)	1.4 (2)	20.6 (8)	7.2 (4)
Lower salaried employees.....	13.9 (7)	16.2 (8)	18.4 (3)	41.9 (10)	3.9 (6)
Skilled workers.....	2.8 (3)	0.4 (3)	1.5 (1)	8.3 (5)	4.1 (4)
Unskilled workers.....	5.3 (4)	5.4 (5)	6.0 (2)	24.3 (8)	4.3 (5)
Rural districts:					
Lower salaried employees.....	20.8 (9)	28.3 (7)	45.3 (5)	30.7 (10)	27.9 (7)
Skilled workers.....	10.9 (4)	3.4 (3)	2.2 (2)	13.3 (6)	12.1 (5)
Unskilled workers.....	1.8 (6)	5.1 (5)	1.2 (2)	25.2 (9)	4.4 (6)
Agricultural workers.....	9.9 (6)	7.1 (3)	6.2 (2)	13.6 (6)	5.0 (5)

Significant values are italicized. Figures in brackets are degrees of freedom.

As it is now assumed that $V \{y | x\} = \sigma^2 \cdot \eta^2 \cong \sigma^2 \cdot \frac{1}{[f(x)]^2}$ this will mean that the weight is $\frac{1}{\sigma^2 [f(x)]^2}$ or since σ^2 is constant, simply $w = \frac{1}{[f(x)]^2}$

Now, however, this weight depends on the parameters which are to be estimated and therefore it is necessary to proceed step by step by means of an iterative process. The initial values for the parameters α and β are calculated by a simple, unweighted regression (which yields unbiased, but not efficient estimates) and on this basis the values of $f(x)$ and thus of w for the first stage of the iteration are calculated; thereafter these values of w are used at the next stage of the iteration, which consists in a weighted regression analysis¹⁵). This stage gives new estimates of α and β , which are used to compute new values for $f(x)$ and thus for w , which again are used in the next stage, etc. The iteration process is carried on until the changes in the estimates α and β become sufficiently small (in terms of s_{α} and s_{β}), a situation which will often occur very quickly since the calculated initial values are not very far off the mark.

2. The log-normal distribution function.

The computational procedure adopted in deriving the maximum-likelihood estimates for the three parameters α , β and κ of the function $\eta = \kappa \Phi(\alpha + \beta \log v)$ has been dealt

¹⁵ Cf. A. Hald, (8) § 18,6, p. 552.

of coefficients of variation.

groups							
Footwear	Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscription etc.
8.7 (6)	23.6 (7)	34.5 (10)	8.7 (6)	9.9 (7)	10.9 (7)	55.8 (10)	14.4 (6)
18.1 (8)	15.2 (8)	100.0 (11)	14.2 (6)	23.0 (9)	44.9 (11)	23.5 (11)	20.9 (8)
6.6 (5)	8.9 (4)	20.1 (8)	8.4 (5)	2.3 (5)	12.5 (6)	7.8 (7)	0.7 (4)
8.8 (5)	8.3 (6)	40.5 (10)	4.2 (5)	8.7 (7)	11.3 (8)	40.7 (10)	15.7 (5)
7.0 (5)	10.9 (6)	34.7 (8)	5.8 (5)	4.0 (6)	16.5 (6)	17.2 (7)	5.2 (4)
11.8 (6)	7.1 (8)	52.7 (11)	7.6 (6)	10.5 (7)	32.8 (9)	53.7 (11)	13.9 (5)
0.9 (3)	2.2 (4)	14.7 (5)	6.3 (3)	5.8 (5)	5.3 (5)	24.8 (5)	5.1 (3)
4.1 (4)	7.5 (5)	35.5 (8)	5.0 (5)	12.4 (6)	24.7 (6)	52.9 (8)	5.7 (4)
16.0 (6)	16.5 (8)	57.5 (11)	9.2 (6)	27.4 (9)	37.5 (9)	84.3 (11)	7.9 (5)
2.3 (4)	14.1 (5)	15.5 (5)	4.2 (3)	5.9 (5)	30.3 (6)	26.0 (5)	2.3 (3)
6.1 (4)	1.9 (6)	44.0 (10)	3.8 (6)	12.5 (7)	48.2 (9)	59.3 (10)	9.5 (3)
4.1 (4)	4.9 (5)	15.9 (5)	4.7 (3)	4.5 (5)	21.9 (6)	15.6 (6)	3.6 (4)

with by Aitchison and Brown¹⁶). Inserting for η and ν the observations y and x and assuming also in this case that $V \{y|x\} = \sigma^2 \eta^2$ the function can be logarithmically transformed into the computationally more convenient form

$$(IV, 10) \quad \log y = \log \kappa + \log \Phi (a + \beta \log x) + \varepsilon.$$

Also here the calculations must be carried out by means of iteration, but in this case, unlike that of functions (3) and (4), it is difficult to achieve convergence as it is not possible to find good initial values for the three parameters in any simple way. Aitchison and Brown suggest that one should guess at an initial value for κ , k_0 say. By plotting $\frac{y}{k_0}$ against $\log x$ on probability paper one should obtain a straight line ($\Phi (a + \beta \log x)$ representing the cumulated log-normal distribution). If the value for k_0 has been fixed wrongly the curve depicted will not be a straight line, and new k_0 values should then be guessed until according to a graphical inspection the curve in the diagram seems to be a straight line. Initial values of a and β are then read from the diagram, after which the iteration can be commenced. As the computations include estimation of parameters separately for 12 social groups and 13 expenditure items, the work in connection with this graphic "targeting" would become of quite considerable dimensions; moreover, examples which have been worked out seem to show that the shape of the curve on probability paper was almost unaffected by large variations of k_0 . The variances of these estimates

¹⁶ Cf. J. Aitchison and J. A. C. Brown, (1) p. 82.

of initial parameter values are thus very big and it was therefore deemed desirable to work out a method of estimation by which the initial values could be computed mechanically.

As initial value for k was chosen $y_{\max} =$ the highest average value observed for y in the groups of three households into which the basic material had been grouped.

Then initial values for a and β were calculated by linear regression since (IV, 5) implies that $u = a + \beta \log x$, where $\Phi(u) = \frac{y}{x}$.

As a result of the method of calculation adopted the whole computation programme for function (IV, 5) became "automatic". Naturally, there was no guarantee that the iteration would always converge; there was no possibility of ensuring in advance that the initial parameter estimates a_0 , b_0 and k_0 would fall within the region of convergence¹⁷). It turned out in fact that in some cases (19 out of 156) the iteration process diverged. As will be shown later in this chapter it also turned out that a fixed value of the parameter β had to be chosen to ensure workability of the estimation procedure.

All estimates of the parameters have been shown in appendix A; extracts of the results are shown and commented upon in chapter V.

IVe. Tests for Goodness of Fit.

1. The tests used.

One of the purposes of the present analysis was, as already mentioned, to find that one of the chosen relations which according to the available observations would show—for each expenditure item—the best goodness of fit. On the basis of such an investigation it would be possible to conclude that among the five given types of Engel curves, one type gives the best fit if we consider the i 'th expenditure item; in the case of the j 'th expenditure item it may be another function type which gives the best fit, and so on. To be able to draw such a conclusion one must test the goodness of fit of the five functions. These tests consist in various comparisons of the calculated function values, $f(x)$, with the observed values of household expenditures y . The function which passes most of these tests can then be said to give the best description of the relationship between ν and η from the point of view of the available observations x and y .

In the present analysis the following tests have been used in evaluating the chosen functions:

- 1) Test for number of runs above and below the curve and test for length of run.
- 2) Durbin and Watson's d -test¹⁸).
- 3) F-test for the ratio between the variance in the distribution of deviations from the curve and the variance within groups,
- 4) χ^2 -test for normality of residuals.

¹⁷) Cf. J. Aitchison and J. A. C. Brown (1) p. 75.

¹⁸) Cf. Durbin and Watson: (4).

Moreover, the coefficient of correlation, R , between observed and calculated expenditures was computed to give a rough indication of closeness of fit; it should also be mentioned that the estimate of the standard deviation, s_b , of the parameter estimate b makes it possible by means of a t -test to test in a simple manner the hypothesis $\beta = 0$ against the alternative hypothesis $\beta \neq 0$.

In the following a brief description of the various tests will be given.

2. Test for number of runs and for length of run and the d -test.

If a given function expresses the true relationship between ν and η the observed deviations from this relationship shown by the observations x and y are of a purely random nature. In that case the number of runs of residuals with the same sign, runs below and above the curve, will follow a distribution¹⁹⁾ which is approximately normal when both the number of positive residuals, P , and that of negative residuals, Q , exceed 10, and in which mean value and standard deviation depend solely on the number of observations. Given the number of observations, therefore, significance limits for runs above and below the curve can be estimated. Similarly, limits of significance can be derived for the longest run²⁰⁾. If the upper limit of significance is exceeded by the test for the number of runs (which is analogous with the lower limit of significance being exceeded by the test for the longest run), this means that the residuals change signs "too often"; this may be caused by a negative correlation between two successive observations. Since such a hypothesis is not relevant for the present survey, moderate transgressions of the upper limit of significance are not considered important. On the other hand, transgressions of the lower limit of significance (or for the second test, the occurrence of too long a run) must be considered more important because this may mean that the calculated curve deviates systematically from the observations over greater or smaller parts of the range of variation.

These tests give the same result whether the residuals in question are large or small; the tests respond only to their signs. Durbin and Watson's²¹⁾ d -test has been designed so as to cover both the sign and the size of the residuals. The test is based on the quantity d , which is defined as $d = \frac{\sum (t_k - t_{k-1})^2}{\sum t_k^2}$, where $t_k = y_k - f(x_k)$. A high d -value means frequent changes of signs, and the transgression of the upper limit of significance is thus evidence in favour of a hypothesis of negative correlation between successive observations, a hypothesis which, as already mentioned, is not considered relevant in this case. A low d -value, on the other hand, indicates too few changes of signs, and the transgression of the lower limit of significance will therefore tend to substantiate that the model in question does not express the true functional relationship but deviates systematically from it. The limit of significance is given as a zone; d -values above and below that zone give clear evidence, whereas d -values within the zone do not allow of any universal conclusion.

¹⁹⁾ Cf. A. Hald (8), p. 346 and Prais and Houthakker (10), p. 53.

²⁰⁾ Cf. A. Hald (8), table 13.5, p. 348.

²¹⁾ Durbin and Watson (4).

The tests for runs and the d-test will, of course, point in the same direction since they aim at the same alternative hypothesis.

3. The F-test.

The F-test which compares the variance in the distribution of the residuals, $y - f(x)$, y being the average in the groups of three observations, with the average variance within the groups, is suitable as a test of different alternative hypotheses although in this case, too, only one limit of significance (the upper one) is relevant.

The test hypothesis is here again that the chosen model $\eta = f(x)$ expresses the true relationship between y and x and that the residuals, $y - f(x)$, are everywhere distributed with mean value 0 and variance $\sigma^2 = V\{y|x\}$. If the test hypothesis is correct, the estimate, s_2^2 , of the variance of the residuals will have the same true value as the estimate, s_1^2 , of the variance within groups, and the ratio between the two variances $F = \frac{s_2^2}{s_1^2}$ will follow a F-distribution.

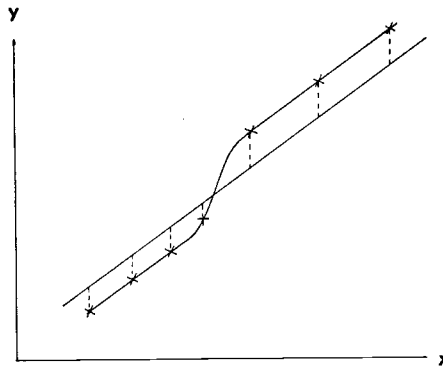


Figure IV, 3a. Type of systematic deviations which will be revealed by the run tests, and may be not by other tests.

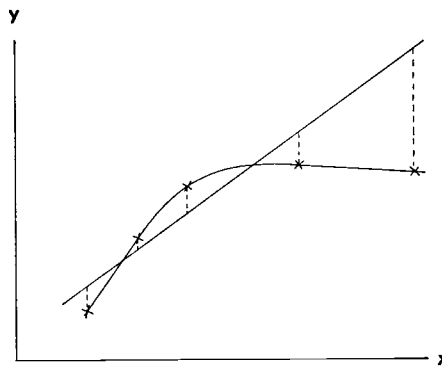


Figure IV, 3b. Type of systematic deviations which will be revealed by the F-test, but may be not by the run tests.

Significant F-values can now be cited in support of several alternative hypotheses. Firstly, the occurrence of the type of systematic deviations which is tested directly through run tests and the d-test will also manifest itself in significant F-values. However, the two types of tests do not measure the deviations from the hypothesis in the same way; confer the example below²²). The run tests may reveal a systematic tendency in the deviations of the observations from the curve—even if the individual deviations are very small as illustrated in figure IV,3a, where the true “relationship” has been plotted together with the chosen function. It is not certain that the F-test will be able to reveal such small systematic deviations. Conversely, it is possible that the run tests will not reveal the type of deviation between the observations and the chosen function which has been illustrated in figure IV,3b, whereas it is likely that this pattern of deviations will lead to significant results of the F-test.

However, if it is assumed that this type of alternative hypothesis is not relevant, significant F-values can, secondly, substantiate hypotheses which say something of the distribution of the stochastic element, ϵ . If the hypothesis of constant residual variance cannot be upheld then the ratio between s_2^2 , and s_1^2 will not follow a F-distribution, and significant F-values can then be taken as an expression of the fact that the assumptions of applying this test have not been fulfilled.

It is obvious that the F-test cannot be used without qualification in the case of the function types with untransformed expenditure, y , as dependent variable since it appeared from the analysis made that $V \{y|x\}$ was not constant; in that case this variance was expressed as a simple function of the level of expenditure namely $V \{y|x\} = \sigma^2 \cdot \eta^2 \simeq \sigma^2 [f(x)]^2$.

In estimating the parameters this variance assumption was taken into account, and it is therefore necessary to do the same thing here so that in the calculation of s_2^2 and s_1^2 the observations are weighted by their reciprocal variance, i.e., the same weight as was used in the parameter estimation $w = \frac{1}{[f(x)]^2}$ ²³).

Thus significantly high F-values may be taken to indicate that the chosen variance assumption $V \{y|x\} = \sigma^2 \cdot \eta^2$ has not been correct, but even then, there remains the alternative hypothesis that the chosen function deviates systematically from the “true” one, the hypothesis shown in figure IV, 3b.

²²) Cf. S. J. Prais and H. S. Houthakker (10) p. 52.

²³) As regards the actual calculation of s_2^2 and s_1^2 it should be mentioned that whereas s_2^2 can naturally always be taken direct from the regression analysis, the matter is a little more difficult as regards the estimation of s_1^2 . The estimate s_1^2 cannot be calculated direct in those cases where $\log y$ is the dependent variable as a number of individual observations are zero. However, it holds good, with good approximation, that $s_1^2 = s_1^2(\log y) \simeq M^2 \cdot \bar{c}^2$ where \bar{c}^2 is the average square of coefficient of variation in the distribution of y -values and $M = 0.4343$. The approximation is satisfactory if $\bar{c} < 0.3$, which is not always the case in our material.

In cases where the untransformed y -values appear as dependent variable it would be possible to form estimates of the inner variance s_1^2 direct on the basis of individual observations. But this does not become necessary since we have an estimate of the coefficient of variation which is also an estimate of σ_1^2 cp. (IV, 6).

4. The χ^2 -test.

To test the hypothesis that the residuals are normally distributed the χ^2 -test has been used. The observed deviations are grouped, and by comparing this grouped distribution of deviations with a normal distribution with the same mean value (0) and variance is it possible to calculate a χ^2 -quantity²⁴). Also in this case it will, of course, be necessary to insert the variance assumption used in the parameter estimation. Significantly high χ^2 -values support the alternative hypothesis that the deviations of the observations from the function are not normally distributed. Assuming that the tests referred to above have given insignificant results, it has, prior to the χ^2 -test, been possible to substantiate the following hypothesis: The relationship suggested by the observations does not deviate systematically from the proposed functional relationship (test for number of runs and size of longest run, d-test and F-test), and the deviations of the observations from this function have everywhere a variance which is of the same magnitude as the variance within groups (F-test). A significant χ^2 -value will then indicate that the residuals are not normally distributed.

This alternative hypothesis can be further specified in the present case. For if it is true—as the test calculations seem to show—that the residuals are normally distributed in the case of the functions where the dependent variable is a logarithmic transformation of the expenditure, then the deviations from the function in the cases where the dependent variable is the untransformed value of expenditure will be log-normally distributed. And this is true even if one uses the weighted calculation method in the parameter estimation. Use of weights in the calculation influences the magnitude of the residuals, but not the *form* of their distribution. One can therefore—with good support in the other results of the test calculations—advance the assertion that significant χ^2 -values support the alternative hypothesis that the residuals are log-normally distributed²⁵).

5. The coefficient of correlation.

A comparison of the calculated values of the coefficient of correlation for the different functions gives an impression of which of the five functions has the closest fit for each expenditure item in each group of wage and salary earners. However, such a direct comparison will only be possible if the residuals are at the same level, i.e., if the variance assumption used in the parameter estimation, $V \{y|x\} = \sigma^2 [f(x)]^2$, is also used here as a basis for assigning weights. In the following chapter the results of this calculation as well as of all the test calculations referred to above will be shown.

IV. f. Planning the Computation Programme and Carrying Out The Computations.

After specifying the five functional relationships and the procedure to be used in their estimation the practical part of the analysis can be started. This part comprises working out a detailed computation programme and the corresponding code for feeding it

²⁴) Chapter V, p. 60.

²⁵) Chapter V, p. 64.

into DASK²⁶) and (at length) computation and printing of parameter estimates and their standard errors and of test results—according to the tests referred to above.

It will be understood from what has been mentioned above that the computation programme is rather comprehensive. For each of the twelve groups of wage and salary earners into which the basic material has been divided, parameter estimates and their standard errors are to be calculated separately for five Engel functions for a total of 13 expenditure items. This gives a total of $12 \times 5 \times 13 = 780$ sets of calculations; the same number of tests have to be made. Coding the programme, therefore, was bound to require a great amount of work, and the fact that part of the computation programme (especially the part concerning the log-normal distribution and the regression analysis with the above-mentioned special variance assumptions and some of the test calculations) had not previously been performed on the DASK rendered the coding even more difficult.

In planning the computation programme it was, of course, necessary to consider how the individual operations were to be performed on the DASK, so that the computation programme could as far as possible be adapted to the capacity of that electronic computer. During the greater part of the analysis a very useful contact was established with the division concerned at the Danish Institute of Computing Machinery.

In the present investigation it had been attempted to guard against “unforeseen” difficulties by working out in advance, at the consumer survey section of The Statistical Department, examples of all the computing operations which were to be performed according to the computation programme.

In the course of this preliminary work several sources of error were traced, and several corrections had to be made in the computation programme and in the input tape—but nevertheless the performance of the computation programme on the DASK presented several unpleasant surprises, two of which deserve to be mentioned because they seem to be of a certain theoretical interest.

The code for the computation programme was made so flexible that the computation process could be stopped at all “vital” points and any necessary corrections be made without re-running the whole programme. As a control measure which could be expected to be very effective an inspection was introduced after all computations had been run for the first of the twelve groups of wage and salary earners. Hereby it was expected that the weak spots which might have escaped the attention during the calculations of the examples mentioned above would be revealed.

This inspection of the results of the computations for the first group of wage and salary earners showed that as regards function type (IV, 5) $\log \eta = \log \kappa + \log \Phi(a + \beta \log \nu)$ the computation programme did not work in five cases out of the thirteen expenditure items comprised by the programme because the iterative process for the calculation of estimates of κ , a and β did not converge²⁷). This made essential changes in the programme necessary, see below. For the other four functions the programme performance seemed fully satisfactory. Thanks to the flexibility of the code worked out for the computation programme the computations could be continued for these four functions, where the

²⁶) i. e. DAnsk Sekvens Kalkulator.

²⁷) Cf. J. Aitchison and J. A. C. Brown (1) p. 82, and above p. 47.

results for the first group of wage and salary earners had been found satisfactory, while the corrections to the programme for the fifth function type were considered. But in the course of the continued run of the accepted programme for the four linear functions it proved impossible in several cases as regards, functions (IV, 3) and (IV, 4), to obtain convergence in the estimation of the parameters.

The reason for the lack of convergence in the case of function (IV, 5), must be found in the fact that the variance $V \{y|x\}$ in the distribution of the observed expenditure values is so big that the functional relationship between x and y can be satisfactorily described by means of two parameters. Or in other words: With the given variance in the distribution of the observations one of the three parameters can be selected arbitrarily within a wide interval, and the other two parameters can be estimated conditioned by this arbitrarily selected parameter value without this causing any appreciable rise in the unexplained variance of y . The iterative procedure for determining the three parameter estimates becomes highly unstable, and in several cases convergence will consequently not be obtained. And equally regrettable: in the cases where the iteration process did converge the standard deviations of the parameter estimates were so great that the estimates had to be considered almost useless, cf. table IV,4, which shows the result of the computations for eight expenditure items in the group of higher public servants and salaried employees in the capital.

The table emphatically demonstrates that the results obtained are not very useful; the standard deviation of the estimates, apart from the food item, are of the same order as the estimates themselves.

The "solution" chosen to this problem consisted in arbitrarily fixing in advance a convenient value of the parameter β , namely unity; it should be added that unity does not significantly deviate from any of the calculated β -values in the three-parameter calculation, cf. table IV,4. This solution was also chosen by Aitchison and Brown in their analysis of British household budgets²⁸). However, it does not appear from their report whether they had previously experienced just as disappointing results in the three parameter estimation as was the case in the present analysis.

The reason for the cases of lacking convergence which occurred in the iterative calculation of parameter estimates in functions

$$(IV, 3) \quad \eta = a + \beta (\log v - \overline{\log v})$$

and

$$(IV, 4) \quad \eta = a + \beta \left(\frac{1}{v} - \bar{\frac{1}{v}} \right)$$

was to be found in an unfortunate property of the estimation procedure adopted. As mentioned above, p. 44, it was found that it applied to the functions in which the untransformed value y was the dependent variable that $V \{y|x\} = \sigma^2 \cdot \eta^2$ for which reason it was decided to carry through the regression analysis with $\frac{1}{[f(x)]^2}$ as weights; the test calculations were carried through in accordance with the same principle in order to

²⁸) Cf. J. Aitchison and J. A. C. Brown (1) p. 130.

Table IV,4. Parameter estimates in the three-parameter case of the function $\log \eta = \log \kappa + \log [\Phi (a + \beta \log \nu)]$. Higher public servants and salaried employees in the Capital.

Expenditure item	k	s _k	a	s _a	b	s _b
Dwelling.....			convergence not obtained			
Fuel and light.....	7600	63000	-4.796	1.1	0.793	1.4
Food.....	2900	950	-6.192	2.0	1.62	0.64
Tobacco.....	6700	820	-7.517	5.3	1.87	1.7
Clothing.....	3200	5600	-6.431	2.6	1.44	1.0
Foot wear.....	230	220	-5.084	4.8	1.34	1.6
Washing and cleaning.....	380	320	-7.414	3.3	1.84	1.1
Durables.....			convergence not obtained			
Personal hygiene.....	1500	4800	-5.460	2.2	1.10	1.1
Books, newspapers etc.....			convergence not obtained			
Sports, holidays, hobbies.....	13000	30000	-7.834	2.3	1.63	0.93
Transport (incl. own car).....			convergence not obtained			
Union fees, subscriptions etc.....	660	2100	-4.861	3.0	1.03	1.4

obtain everywhere as efficient estimators as practically possible. If, however $f(x)$ assumes very low values, in the extreme case zero, the weight factor for the value or values in question will completely dominate the calculation, and the iteration process, which was mentioned on p. 46, will become quite unstable. If the initial parameter estimates or one of the subsequent estimates in the iteration process results in such low values of $f(x)$, there will be a risk that the iteration will stop. The reason why the group of wage and salary earners which was computed first, viz. higher public servants and salaried employees, passed through without any stop as the only one of all the groups, is that the x -values in this group are rather large so that no $f(x)$ values came close to zero.

The correction which was made in the computation programme to enable calculations of parameter estimates to be made in the 41 cases (out of a total of 143) in the other 11 groups of wage and salary earners in which the iteration process did not converge consisted in rejecting the special variance assumption and estimating on the assumption of constant variance, i.e. applying the estimation procedure from the function types in which the logarithmically transformed expenditure, $\log y$, is the dependent variable. The parameter values and test values thus estimated are not efficient, but this estimation procedure were preferred rather than rejecting observations or omitting estimation²⁹⁾.

²⁹⁾ As mentioned p. 40, Prais and Houthakker, in their analysis, have everywhere used this estimation procedure.

Chapter V.

MAIN RESULTS.

Va. Introductory remarks.

In this chapter some of the main results of the Engel curve analysis will be discussed. In the next chapter some results of certain further calculations will be put forward, but as already mentioned the main object of this inquiry has been the Engel curve analysis.

In the appendix is shown, separately for each of the twelve groups of wage and salary earners, parameter estimates and test results for five Engel functions (I,1) to (I,5) of the thirteen expenditure items.

However, it may perhaps be natural to try to boil down the somewhat overwhelming abundance of figures in these tables.¹⁾ How can the result of the analysis be summed up?

In doing so the point of departure will be taken in the description of the object of the analysis which was given in chapter IV, p. 31, where the object was characterized by the following two steps:

- 1) calculation of parameter estimates in the selected models and
- 2) testing these models by various tests for goodness of fit.

It will first be examined whether the tests for goodness of fit point in the same direction or, in other words, whether it is possible to classify the selected Engel functions on this basis. Then follows an interpretation of the calculated parameter values.

The following tests were used (cf. p. 48 above):

- 1) The coefficient of correlation between calculated and observed values (no proper test, but the size of the coefficient of correlation can be taken as a measure of the "closeness of fit" of the relations proposed.²⁾
- 2) Test for number of runs.
- 3) Test for longest run (where a run is defined as a series of positive or negative deviations between calculated and observed values).
- 4) The d-test, in which both the signs and the numerical values of the deviations enter.
- 5) The F-test, the ratio of the variance within groups and the variance of the residuals.
- 6) The χ^2 -test for the normality of the distribution of the residuals.

The proper tests fall into three categories. 1°. The χ^2 -test is concerned with the form of the distribution of the deviations between the calculated and the observed values, the test hypothesis being that these deviations are normally distributed. 2°. The variance-

¹⁾ More specifically 5 (models) \times 12 (groups of wage and salary earners) \times 13 (expenditure items) = 780 less 19 cases in which the estimation procedure failed (model (I,5)) = 761 sets of parameter estimates and corresponding test results.

²⁾ Cf. Prajs & Houthakker, (10), p. 95.

ratio test, the F-test, compares the deviations of the observed group averages from the calculated function values with the variation within the groups. The test hypothesis is that the corresponding variances are equal, and a significant F-value is interpreted as an indication that the calculated Engel function deviates systematically from the "true" Engel curve.³ The tests for number of runs and length of runs and the d-test, measure systematic tendencies in the signs of the deviations. The test hypothesis states that there is no such systematic tendency, and that the signs of successive deviations change at random. In the following the result of these tests will be examined separately for each test.

However, it may be disclosed already here that the double-logarithmic function clearly stands out as the one of the five selected functions which gives the best fit for almost all expenditure items. A considerable part of the following interpretation of the results will therefore be based on the estimated parameters of this function.

Vb. Examination of test results.

1. Coefficient of correlation, *R*, between calculated and observed values.

Table V,1 shows the calculated coefficients of correlation between the observed and calculated expenditures.

The *R*-values have been calculated by one of the following formulae:

$$(V,1) \quad R_1 = \frac{\sum_j^N (\log y_j - \overline{\log y}) (\log Y_j - \overline{\log Y})}{\sqrt{\sum_j^N (\log y_j - \overline{\log y})^2 \cdot \sum_j^N (\log Y_j - \overline{\log Y})^2}}$$

$$(V,2) \quad R_2 = \frac{\sum_j^N (y_j - \bar{y}) (Y_j - \bar{Y}) w_j}{\sqrt{\sum_j^N (y_j - \bar{y})^2 w_j \cdot \sum_j^N (Y_j - \bar{Y})^2 w_j}}$$

where *N* is the number of groups of three observations in the given social group, $Y_j = f(x_j)$ for the models with untransformed dependent variable and $\log Y_j = f(x_j)$ for the models with logarithmically transformed dependent variable; $w_j = \frac{1}{[f(x_j)]^2}$.

Formula (V,1) was used in the case of the Engel functions in which $\log y$ is the dependent variable and formula (V,2) in the case of the other functions, in which the untransformed *y* values appear as dependent variables.³⁾

³⁾ For estimation purposes the function $\eta = \alpha \Phi (\alpha + \beta \log v)$ was logarithmically transformed into the form $\log \eta = \log \alpha + \log \Phi (\alpha + \beta \log v)$ and all test calculations, accordingly, were based on the deviations of logarithmically transformed expenditures. The *R*-values in table V,1, however are based on formula (V,2).

Table V.1. Correlation coefficients (R × 100)

	Groups of wage and salary earners*											
	1	2	3	4	5	6	7	8	9	10	11	12
	(1) $\log y = a + b (\log x - \overline{\log x})$											
Dwelling	79	81	79	80	83	80	81	68	77	81	80	70
Fuel & light	76	49	57	34	69	51	36	60	57	45	76	78
Food	82	84	94	91	89	88	86	89	85	92	94	88
Tobacco	53	51	72	72	57	60	73	80	65	70	82	71
Clothing	80	84	84	80	84	86	83	76	86	82	84	83
Footwear	54	70	70	69	56	69	79	67	81	69	64	66
Washing & cleaning	76	77	72	75	75	78	75	71	77	78	82	67
Durables excl. vehicles	36	46	67	56	54	55	64	72	58	52	52	54
Personal hygiene	82	84	77	78	83	80	85	88	86	76	87	81
Books, newspapers etc.	81	74	76	75	76	81	76	84	70	74	80	75
Sports, holidays, hobbies	90	84	91	84	90	89	91	85	85	83	86	77
Transport incl. own car	41	56	72	70	62	64	66	70	65	73	76	51
Union fees, subscriptions etc.	69	66	90	85	85	81	88	90	81	88	93	80
	(2) $\log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$											
Dwelling	75	78	76	78	82	79	80	64	74	82	77	65
Fuel & light	74	46	62	41	68	49	45	54	61	57	79	74
Food	83	84	92	87	87	86	84	86	83	88	91	89
Tobacco	53	54	69	66	56	59	71	78	68	67	78	67
Clothing	79	82	86	78	82	85	83	76	81	81	79	83
Footwear	54	67	72	68	55	68	79	69	77	68	61	66
Washing & cleaning	77	77	66	67	69	75	71	68	69	74	77	54
Durables excl. vehicles	40	47	65	58	51	56	65	73	60	52	57	58
Personal hygiene	80	87	78	80	81	80	86	86	82	73	83	80
Books, newspapers etc.	76	74	73	71	73	77	76	83	68	73	79	80
Sports, holidays, hobbies	89	82	88	83	88	85	92	83	81	77	83	70
Transport incl. own car	46	56	69	67	59	64	64	63	60	71	67	41
Union fees, subscriptions etc.	68	68	87	85	84	79	87	83	77	82	90	79
	(3) $y = a + b (\log x - \overline{\log x})$											
Dwelling	73	76	79	79	87	84	79	63	74	81	77	63
Fuel & light	77	57	66	57	74	62	56	52	77	63	82	72
Food	84	86	93	90	88	89	85	88	86	90	93	90
Tobacco	62	67	72	73	69	70	73	84	81	67	68	71
Clothing	79	84	87	85	85	84	82	87	83	83	79	80
Footwear	58	69	70	74	58	72	80	72	79	70	68	67
Washing & cleaning	77	76	70	74	74	82	73	73	72	73	78	53
Durables excl. vehicles	46	58	63	72	51	47	74	82	76	21	66	66
Personal hygiene	79	84	78	86	84	76	86	90	84	78	86	82
Books, newspapers etc.	66	75	73	78	80	42	75	91	68	69	82	90
Sports, holidays, hobbies	85	45	84	85	75	74	91	78	77	41	81	71
Transport incl. own car	46	47	71	70	65	64	62	55	45	73	16	37
Union fees, subscriptions etc.	67	72	88	88	86	81	89	82	79	82	90	83

between observed and calculated expenditures.

	Groups of wage and salary earners*)											
	1	2	3	4	5	6	7	8	9	10	11	12
	(4) $y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{\bar{x}} \right)$											
Dwelling	70	67	76	78	85	81	73	50	66	79	67	64
Fuel & light	73	53	69	53	69	56	60	42	73	66	80	71
Food	81	83	89	84	79	83	80	79	79	84	87	87
Tobacco	62	72	67	60	63	67	69	78	74	64	73	66
Clothing	74	68	84	83	80	80	78	86	76	77	69	72
Footwear	57	66	73	75	60	72	78	79	71	71	66	64
Washing & cleaning	75	72	65	67	70	76	69	71	66	65	73	46
Durables excl. vehicles	47	62	60	71	48	52	73	77	76	42	68	66
Personal hygiene	75	78	79	88	79	73	85	84	79	73	80	76
Books, newspapers etc.	56	70	68	70	76	41	73	85	67	66	75	88
Sports, holidays, hobbies	79	69	77	81	69	65	84	71	74	50	52	64
Transport incl. own car	54	42	67	66	60	58	24	52	46	65	45	29
Union fees, subscriptions etc.	63	67	81	86	83	74	84	78	86	72	88	78

	(5) $\log y = \log k + \log \Phi (a + \log x)$											
Dwelling	72	78	78	79	83	81	80	65	76	27	75	59
Fuel & light	76	56	64	55	73	62	53	53	74	59	80	71
Food	84	85	94	-	88	89	85	89	86	-	94	90
Tobacco	62	65	72	-	68	68	70	79	76	66	47	70
Clothing	79	-	84	82	-	84	82	76	61	-	82	82
Footwear	58	69	69	73	58	71	79	68	81	68	68	67
Washing & cleaning	77	76	71	76	72	79	71	69	73	76	79	52
Durables excl. vehicles	43	50	66	68	48	46	-	74	58	59	59	60
Personal hygiene	79	83	76	81	82	76	84	88	86	78	86	83
Books, newspapers etc.	70	75	73	79	72	79	74	-	71	21	82	84
Sports, holidays, hobbies	64	55	67	-	-	65	89	-	-	56	58	-
Transport incl. own car	34	43	42	66	55	-	-	-	30	-	-	34
Union fees, subscriptions etc.	67	72	49	61	84	82	89	62	80	86	-	82

*) 1. Higher public servants and salaried employees. The capital. 2. Lower public servants and salaried employees. The capital. 3. Skilled workers. The capital. 4. Unskilled workers. The capital. 5. Higher public servants and salaried employees. Provincial towns. 6. Lower public servants and salaried employees. Provincial towns. 7. Skilled workers. Provincial towns. 8. Unskilled workers. Provincial towns. 9. Lower public servants and salaried employees. Rural districts. 10. Skilled workers. Rural districts. 11. Unskilled workers. Rural districts. 12. Agricultural workers. Rural districts.

When considering the R-values given in table V,1 it must be borne in mind that the R-values from the two formulae cannot be compared directly. Whether one of the formulae generally leads to systematically higher or lower R-values than the other one is, however, very difficult to determine. According to an unpublished paper by Theil, which is referred to in Prais and Houthakker⁴), the logarithmic form seems to result in higher R-values than does the use of untransformed y observations, but Theil's calculations do not aim at weighted R-values calculated by means of formula (V,2), and his conclusions do not, therefore, apply to our case. Since the "transformation" effect seems to be moderate even when unweighted calculations are used, any systematic differences in the R-values caused by this transformation effect will be disregarded in the following.

Going through the table item by item it will be found that none of the five Engel functions stands out as the best in all cases, but on the other hand it is noteworthy that the double-logarithmic function more frequently than any of the other functions has the highest R-value. Counting for each of the 13 categories of expenditures the number of social groups (out of a total of 12) in which this function has the highest R-value, it ranks first in 6 categories and tied first in another 3.

It must be emphasized, however, that, considered in isolation, the R-values shown are not a suitable criterion for deciding which of the five functions offers the best description of the observations.

For one thing, the above-mentioned reservations regarding comparisons between the R-values of the different types of functions must be taken into account, and for another it must again be emphasized that the R-test is no proper test, since it is not possible to set up test hypotheses as regards closeness of fit, which may be accepted or rejected at a given significance level for R.

2. The χ^2 -test.

By grouping the differences, t , between the observed and calculated expenditures into k groups of size $\frac{1}{3} s_t$, where s_t^2 is the variance in the distribution of these differences or residuals, the following grouped distribution will appear:

<i>Interval</i>	<i>Number of residuals</i>
$t < - 2 s_t$	n_1
$- 2 s_t < t < - 1\frac{2}{3} s_t$	n_2
.	.
.	.
.	.
.	.
$t > 2 s_t$	n_k

⁴) Cf. J. Prais and H. S. Houthakker (10), p. 96.

This distribution is then compared with a similarly grouped, normal distribution with the same mean and variance and with the same number of elements as the empirical distribution, $L = \sum_j^k n_j$.

For each group in the distribution of the residuals is calculated the difference $(n_j - L \theta_j)$ between number of elements in the empirical and in the theoretical, normal distribution, θ_j denoting the expected frequency in the j 'th group. The quantity

$$\sum_j^k (n_j - L \theta_j)^2 / L \theta_j$$

will then be approximately χ^2 -distributed with $k-m-1$ degrees of freedom if the test hypothesis concerning the normality of the residuals is correct; m is the number of parameters in the given Engel function, and k is the number of groups after it has been ensured, through a suitable merging of too small groups, that everywhere

$$L \theta_j > 5$$

The χ^2 -values have been shown in table V,2.

It holds good for all expenditure items, with the exception of the expenditure on tobacco, that the functions in which $\log y$ is the dependent variable, show the fewest significant χ^2 -values.

In the case of several expenditure items there is only one social group out of twelve which has significant χ^2 -values for these two functions.

It thus seems evident that this test points to one of the logarithmically transformed Engel functions as being the best—which of them must be left open until further evidence can be put forward as they do almost equally well.

In the case of the types of function in which the untransformed value, y , appears as the dependent variable it does not seem possible, however, to accept the hypothesis that t is normally distributed. It has been taken into account, in connection with the parameter estimation and subsequent tests for these two functions, that the variance of y (for given x) increases with the value of y . According to the investigations made this relationship could be described with good approximation by the formula

$$V \{y | x\} = \sigma^2 [f(x)]^2 \cong s_t^2 [f(x)]^2,$$

wherefore the tests have been based on the quantity:

$$t = \frac{f(x) - y}{f(x)}$$

Hereby was obtained that the variance in the distribution of t could be considered constant and independent of x . On the other hand, as the χ^2 -tests show, the distributions obtained are evidently not normal.

It will be found, however, that the results achieved correspond closely to what was to be expected from the discussion on this subject in chapter IV, p. 52. It was concluded

	Groups of wage and salary earners*											
	1	2	3	4	5	6	7	8	9	10	11	12
	(1) $\log y = a + b (\log x - \overline{\log x})$											
Dwelling.....	9.9	5.2	4.5	4.1	5.4	9.4	7.0	4.6	5.0	0.6	3.8	6.6
Fuel & light.....	6.3	25.5	10.8	8.0	3.8	15.0	6.5	3.1	23.4	2.3	17.7	8.6
Food.....	8.9	10.2	1.4	8.5	7.1	5.8	3.9	2.8	2.8	9.4	9.0	5.8
Tobacco.....	21.5	30.4	6.1	14.6	9.0	32.0	5.8	4.9	11.1	5.9	11.7	1.6
Clothing.....	8.4	8.5	8.7	13.3	6.4	6.1	2.1	6.3	5.5	3.0	3.6	3.2
Footwear.....	10.0	9.4	9.8	7.3	5.0	10.9	2.8	1.6	6.7	8.6	8.9	2.4
Washing & cleaning.....	12.4	13.8	5.0	9.0	3.8	5.6	0.7	7.8	3.7	1.1	9.0	8.8
Durables excl. vehicles.....	3.9	8.2	12.8	9.7	14.6	12.6	2.2	3.2	6.8	3.7	7.3	1.3
Personal hygiene.....	5.1	9.1	6.6	5.0	6.8	7.0	3.7	7.0	14.9	11.5	3.8	5.6
Books, newspapers etc.....	10.6	9.9	0.9	3.4	7.4	12.6	4.3	2.8	8.3	1.5	5.2	6.1
Sports, holidays, hobbies...	6.0	3.9	4.5	3.3	10.2	8.5	5.0	14.0	4.6	7.1	5.2	1.8
Transport incl. own car....	25.7	40.3	3.8	5.1	6.7	7.1	0.5	4.6	5.7	4.9	4.0	9.7
Union fees, subscriptions etc.	3.3	10.2	5.0	8.0	4.1	6.4	13.0	4.9	8.1	6.6	10.6	3.5
95% significance level.....	14.1	16.9	11.1	14.1	11.1	14.1	7.8	11.1	14.1	7.8	14.1	7.8
Degrees of freedom.....	7	9	5	7	5	7	3	5	7	3	7	3
	(2) $\log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$											
Dwelling.....	5.9	4.8	3.3	9.9	4.6	3.9	5.3	3.2	7.7	4.1	7.4	2.6
Fuel & light.....	7.4	16.7	4.5	13.7	6.0	9.6	9.6	3.2	20.7	2.9	8.2	4.7
Food.....	8.8	10.9	0.9	7.3	14.0	4.3	0.9	1.8	2.4	2.2	2.9	1.8
Tobacco.....	12.9	38.8	1.1	14.6	10.6	18.9	6.4	9.7	13.4	3.7	10.6	0.6
Clothing.....	9.2	3.2	6.6	19.7	5.4	7.7	3.2	9.4	4.5	6.2	1.5	6.3
Footwear.....	6.5	5.8	3.6	7.0	3.0	15.5	5.4	1.9	9.0	3.7	11.7	2.9
Washing & cleaning.....	8.6	7.8	5.0	9.8	13.5	5.8	1.2	1.4	3.5	3.2	10.5	1.7
Durables excl. vehicles.....	11.5	9.1	5.5	15.0	10.6	17.6	0.1	3.0	2.9	2.6	7.8	1.1
Personal hygiene.....	6.6	7.0	8.4	8.8	4.5	10.4	2.7	2.9	6.2	10.6	14.1	5.3
Books, newspapers etc.....	11.6	6.7	3.0	9.0	6.7	15.2	4.1	3.3	8.1	1.3	8.8	3.3
Sports, holidays, hobbies...	4.6	5.0	4.0	2.2	3.0	2.5	0.4	4.2	6.7	2.3	4.9	6.6
Transport incl. own car....	16.9	34.0	5.1	7.6	11.8	8.7	1.2	3.4	5.5	2.0	4.5	8.1
Union fees, subscriptions etc.	6.9	8.2	4.2	10.2	11.9	7.0	7.8	3.1	10.8	2.2	9.4	3.4
95% significance level.....	14.1	16.9	11.1	14.1	11.1	14.1	7.8	11.1	14.1	7.8	14.1	7.8
Degrees of freedom.....	7	9	5	7	5	7	3	5	7	3	7	3
	(3) $y = a + b (\log x - \overline{\log x})$											
Dwelling.....	17.2	18.9	2.7	12.9	12.6	13.5	2.5	12.9	9.5	5.4	11.3	4.9
Fuel & light.....	14.8	22.0	8.0	8.8	3.6	3.8	6.9	12.0	8.1	3.3	17.7	20.2
Food.....	13.1	10.7	7.1	3.5	12.5	4.0	1.4	1.5	8.9	3.4	5.9	0.8
Tobacco.....	3.6	11.6	6.9	10.7	6.5	6.0	6.9	3.8	12.5	5.4	-	3.2
Clothing.....	4.3	13.2	2.4	9.0	7.1	11.9	2.6	9.0	19.4	0.5	15.2	2.1
Footwear.....	8.6	13.8	18.9	2.2	3.1	21.5	2.7	4.7	13.9	4.0	5.1	8.5
Washing & cleaning.....	10.4	25.1	13.5	14.7	8.0	14.3	1.3	11.1	14.0	7.0	31.4	3.2
Durables excl. vehicles.....	39.6	60.0	3.9	5.4	31.9	91.1	9.3	10.4	38.8	-	26.2	2.7
Personal hygiene.....	17.7	36.9	1.8	13.9	13.8	36.9	4.8	7.9	18.6	4.5	27.4	3.6

χ^2 -test.

	Groups of wage and salary earners*)											
	1	2	3	4	5	6	7	8	9	10	11	12
Books, newspapers etc.	50.5	19.7	8.1	18.2	16.8	-	11.0	3.9	43.2	4.8	13.9	4.0
Sports, holidays, hobbies. . .	8.6	-	14.2	9.7	-	-	2.8	-	-	-	35.7	29.5
Transport incl. own car.	80.6	229.8	44.3	60.7	64.5	100.5	7.8	52.5	-	4.1	-	34.4
Union fees, subscriptions etc.	8.6	14.2	2.7	11.7	8.3	8.5	5.4	1.8	11.5	1.5	12.7	3.2
95% significance level.	14.1	16.9	11.1	14.1	11.1	14.1	7.8	11.1	14.1	7.8	14.1	7.8
Degrees of freedom.	7	9	5	7	5	7	3	5	7	3	7	3

$$(4) y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$$

Dwelling.	27.8	33.4	6.8	-	4.8	14.7	8.2	-	14.1	8.4	-	3.3
Fuel & light.	25.0	34.8	9.6	10.9	1.2	12.4	10.3	11.8	14.7	3.3	20.7	3.2
Food.	8.0	10.1	1.7	9.7	33.4	9.0	3.1	4.6	14.8	5.9	33.7	8.3
Tobacco.	15.0	-	12.8	-	11.9	12.2	5.5	18.3	-	3.9	-	2.9
Clothing.	7.6	-	1.0	9.4	9.9	19.5	1.1	8.0	47.1	8.2	-	-
Footwear.	5.8	14.9	13.3	8.3	10.7	27.3	1.3	5.8	22.1	3.8	3.6	7.0
Washing & cleaning.	7.6	29.6	16.9	16.7	17.9	37.7	5.5	31.9	36.8	19.3	42.1	10.0
Durables excl. vehicles.	42.1	-	-	9.8	32.7	98.4	11.8	19.6	47.0	52.0	-	5.1
Personal hygiene.	35.7	54.5	6.9	8.6	12.4	42.7	8.1	12.2	-	6.4	30.6	9.3
Books, newspapers etc.	68.4	-	13.7	32.6	25.1	-	9.2	6.6	-	10.0	28.1	2.4
Sports, holidays, hobbies. . .	35.1	-	20.7	16.8	-	-	13.6	25.2	-	-	-	45.4
Transport incl. own car.	86.4	-	56.7	77.4	47.2	-	-	88.7	-	5.5	-	44.3
Union fees, subscriptions etc.	23.1	31.5	3.3	-	23.4	18.4	10.3	-	-	11.0	-	13.1
95% significance level.	14.1	16.9	11.1	14.1	11.1	14.1	7.8	11.1	14.1	7.8	14.1	7.8
Degrees of freedom.	7	9	5	7	5	7	3	5	7	3	7	3

$$(5) \log y = \log k + \log \Phi (a + \log x)$$

Dwelling.	6.0	6.6	7.5	7.9	2.5	6.5	3.5	4.5	2.8	25.2	4.2	2.9
Fuel & light.	6.3	20.8	8.4	8.8	4.1	15.0	2.3	4.1	20.9	1.4	14.5	10.6
Food.	13.7	9.4	2.4	-	9.0	10.7	0.6	4.8	8.9	-	14.8	3.6
Tobacco.	19.0	32.7	4.9	-	8.8	25.5	5.6	8.5	11.0	2.8	14.0	0.1
Clothing.	12.3	-	4.4	12.5	-	8.4	1.5	5.2	34.5	-	4.2	3.1
Footwear.	13.2	10.1	9.4	9.1	2.0	14.2	5.5	1.8	4.9	4.2	9.1	4.0
Washing & cleaning.	9.1	12.4	6.1	11.6	5.8	9.6	0.5	7.4	8.8	2.5	8.1	4.9
Durables excl. vehicles.	5.9	8.9	4.8	10.4	13.0	13.5	-	2.2	7.9	3.1	7.9	1.9
Personal hygiene.	8.7	8.2	7.2	2.7	8.4	7.1	3.4	1.9	13.8	5.9	5.0	13.7
Books, newspapers etc.	9.4	8.4	2.3	3.6	9.8	11.8	7.8	-	8.9	23.2	11.2	6.6
Sports, holidays, hobbies. . .	19.6	16.4	6.3	-	-	8.0	3.7	-	-	8.8	11.3	-
Transport incl. own car.	29.3	39.3	3.6	6.0	8.8	-	-	-	3.7	-	-	10.3
Union fees, subscriptions etc.	4.4	12.0	29.4	23.4	2.1	10.5	7.9	34.5	11.2	2.3	-	3.0
95% significance level.	14.1	16.9	11.1	14.1	11.1	14.1	7.8	11.1	14.1	7.8	14.1	7.8
Degrees of freedom.	7	9	5	7	5	7	3	5	7	3	7	3

*) 1. Higher public servants and salaried employees. The Capital. 2. Lower public servants and salaried employees. The Capital. 3. Skilled workers. The Capital. 4. Unskilled workers. The Capital. 5. Higher public servants and salaried employees. Provincial towns. 6. Lower public servants and salaried employees. Provincial towns. 7. Skilled workers. Provincial towns. 8. Unskilled workers. Provincial towns. 9. Lower public servants and salaried employees. Rural districts. 10. Skilled workers. Rural districts. 11. Unskilled workers. Rural districts. 12. Agricultural workers. Rural districts.

there that the logarithmically transformed dependent variables, $\log y$, were normally distributed, and the untransformed values, y , consequently followed the log-normal distribution; a graphic presentation of these two cases has been given in fig. V,1a and V,1b, where the "true" Engel curves have also been drawn.

Expenditure on a given item

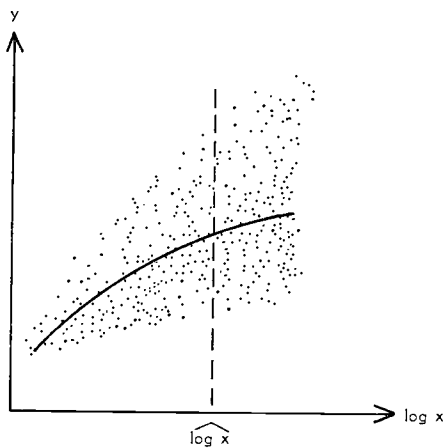


Fig. V, 1a

Expenditure on a given item

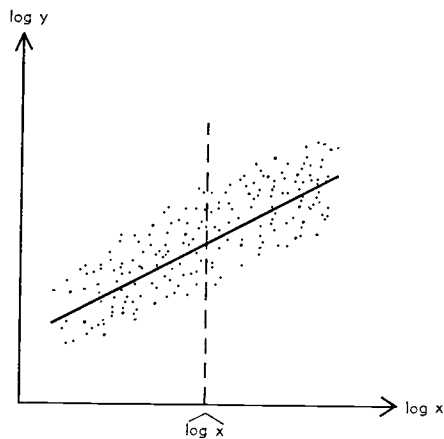


Fig. V, 1b

The distributions of residuals from the Engel curve for a given value of x can be illustrated as shown in fig. V,2a and V,2b.

The distribution of residuals according to fig. V,1b is normal and independent of the chosen value of x . The distribution according to V,1a is log-normal and the variance in the distribution, $V\{y | x\}$, increases proportionally to $[f(x)]^2$. The correction made in the tests consisted, as mentioned, in a division of all $[f(x) - y]$ by $f(x)$. Thereby it is achieved that the variance in the corrected distribution of residuals becomes independent of x , but the *distribution form* of the deviations is not changed.

We are thus in the situation of having set up an explicit alternative hypothesis which may be assumed to hold good for the functions for which the test hypothesis was rejected.

3. The F-test.

The variance-ratio test has been calculated as the ratio of the variance of the deviations of the group averages from the calculated values to the variance within groups,

$$F \cong \frac{s_2^2}{s_1^2}, \text{ say.}$$

The number of degrees of freedom for the two independent variance estimates are $L-2$ for s_2^2 , where L is the number of group averages, and $\sum_{j=1}^L (n_j - 1) = 2L$ for s_1^2 ,

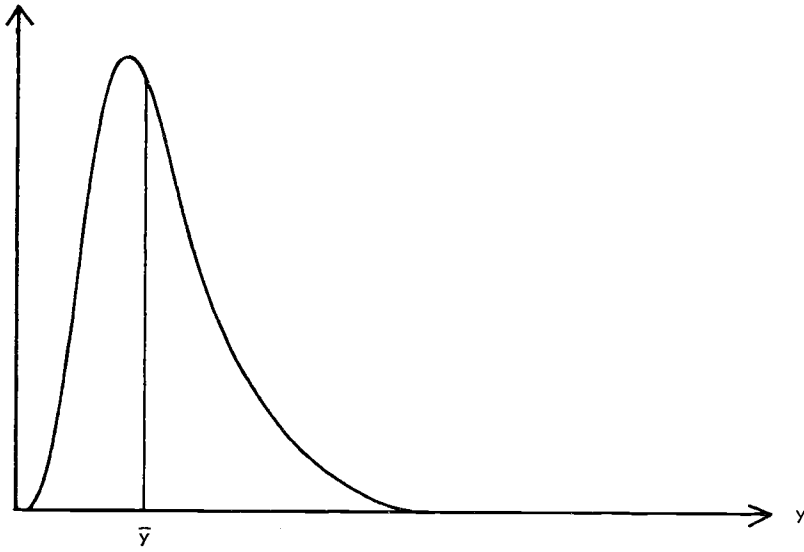


Fig. V, 2a Frequency of expenditure for given income

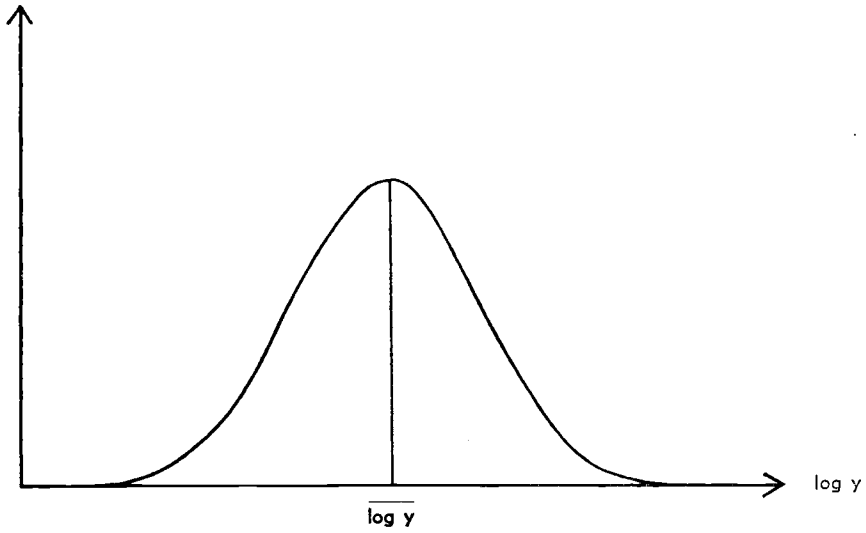


Fig. V, 2b Frequency of log expenditure for given income

Table V,3. F-test for linearity (ratio between

	Groups of wage and salary earners*											
	1	2	3	4	5	6	7	8	9	10	11	12
	(1) $\log y = a + b(\log x - \overline{\log x})$											
Dwelling.....	1.45	1.24	1.05	1.14	0.73	1.09	0.83	1.38	0.92	0.95	1.31	1.31
Fuel & light.....	1.30	1.31	0.78	1.92	0.96	1.00	1.21	1.12	1.25	1.30	1.04	1.36
Food.....	1.59	1.19	0.67	1.02	0.78	1.20	1.67	0.83	1.12	1.17	0.85	1.21
Tobacco.....	1.57	1.43	1.27	1.32	1.45	1.56	1.07	0.86	1.60	0.81	1.22	1.56
Clothing.....	1.20	1.14	1.23	1.06	1.07	1.10	1.25	1.84	0.95	0.99	1.47	1.54
Footwear.....	1.19	0.99	0.96	0.92	1.34	1.10	0.79	1.46	0.79	0.89	1.25	0.87
Washing & cleaning.....	1.10	1.02	1.27	1.11	1.29	1.22	1.52	1.21	1.08	0.99	0.91	2.07
Durables excl. vehicles.....	2.16	1.89	1.76	1.80	2.10	1.97	2.88	1.33	1.85	2.41	1.75	1.39
Personal hygiene.....	1.05	1.37	1.25	1.30	0.89	1.47	0.68	1.01	0.94	1.48	0.94	0.94
Books, newspapers etc.....	0.97	1.46	0.91	1.39	1.48	1.14	1.04	0.90	1.37	1.20	0.96	1.54
Sports, holidays, hobbies.....	0.91	1.20	0.86	1.14	0.95	1.28	0.87	1.20	1.55	1.51	1.23	1.54
Transport incl. own car.....	2.84	2.50	2.04	1.36	2.97	2.24	1.92	1.85	2.05	1.91	1.54	2.20
Union fees, subscriptions etc...	1.17	1.29	0.77	1.28	0.95	1.50	0.66	0.91	1.38	1.05	0.79	1.27
95% significance level.....	1.31	1.26	1.36	1.41	1.40	1.31	1.48	1.40	1.32	1.48	1.34	1.47
f_1 ...	110	152	81	66	68	109	49	68	100	49	91	51
f_2 ...	224	308	166	136	140	222	102	140	204	102	186	106
	(2) $\log y = a + b\left(\frac{1}{x} - \frac{1}{\bar{x}}\right)$											
Dwelling.....	1.69	1.43	1.16	1.21	0.76	1.15	0.89	1.55	1.02	0.91	1.51	1.47
Fuel & light.....	1.37	1.36	0.70	1.81	0.99	1.04	1.10	1.25	1.18	1.11	0.93	1.58
Food.....	1.53	1.18	0.87	1.48	0.93	1.40	1.86	1.03	1.22	1.66	1.20	1.17
Tobacco.....	1.56	1.38	1.38	1.54	1.48	1.60	1.15	0.94	1.51	0.87	1.43	1.74
Clothing.....	1.26	1.26	1.14	1.13	1.16	1.23	1.29	1.82	1.32	1.05	1.85	1.55
Footwear.....	1.19	1.07	0.90	0.96	1.36	1.13	0.82	1.42	0.95	0.92	1.33	0.88
Washing & cleaning.....	1.09	1.04	1.47	1.38	1.53	1.38	1.71	1.29	1.38	1.13	1.13	2.65
Durables excl. vehicles.....	2.08	1.88	1.86	1.73	2.19	1.92	2.78	1.30	1.80	2.38	1.61	1.31
Personal hygiene.....	1.14	1.37	1.18	1.22	1.00	1.49	0.63	1.10	1.18	1.64	1.20	0.95
Books, newspapers etc.....	1.21	1.45	1.01	1.59	1.65	1.39	1.03	0.95	1.47	1.21	1.01	1.25
Sports, holidays, hobbies.....	0.98	1.35	1.16	1.21	1.09	1.63	0.85	1.34	1.92	1.99	1.53	1.91
Transport incl. own car.....	2.70	2.50	2.20	1.47	3.15	2.25	1.99	2.18	2.30	2.03	2.02	2.47
Union fees, subscriptions etc...	1.20	1.24	0.98	1.32	1.06	1.60	0.75	1.47	1.69	1.47	1.13	1.33
95% significance level.....	1.31	1.26	1.36	1.41	1.40	1.31	1.48	1.40	1.32	1.48	1.34	1.47
f_1 ...	110	152	81	66	68	169	49	68	100	49	91	51
f_2 ...	224	308	166	136	140	222	102	140	204	102	186	106
	(3) $y = a + b(\log x - \overline{\log x})$											
Dwelling.....	1.63	1.61	1.03	1.30	0.64	1.07	0.87	1.65	1.04	1.02	1.58	1.63
Fuel & light.....	1.18	1.11	0.68	1.18	0.84	0.75	0.87	1.40	0.74	1.03	0.91	1.72
Food.....	1.38	1.10	0.75	1.23	0.92	1.21	1.81	0.94	1.15	1.35	0.94	1.09
Tobacco.....	0.96	0.83	1.29	1.26	1.00	1.10	1.12	0.89	1.21	0.98	-	1.72
Clothing.....	1.11	1.25	1.09	0.95	1.07	1.46	1.24	1.74	1.39	0.84	2.02	1.78
Footwear.....	1.07	1.00	1.03	0.81	1.32	1.09	0.76	1.38	0.94	0.78	1.02	0.91
Washing & cleaning.....	1.00	1.10	1.43	1.19	1.61	1.33	1.73	1.72	1.47	1.37	1.21	3.24
Durables excl. vehicles.....	2.16	1.96	1.71	1.24	2.75	2.60	2.57	1.33	1.96	-	1.77	1.24
Personal hygiene.....	1.21	1.66	1.23	1.21	0.99	2.09	0.66	1.22	1.36	1.34	1.17	1.00
Books, newspapers etc.....	1.77	1.42	1.12	1.64	1.74	-	1.29	0.88	2.38	1.39	1.14	1.15

variances "around" the regression and variances within groups).

	Groups of wage and salary earners*)											
	1	2	3	4	5	6	7	8	9	10	11	12
Sports, holidays, hobbies.....	1.10	-	1.62	1.39	-	-	1.19	-	-	-	2.53	3.10
Transport incl. own car.....	4.14	5.34	2.50	2.24	3.03	3.72	2.09	4.39	-	1.96	-	3.49
Union fees, subscriptions etc...	1.26	1.16	0.84	1.33	1.05	1.60	0.61	1.40	1.76	1.38	1.22	1.28
95% significance level.....	1.31	1.26	1.36	1.41	1.40	1.31	1.48	1.40	1.32	1.48	1.34	1.47
$f_1...$	110	152	81	66	68	169	49	68	100	49	91	51
$f_2...$	224	308	166	136	140	222	102	140	204	102	186	106
(4) $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$												
Dwelling.....	2.07	2.39	1.37	-	0.95	1.18	1.20	-	1.47	1.26	-	1.83
Fuel & light.....	1.55	1.28	0.67	1.32	1.12	1.83	0.85	1.70	1.00	1.02	1.20	2.02
Food.....	1.81	1.43	1.42	2.33	1.71	1.18	2.53	1.73	1.79	2.60	2.22	1.60
Tobacco.....	1.08	-	1.73	-	1.36	1.57	1.53	1.41	-	1.22	-	2.25
Clothing.....	1.48	-	1.44	1.34	1.67	1.16	1.73	2.38	2.28	1.29	-	-
Footwear.....	1.15	1.15	1.04	0.91	1.41	1.76	0.90	1.50	1.42	0.85	1.21	1.00
Washing & cleaning.....	1.21	1.38	1.96	1.69	2.42	0.88	2.20	2.48	2.15	2.20	1.82	4.53
Durables excl. vehicles.....	2.41	-	-	1.45	3.01	1.69	3.21	1.86	2.14	6.03	-	1.31
Personal hygiene.....	1.60	2.36	1.30	1.39	1.48	1.41	0.79	2.21	-	1.99	2.09	1.43
Books, newspapers etc.....	3.13	-	1.44	2.51	2.40	2.02	1.59	1.44	-	1.63	1.70	1.28
Sports, holidays, hobbies.....	1.99	-	2.86	2.01	-	1.78	2.50	2.84	-	-	-	4.99
Transport incl. own car.....	3.73	-	3.40	2.96	3.34	1.80	-	6.38	-	3.10	-	4.34
Union fees, subscriptions etc...	1.51	1.45	1.44	-	1.67	1.60	1.01	-	-	2.61	-	1.90
95% significance level.....	1.31	1.26	1.36	1.41	1.40	1.31	1.48	1.40	1.32	1.48	1.34	1.47
$f_1...$	110	152	81	66	68	169	49	68	100	49	91	51
$f_2...$	224	308	166	136	140	222	102	140	204	102	186	106
(5) $\log y = \log k + \log \Phi (a + \log x)$												
Dwelling.....	1.45	1.24	1.04	1.11	0.75	1.07	0.80	1.38	0.91	8.07	1.31	1.29
Fuel & light.....	1.29	1.32	0.74	1.85	0.95	1.00	1.14	1.12	1.21	1.21	0.97	1.35
Food.....	1.47	1.13	0.65	-	0.76	1.16	1.61	0.80	1.07	-	0.84	1.11
Tobacco.....	1.54	1.40	1.27	-	1.43	1.54	1.05	0.85	1.55	0.79	3.83	1.54
Clothing.....	1.16	-	1.16	1.04	-	1.08	1.20	1.79	7.35	-	1.49	1.49
Footwear.....	1.17	0.98	0.91	0.90	1.33	1.09	0.77	1.42	0.79	0.87	1.24	0.85
Washing & cleaning.....	1.06	1.00	1.29	1.15	1.31	1.21	1.52	1.20	1.10	0.99	0.91	2.14
Durables excl. vehicles.....	2.11	1.86	1.73	1.75	2.07	1.94	-	1.30	1.82	2.33	1.70	1.33
Personal hygiene.....	1.03	1.32	1.19	1.25	0.89	1.41	0.63	0.98	0.93	1.44	0.94	0.90
Books, newspapers etc.....	0.99	1.42	0.91	1.38	1.47	1.14	1.02	-	1.36	8.37	0.94	1.39
Sports, holidays, hobbies.....	3.48	3.42	2.86	-	-	2.89	0.82	-	-	2.58	2.52	-
Transport incl. own car.....	2.79	2.47	2.82	1.34	2.91	-	-	-	2.64	-	-	2.21
Union fees, subscriptions etc...	1.15	1.26	1.78	8.14	0.93	1.45	0.64	20.7	1.38	1.06	-	1.24
95% significance level.....	1.31	1.26	1.36	1.41	1.40	1.31	1.48	1.40	1.32	1.48	1.34	1.47
$f_1...$	110	152	81	66	68	169	49	68	100	49	91	51
$f_2...$	224	308	166	136	140	222	102	140	204	102	186	106

*) 1. Higher public servants and salaried employees. The Capital. 2. Lower public servants and salaried employees. The Capital. 3. Skilled workers. The Capital. 4. Unskilled workers. The Capital. 5. Higher public servants and salaried employees. Provincial towns. 6. Lower public servants and salaried employees. Provincial towns. 7. Skilled workers. Provincial towns. 8. Unskilled workers. Provincial towns. 9. Lower public servants and salaried employees. Rural districts. 10. Skilled workers. Rural districts. 11. Unskilled workers. Rural districts. 12. Agricultural workers. Rural districts.

Groups of wage and salary earners*)

1 2 3 4 5 6 7 8 9 10 11 12

(1) $\log y = a + b (\log x - \overline{\log x})$

Dwelling	49 46.3	76 65.5	36 26.6	38 33.2	35 27.3	48 46.0	24 18.9	34 27.5	44 41.9	25 19.4	44 37.9	21 20.3
Fuel & light	63 46.3	67 65.4	33 26.3	38 32.9	39 27.3	63 45.2	25 19.3	32 27.3	42 40.4	19 19.4	39 37.9	27 20.1
Food	63 46.5	73 65.5	35 26.8	40 32.9	36 27.6	48 45.8	28 19.4	32 27.6	42 41.9	29 18.9	46 37.9	27 20.1
Tobacco	47 45.8	70 61.7	37 26.7	50 33.4	43 27.3	54 44.7	18 19.3	43 27.2	47 41.7	21 18.7	49 37.8	26 20.1
Clothing	48 46.1	81 65.6	40 26.5	46 32.6	36 27.7	60 45.9	30 19.3	35 27.5	55 41.9	22 19.3	46 37.7	28 20.3
Footwear	58 46.2	66 65.6	33 26.7	43 32.9	34 27.7	57 46.0	29 19.4	45 27.7	51 41.8	28 19.1	47 37.6	22 20.3
Washing & cleaning	45 46.3	81 65.2	37 26.8	33 33.4	36 27.5	45 46.0	21 19.4	35 27.6	48 41.9	30 19.4	46 37.6	23 20.3
Durables excl. vehicles	59 46.5	81 65.6	36 26.3	49 33.2	39 26.9	47 46.0	23 19.3	33 27.6	50 41.8	23 19.4	45 37.9	30 20.3
Personal hygiene	57 46.4	95 65.4	35 26.8	41 33.4	37 27.6	51 45.7	21 19.1	33 27.5	64 41.6	21 18.6	51 37.9	32 19.8
Books, newspapers etc.	61 45.9	70 65.4	34 26.7	40 33.4	36 27.3	59 45.9	29 19.4	35 27.6	45 41.8	28 19.4	47 37.9	31 20.3
Sports, holidays, hobbies	55 46.4	76 65.6	34 26.7	49 33.2	41 27.6	62 45.6	28 19.3	36 27.3	58 41.9	29 19.3	51 37.9	22 20.2
Transport incl. own car	49 46.1	80 62.9	36 26.8	51 33.2	39 27.5	45 45.7	23 19.4	39 27.6	50 41.9	30 19.4	49 37.9	25 20.3
Union fees, subscriptions etc.	58 46.5	72 64.9	36 26.5	31 32.9	35 27.7	60 46.0	32 18.2	34 27.5	51 41.8	20 19.3	36 37.9	22 19.8

(2) $\log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$

Dwelling	49 46.3	69 65.6	35 26.8	39 33.2	37 27.5	48 46.0	28 19.4	30 27.6	45 41.7	25 19.3	35 37.9	21 20.1
Fuel & light	57 46.3	67 65.5	33 26.5	42 32.4	37 27.3	61 45.4	26 19.1	28 27.3	43 40.7	20 19.4	40 37.9	19 20.1
Food	52 46.5	70 65.1	31 26.8	33 33.3	28 27.2	53 46.0	28 19.4	29 27.7	43 41.9	25 19.4	35 37.9	29 20.2
Tobacco	51 45.5	72 61.3	37 26.7	43 33.4	42 27.2	55 43.7	19 19.3	41 26.9	55 41.4	25 18.9	51 37.8	25 20.3
Clothing	48 45.8	77 65.6	33 26.8	43 32.9	33 27.3	55 45.8	28 19.4	36 27.3	39 41.9	25 18.9	45 37.9	29 20.3
Footwear	62 46.1	70 65.6	34 26.8	42 32.9	33 27.7	58 46.0	28 19.4	45 27.7	49 41.9	28 19.1	47 37.4	23 20.1
Washing & cleaning	44 46.2	80 64.9	33 26.7	33 33.4	35 27.3	43 46.0	25 19.4	33 27.5	43 41.7	30 19.3	39 37.6	23 20.3
Durables excl. vehicles	62 46.3	82 65.6	35 26.5	48 32.9	39 27.2	57 46.0	26 19.4	34 27.7	50 41.9	23 19.3	44 37.9	35 20.3
Personal hygiene	55 46.4	87 65.6	37 26.7	44 33.4	29 27.7	54 45.8	19 19.4	34 27.6	43 41.6	22 18.9	40 37.7	24 20.1
Books, newspapers etc.	51 45.9	72 65.6	33 26.7	35 33.4	35 27.5	51 45.8	25 19.3	31 27.7	37 41.6	26 19.4	40 37.8	28 20.3
Sports, holidays, hobbies	49 46.5	65 65.5	29 26.8	45 33.4	31 27.6	41 46.0	26 19.4	37 27.6	49 41.9	25 19.1	45 37.9	24 20.3
Transport incl. own car	54 46.2	82 63.5	34 26.6	43 33.1	40 27.6	50 45.8	23 19.4	33 27.7	49 41.9	29 19.1	39 37.6	23 20.2
Union fees, subscriptions etc.	61 46.3	78 65.1	33 26.7	30 32.9	28 26.9	51 45.8	29 18.6	25 27.7	51 41.9	21 19.4	37 37.8	25 20.1

(3) $y = a + b (\log x - \overline{\log x})$

Dwelling	45 45.5	70 64.3	37 26.7	37 32.9	39 27.7	52 44.7	26 19.4	36 26.6	43 41.2	23 19.4	35 37.0	23 19.1
Fuel & light	57 46.1	65 64.3	31 26.6	42 33.3	35 27.7	63 46.0	24 19.4	30 26.6	47 41.9	20 19.4	44 36.4	21 17.8
Food	56 46.4	76 65.6	33 26.8	37 33.4	32 27.2	53 46.0	28 19.3	29 27.5	42 41.6	26 19.4	43 37.7	31 20.5
Tobacco	55 46.3	74 65.5	37 26.5	49 32.9	38 27.6	61 46.0	23 18.6	41 27.5	52 41.0	23 18.3	39 37.0	25 19.6
Clothing	58 46.2	73 64.8	33 26.8	41 32.9	33 27.7	51 44.7	28 19.1	34 26.6	29 41.0	23 19.4	49 37.6	27 20.0
Footwear	60 46.5	70 64.8	35 26.0	47 33.4	32 27.3	60 45.6	31 19.1	45 27.6	49 41.8	28 19.3	49 37.9	22 19.1
Washing & cleaning	48 46.3	78 65.5	31 25.7	33 31.8	35 26.9	49 44.7	23 19.3	33 26.9	41 41.0	26 18.2	35 34.8	23 20.0
Durables excl. vehicles	64 44.4	80 61.3	35 26.3	48 33.2	29 25.5	35 35.0	25 18.2	31 27.3	44 38.4	17 16.9	42 36.0	31 19.5
Personal hygiene	51 45.8	84 63.2	34 26.7	39 32.9	35 27.3	52 43.4	21 19.1	32 27.5	37 39.3	22 19.3	44 37.4	26 20.5
Books, newspapers etc.	53 43.6	74 64.5	33 25.7	37 33.1	35 26.6	45 44.7	26 18.6	35 27.3	35 39.3	24 18.6	42 37.4	28 19.6
Sports, holidays, hobbies	51 45.8	63 64.5	25 26.5	43 32.6	37 26.9	37 42.0	21 19.3	29 26.3	39 40.1	13 16.9	31 36.4	18 16.0
Transport incl. own car	43 40.1	66 52.3	31 25.4	33 27.7	34 25.9	40 37.2	17 18.2	27 24.6	37 35.0	23 19.4	17 23.1	21 16.7
Union fees, subscriptions etc.	57 46.1	72 65.6	31 26.8	28 33.3	28 26.6	53 45.0	31 18.9	25 27.6	51 41.2	19 19.3	35 37.2	25 20.5

for number of runs.*)

	Groups of wage and salary earners*)											
	1	2	3	4	5	6	7	8	9	10	11	12
	(4) $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$											
Dwelling.....	43 45.3	55 63.2	27 26.5	37 32.4	27 27.5	39 44.7	24 18.2	28 25.1	35 40.7	17 18.2	31 37.0	23 19.5
Fuel & light.....	49 44.0	61 63.5	36 26.6	36 33.3	33 27.5	63 45.0	24 19.4	21 26.9	33 41.0	24 19.1	40 35.7	17 19.8
Food.....	50 45.8	60 65.4	19 26.8	19 33.3	20 23.4	35 45.2	21 18.6	27 27.5	33 41.0	17 19.4	27 35.4	23 19.5
Tobacco.....	61 45.9	72 65.4	31 25.4	37 31.8	30 25.9	49 44.7	23 19.1	31 26.3	53 40.7	25 18.0	39 35.4	21 19.5
Clothing.....	49 46.2	61 63.5	29 26.8	33 33.4	25 27.2	43 43.7	21 19.3	31 26.6	23 39.7	19 18.9	37 33.8	23 18.8
Footwear.....	58 46.5	65 64.9	34 26.3	39 33.4	35 27.6	55 45.0	29 19.3	37 26.6	43 41.0	25 19.4	45 37.9	21 19.5
Washing & cleaning.....	46 46.2	61 64.8	31 26.0	29 31.4	29 25.5	35 42.5	19 18.2	32 26.3	37 39.3	17 16.9	31 34.3	21 19.1
Durables excl. vehicles.....	62 43.6	71 61.7	31 25.0	42 32.9	27 25.1	36 37.2	23 17.8	21 26.9	41 37.9	15 16.3	42 35.4	33 19.5
Personal hygiene.....	37 43.2	57 59.9	35 26.6	35 32.6	27 26.9	39 42.0	17 19.3	21 27.2	33 38.9	21 19.4	31 37.0	16 19.1
Books, newspapers etc.....	49 43.6	74 62.4	25 24.6	37 32.4	21 25.9	31 37.2	23 19.1	27 26.9	35 36.3	24 19.1	35 37.0	27 19.8
Sports, holidays, hobbies.....	39 44.7	55 61.7	15 26.3	31 33.2	33 26.3	33 38.0	19 18.2	27 25.1	33 35.7	15 16.3	27 30.7	17 17.3
Transport incl. own car.....	45 41.2	58 51.5	29 25.0	29 27.7	28 24.6	36 35.0	17 15.7	23 25.1	27 31.4	21 18.9	19 24.1	17 16.0
Union fees, subscriptions etc.	51 46.1	74 65.1	23 26.6	26 33.2	29 25.9	47 45.2	27 19.4	21 26.6	39 40.1	15 18.2	29 35.7	23 19.1
	(5) $\log y = \log k + \log \Phi (a + \log x)$											
Dwelling.....	53 46.1	78 65.6	37 26.5	38 33.2	34 27.3	48 46.0	26 19.1	32 27.5	44 41.9	11 18.6	44 37.9	21 20.3
Fuel & light.....	61 46.1	67 65.1	33 26.5	38 33.1	37 27.5	65 45.0	25 19.3	32 27.3	42 40.4	19 19.4	41 37.7	27 20.1
Food.....	63 46.4	75 65.4	34 26.8	- -	34 27.7	53 46.0	28 19.4	34 27.5	44 41.9	- -	46 37.8	29 20.3
Tobacco.....	49 45.9	70 62.1	37 26.7	- -	43 27.2	54 44.1	18 19.3	44 26.9	51 41.4	25 19.0	33 37.9	28 20.3
Clothing.....	50 46.3	- -	40 26.5	48 32.9	- -	60 45.9	30 19.3	38 27.5	31 40.1	- -	46 37.8	28 20.2
Footwear.....	60 46.2	66 65.6	33 26.7	41 32.9	34 27.7	58 46.0	29 19.4	45 27.7	49 41.9	28 19.1	47 37.4	22 20.3
Washing & cleaning.....	46 46.2	82 65.1	37 26.8	31 33.2	36 27.6	46 46.0	21 19.4	35 27.6	47 41.9	30 19.4	48 37.7	23 20.3
Durables excl. vehicles.....	61 46.5	81 65.6	36 26.3	49 33.2	39 27.2	47 46.0	- -	33 27.7	52 41.6	23 19.4	45 37.9	30 20.2
Personal hygiene.....	55 46.4	94 65.5	36 26.7	41 33.4	39 27.7	49 46.0	23 19.4	33 27.6	62 41.6	21 18.6	47 37.9	32 19.8
Books, newspapers etc.....	59 45.9	70 65.5	34 26.7	38 33.4	36 27.3	55 46.0	29 19.4	- -	42 41.8	15 19.4	49 37.9	31 20.3
Sports, holidays, hobbies.....	33 46.1	51 65.6	29 26.8	- -	- -	45 46.0	28 19.4	- -	- -	- -	27 19.4	33 37.8
Transport incl. own car.....	51 46.2	82 62.8	37 26.8	51 33.2	39 27.5	- -	- -	- -	42 41.9	- -	- -	25 20.3
Union fees, subscriptions etc.	63 46.5	76 64.3	15 26.6	25 32.9	35 27.6	62 46.0	34 18.6	19 27.5	53 41.8	22 19.4	- -	24 20.1

*) The italicized figures denotes the lower 5 per cent significance level. 1. Higher public servants and salaried employees. The Capital. 2. Lower public servants and salaried employees. The Capital. 3. Skilled workers. The Capital. 4. Unskilled workers. The Capital. 5. Higher public servants and salaried employees. Provincial towns. 6. Lower public servants and salaried employees. Provincial towns. 7. Skilled workers. Provincial towns. 8. Unskilled workers. Provincial towns. 9. Lower public servants and salaried employees. Rural districts. 10. Skilled workers. Rural districts. 11. Unskilled workers. Rural districts. 12. Agricultural workers. Rural districts.

where $n_j = 3$ is the number of observations in the j 'th group. s_2^2 has been calculated direct on the basis of $[\log y - f(x)]$ and $\left(\frac{y - f(x)}{f(x)}\right)$, respectively for the two types of functions.

As regards s_1^2 it has not been possible to use the individual observations in those cases where $\log y$ was the dependent variable since several observations of y were zero. This zero-observation problem, which was apparently solved by the grouping of the observations, cf. the discussion in chapter IV, p. 36, thus crops up again here in a new form. However, for moderate values of the average coefficient of variation $\bar{c}^2 = \frac{1}{L} \sum \frac{s_{y_j}^2}{\bar{y}_j^2}$

s_1^2 , may be found with good approximation as $s_1^2(\log y) \cong M^2 \cdot \bar{c}^2$, where $M = 0.4343$.

This approximation is not very good in those cases where $\bar{c} > 0.5$, which must be borne in mind when the F-tests are examined below. For the three items *durable goods*, *transport* and *sports, holidays and hobbies* several of the calculated c -values are very high, and the F-test in the case of these items must be considered dubious as regards the two functions in which $\log y$ is the dependent variable.

As regards the functions in which y is the dependent variable, s_1^2 has everywhere been considered equal to \bar{c}^2 , since

$$s_1^2(y) = \frac{\sum_{j=1}^L \frac{1}{(n_j - 1)} \sum_{k=1}^{n_j} \frac{(y_{jk} - \bar{y}_j)^2}{\bar{y}_j^2}}{L} = \frac{1}{L} \sum_{j=1}^L \frac{s_{y_j}^2}{\bar{y}_j^2} = \bar{c}^2$$

The F-values thus calculated have been shown in table V,3.

Can any conclusions be drawn from the F-test as regards the determination of the "best" algebraic formulation(s) of the Engel curve? Does the F-test point in the same direction as the χ^2 -test and as the more dubious evidence from the correlation coefficients, i.e. to one of those functions in which the dependent variable is the logarithmically transformed expenditure, $\log y$?

As mentioned above there are several reservations to be made. For expenditure items where $\bar{c} > 0.5$ the F-test fades away as regards functions (I,1), (I,2), and (I,5). As regards functions (I,3) and (I,4), parameter estimation has failed in some cases, cf. chapter IV, p. 54, and, accordingly, no testing is possible. These cases coincide with the invalidation due to high \bar{c} -values of the F-test performed on functions (I,1) and (I,2). Despite these substantial reservations concerning the results shown in the table, it nevertheless seems justified to draw the general conclusion that the F-test confirms the conclusion of the χ^2 -test to the effect that the logarithmic functions pass the tests more easily than the functions in which the expenditure enters untransformed.

For all expenditure items (except footwear) there are significant F-values for all five types of functions in one or more of the twelve social groups, but in the case of functions (I,1) and (I,2) frequently only one of the social groups gives significance. For the double-logarithmic function (I,1) the items of *dwelling and footwear* and *fuel and lighting* thus show

only one significant F-value, *food, house-cleaning* and *washing*, two significant values, and *clothing, personal hygiene, sports, holidays and hobbies*, and *subscriptions, etc.* three significant values. The semi-logarithmic function (I,2) displays somewhat poorer results. For the remaining functions the number of significant results are much higher.

4. The test for number of runs and for the longest run; the d-test.

The last category of tests for goodness of fit tests the hypothesis that the observations are distributed at random around the Engel curve determined by the calculated parameter estimates.

The two runs test consist of a simple count of the number of changes of signs (test for number of runs, here called N-test) and of the number of elements in the longest run (test for longest run, here called l-test). A run is accordingly defined as a series of successive deviations with the same sign, cf. e.g. A. Hald, (8), p. 342, and Prais and Houthakker, (10), pp. 53-55. If the number of positive and negative deviations, P and Q, respectively are greater than about ten, D is approximately normally distributed

with mean

$$M \{D\} \simeq 1 + \frac{2 PQ}{N}$$

and variance

$$V \{D\} \simeq \frac{2 PQ (2 PQ - N)}{N^2 (N - 1)}$$

where D is number of runs and N the number of observations.

If the test hypothesis is rejected in favour of an alternative hypothesis because the calculated Engel curve "misses the mark" in a greater or smaller area of the field of observation, then the number of runs will be too small.

The limit of significance can be chosen as

$$M \{D\} - 2 \sqrt{V \{D\}}$$

In table V,4 the number of runs, D, and the limit of significance thus found has been shown.

The test for the longest run, the l-test, which is not, of course, unrelated to the N-test, is derived from the knowledge of the distribution function for the number of runs of a given length.

In table V,6 all the longest runs have been shown, and for each set of calculations has been given the 5 per cent significance limit.

A glance at table V,4 and table V,5 will show how the results of the two tests of runs correspond so that they should rightly be considered as one test. The conclusion of this combined test points in the same direction as the previous tests: among the five relations tested the two in which the dependent variable is a logarithmic transformation of the expenditure performs better than the other three functions, and as indicated also by the F-test, the double-logarithmic function seems to do best.

The final test, the d-test, is based on the following quantity, cf. Prais and Houthakker (10), page 53

$$d = \frac{\sum (t_k - t_{k-1})^2}{\sum t_k^2}$$

Table V,5. 1-test for

Groups of wage and salary earners*)												
	1	2	3	4	5	6	7	8	9	10	11	12
	(1) $\log y = a + b (\log x - \overline{\log x})$											
Dwelling	9	8	4	5	6	7	6	6	10	7	5	6
Fuel & light	7	7	4	7	5	9	5	7	9	10	10	6
Food	8	7	4	5	8	6	5	13	11	6	6	7
Tobacco	7	9	5	6	4	6	6	5	8	6	6	8
Clothing	7	9	5	6	4	6	4	7	12	5	6	5
Footwear	10	9	6	7	5	6	6	6	5	5	8	5
Washing & cleaning	9	6	5	7	5	10	7	7	7	4	9	8
Durables excl. vehicles	10	5	5	7	7	7	5	10	6	9	6	5
Personal hygiene	11	5	6	6	5	9	6	5	7	7	8	6
Books, newspapers etc.	7	8	6	6	6	6	5	5	6	4	6	4
Sports, holidays, hobbies	6	5	6	4	7	6	8	7	5	5	6	7
Transport incl. own car	10	7	4	4	6	8	6	7	8	5	9	6
Union fees, subscriptions etc.	6	9	7	9	7	6	5	8	6	6	6	6
95% significance level	11	12	10	10	10	11	9	10	11	9	10	9
	(2) $\log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$											
Dwelling	9	11	6	6	6	7	4	6	8	5	11	6
Fuel & light	7	7	4	7	5	9	5	10	9	10	14	14
Food	8	8	6	11	8	9	7	9	9	6	9	5
Tobacco	7	9	5	6	4	6	6	5	6	5	8	6
Clothing	8	9	6	5	6	6	5	7	15	4	7	4
Footwear	10	9	6	6	5	6	6	6	10	5	8	6
Washing & cleaning	12	6	4	7	7	10	7	7	7	4	9	6
Durables excl. vehicles	8	6	5	7	7	7	4	10	5	9	7	4
Personal hygiene	11	6	5	6	5	6	6	5	8	5	10	7
Books, newspapers etc.	11	8	11	7	6	8	4	6	7	4	6	6
Sports, holidays, hobbies	6	15	13	6	8	8	6	5	12	7	6	7
Transport incl. own car	9	6	5	7	6	8	6	7	9	5	10	8
Union fees, subscriptions etc.	6	6	6	10	8	12	6	7	7	6	6	4
95% significance level	11	12	10	10	10	11	9	10	11	9	10	9
	(3) $y = a + b (\log x - \overline{\log x})$											
Dwelling	11	18	6	6	6	7	7	6	8	7	11	7
Fuel & light	10	10	4	7	5	8	5	10	9	10	14	19
Food	8	8	6	8	8	9	7	9	9	6	6	5
Tobacco	6	9	6	6	5	7	6	6	6	8	8	6
Clothing	7	9	6	6	6	7	5	9	15	4	8	7
Footwear	10	11	8	5	5	6	6	5	10	5	5	6
Washing & cleaning	12	6	9	7	7	10	7	6	7	4	9	8
Durables excl. vehicles	8	6	5	6	8	13	6	10	11	9	7	4
Personal hygiene	12	10	5	6	5	10	6	5	15	5	8	7
Books, newspapers etc.	9	8	11	7	6	9	4	6	12	8	7	8
Sports, holidays, hobbies	7	14	14	8	9	13	8	12	16	16	14	7
Transport incl. own car	14	11	7	12	12	9	10	11	14	9	20	9
Union fees, subscriptions etc.	10	9	6	11	8	12	5	8	7	6	6	4
95% significance level	11	12	10	10	10	11	9	10	11	9	10	9

the longest run.

Groups of wage and salary earners*)												
	1	2	3	4	5	6	7	8	9	10	11	12
	(4) $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$											
Dwelling.....	11	21	11	6	12	11	7	7	14	12	11	9
Fuel & light.....	10	11	4	10	8	11	6	16	19	9	15	19
Food.....	8	20	13	17	26	21	16	10	23	13	17	10
Tobacco.....	6	9	9	10	10	11	6	8	8	9	10	10
Clothing.....	12	17	7	10	13	18	7	11	24	9	9	7
Footwear.....	10	16	8	7	5	5	6	6	14	6	9	6
Washing & cleaning.....	12	15	9	15	13	17	7	10	10	7	13	8
Durables excl. vehicles.....	8	9	8	9	13	13	7	11	11	11	7	4
Personal hygiene.....	15	31	6	9	10	18	7	17	15	6	12	19
Books, newspapers etc.....	10	8	11	7	11	17	5	10	13	8	14	8
Sports, holidays, hobbies.....	11	14	20	11	9	15	8	10	15	16	14	7
Transport incl. own car.....	14	13	7	14	13	9	10	11	14	12	22	12
Union fees, subscriptions etc.....	10	8	15	11	8	16	6	14	14	15	15	8
95% significance level.....	11	12	10	10	10	11	9	10	11	9	10	9
	(5) $\log y = \log k + \log \Phi (a + \log x)$											
Dwelling.....	9	8	4	5	6	7	4	6	10	24	5	6
Fuel & light.....	7	7	4	7	5	9	5	7	9	10	11	6
Food.....	8	7	6	-	8	7	5	9	10	-	6	5
Tobacco.....	7	9	5	-	4	6	6	5	8	5	18	4
Clothing.....	7	-	5	5	-	6	4	7	17	-	6	5
Footwear.....	10	9	6	7	5	6	6	6	9	5	8	5
Washing & cleaning.....	9	6	5	10	5	10	7	7	7	4	9	8
Durables excl. vehicles.....	6	5	5	7	7	7	-	10	5	9	6	5
Personal hygiene.....	11	5	6	6	5	5	6	5	7	7	8	6
Books, newspapers etc.....	7	8	6	7	6	6	5	-	6	14	6	4
Sports, holidays, hobbies.....	21	23	13	-	-	12	8	-	-	7	19	-
Transport incl. own car.....	9	7	5	4	6	-	-	-	11	-	-	6
Union fees, subscriptions etc.....	6	9	17	18	7	5	5	20	6	6	-	4
95% significance level.....	11	12	10	10	10	11	9	10	11	9	10	9

*) 1. Higher public servants and salaried employees. The Capital. 2. Lower public servants and salaried employees. The Capital. 3. Skilled workers. The Capital. 4. Unskilled workers. The Capital. 5. Higher public servants and salaried employees. Provincial towns. 6. Lower public servants and salaried employees. Provincial towns. 7. Skilled workers. Provincial towns. 8. Unskilled workers. Provincial towns. 9. Lower public servants and salaried employees. Rural districts. 10. Skilled workers. Rural districts. 11. Unskilled workers. Rural districts. 12. Agricultural workers. Rural districts.

Table V,6. d-test for size and

	Groups of wage and salary earners*)											
	1	2	3	4	5	6	7	8	9	10	11	12
	(1) $\log y = a + b (\log x - \overline{\log x})$											
Dwelling.....	2.24	1.76	2.03	1.83	2.15	1.77	2.34	1.77	1.82	1.65	1.91	1.57
Fuel & light.....	2.03	1.97	1.86	1.43	2.26	2.20	1.42	1.64	1.59	1.43	1.54	1.95
Food.....	1.85	2.06	2.11	1.76	1.97	1.90	2.46	1.80	1.57	2.03	1.74	1.58
Tobacco.....	1.86	1.91	1.70	2.06	2.35	2.14	1.57	2.44	2.04	2.12	2.28	2.26
Clothing.....	1.69	2.12	2.04	2.08	1.94	2.08	2.44	2.32	2.21	1.95	2.43	2.05
Footwear.....	2.07	1.91	1.94	2.30	1.86	2.05	2.10	2.11	2.19	1.94	2.17	1.83
Washing & cleaning.....	1.65	1.84	1.69	1.89	1.56	1.68	1.74	1.90	1.79	2.30	1.93	1.43
Durables excl. vehicles.....	2.21	2.07	2.07	1.95	2.22	1.84	2.20	1.99	1.92	1.50	1.76	2.33
Personal hygiene.....	1.84	2.27	2.06	1.83	1.97	1.81	1.69	1.62	2.36	1.47	1.96	1.78
Books, newspapers etc.....	2.04	1.91	2.05	1.88	2.00	1.80	2.14	2.06	1.85	1.55	1.79	1.52
Sports, holidays, hobbies.....	2.14	1.87	2.26	1.97	2.15	2.15	1.90	2.28	2.34	2.37	2.02	1.34
Transport incl. own car.....	1.98	1.96	2.34	2.54	2.41	1.87	1.86	1.79	1.97	2.23	1.85	1.83
Union fees, subscriptions etc....	2.10	1.85	2.04	1.59	2.13	2.13	2.64	1.57	2.21	1.87	1.86	1.85
	(2) $\log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$											
Dwelling.....	1.89	1.52	1.84	1.75	2.04	1.69	2.14	1.58	1.66	1.72	1.66	1.40
Fuel & light.....	1.94	1.90	2.07	1.52	2.22	2.13	1.54	1.49	1.69	1.67	1.71	1.67
Food.....	1.81	2.03	1.62	1.23	1.72	1.65	2.18	1.47	1.45	1.32	1.23	1.65
Tobacco.....	1.87	1.98	1.58	1.77	2.29	2.09	1.50	2.24	2.18	1.95	1.95	2.02
Clothing.....	1.59	1.90	2.23	1.94	1.80	1.88	2.36	2.27	1.58	1.84	1.91	2.04
Footwear.....	2.06	1.76	2.06	2.19	1.83	1.99	2.05	2.10	1.85	1.86	2.03	1.83
Washing & cleaning.....	1.60	1.77	1.48	1.53	1.32	1.48	1.55	1.72	1.40	1.96	1.55	1.15
Durables excl. vehicles.....	2.29	2.08	1.97	2.02	2.13	1.87	2.27	1.97	1.97	1.56	1.90	2.50
Personal hygiene.....	1.70	2.23	2.18	1.91	1.76	1.79	1.80	1.43	1.89	1.31	1.49	1.72
Books, newspapers etc.....	1.72	1.91	1.86	1.64	1.78	1.50	2.13	1.89	1.75	1.52	1.69	1.90
Sports, holidays, hobbies.....	1.96	1.64	1.69	1.84	1.86	1.70	1.90	2.08	1.91	1.84	1.60	1.07
Transport incl. own car.....	2.09	1.95	2.18	2.32	2.28	1.85	1.82	1.57	1.76	2.11	1.42	1.66
Union fees, subscriptions etc....	2.05	1.90	1.63	1.52	1.88	2.06	2.35	1.06	1.82	1.31	1.34	1.75
	(3) $y = a + b (\log x - \overline{\log x})$											
Dwelling.....	1.95	1.51	1.88	1.63	2.12	1.66	2.16	1.71	1.60	1.70	1.67	1.60
Fuel & light.....	2.03	1.88	1.95	1.91	2.22	2.12	1.53	1.53	1.60	1.66	1.69	1.82
Food.....	1.94	2.11	1.88	1.44	1.90	1.83	2.29	1.61	1.57	1.60	1.49	1.68
Tobacco.....	1.75	1.81	1.62	1.80	2.32	1.98	1.39	2.03	2.08	2.08	2.23	2.02
Clothing.....	1.59	1.79	2.16	1.92	1.78	1.77	2.18	2.21	1.46	1.71	1.65	1.90
Footwear.....	2.12	1.86	2.10	2.20	1.83	2.00	2.12	2.04	1.96	1.90	2.08	1.88
Washing & cleaning.....	1.63	1.76	1.33	1.58	1.24	1.31	1.64	1.78	1.42	1.94	1.58	1.17
Durables excl. vehicles.....	2.26	2.15	1.56	2.26	2.31	1.77	2.14	1.74	1.94	2.54	1.97	2.23
Personal hygiene.....	1.70	2.16	2.19	1.87	1.92	1.78	1.83	1.31	1.79	1.24	1.51	1.73
Books, newspapers etc.....	1.57	1.87	1.82	1.36	1.89	2.07	2.11	1.82	1.84	1.53	1.64	1.99
Sports, holidays, hobbies.....	1.50	1.98	1.00	1.46	2.29	2.02	1.12	1.95	1.91	2.35	0.97	0.76
Transport incl. own car.....	2.20	1.95	1.87	2.12	2.12	1.86	1.34	1.25	2.18	1.45	1.62	1.67
Union fees, subscriptions etc....	2.05	1.84	1.56	1.37	2.00	2.09	2.48	1.05	1.95	1.22	1.29	1.81

direction of deviations from the regression.

Groups of wage and salary earners*)												
	1	2	3	4	5	6	7	8	9	10	11	12
(4) $y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$												
Dwelling	1.43	1.18	1.53	1.77	1.50	1.18	1.66	1.57	1.22	1.31	1.73	1.39
Fuel & light	1.61	1.75	2.02	1.86	1.90	1.83	1.64	1.37	1.41	1.82	1.35	1.47
Food	1.50	1.68	1.02	0.82	1.27	1.18	1.71	1.00	1.08	0.81	0.70	1.20
Tobacco	1.64	2.20	1.32	1.91	1.89	1.57	1.18	1.35	2.14	1.81	1.37	1.65
Clothing	1.22	1.91	1.67	1.47	1.13	1.16	1.63	1.86	0.91	1.19	2.31	1.94
Footwear	2.03	1.61	2.06	1.92	1.74	1.76	1.88	1.94	1.41	1.70	1.78	1.73
Washing & cleaning	1.34	1.39	1.01	1.18	0.89	0.88	1.31	1.49	0.99	1.51	1.07	1.00
Durables excl. vehicles	2.22	2.07	2.10	2.02	2.16	1.69	2.02	1.50	1.82	1.17	1.78	2.12
Personal hygiene	1.35	1.67	1.99	1.62	1.35	1.41	1.58	0.79	1.45	0.85	0.90	1.37
Books, newspapers etc.	1.16	2.35	1.55	0.98	1.34	2.02	1.83	1.13	2.01	1.35	1.28	1.85
Sports, holidays, hobbies	0.81	1.99	0.57	1.05	1.73	1.78	0.53	1.36	1.23	2.19	1.38	0.49
Transport incl. own car	2.22	1.66	1.49	1.84	1.98	1.80	2.24	1.10	1.78	0.91	2.27	1.47
Union fees, subscriptions etc.	1.86	1.56	0.87	1.86	1.38	1.60	1.67	1.94	1.54	0.72	1.66	1.44
(5) $\log y = \log k + \log \Phi (a + \log x)$												
Dwelling	2.21	1.78	2.01	1.87	2.22	1.83	2.45	1.80	1.90	0.29	1.92	1.56
Fuel & light	2.05	1.96	1.95	1.48	2.28	2.21	1.47	1.82	1.63	1.51	1.64	1.94
Food	1.93	2.13	2.15	-	2.09	1.97	2.48	1.88	1.65	-	1.74	1.69
Tobacco	1.87	1.94	1.71	-	2.35	2.15	1.57	2.45	2.09	2.10	0.75	2.25
Clothing	1.74	-	2.13	2.11	-	2.13	2.49	2.57	0.30	-	2.38	2.15
Footwear	2.09	1.90	2.10	2.33	1.86	2.10	2.17	2.29	2.20	1.96	2.18	1.89
Washing & cleaning	1.69	1.86	1.64	1.84	1.53	1.67	1.72	1.96	1.72	2.29	1.88	1.37
Durables excl. vehicles	2.24	2.08	2.11	1.98	2.22	1.96	-	2.05	2.05	1.52	1.80	2.37
Personal hygiene	1.86	2.32	2.12	2.06	1.98	1.87	1.82	1.69	2.36	1.44	1.95	1.83
Books, newspapers etc.	2.11	1.95	2.06	1.85	2.00	1.80	2.19	-	1.86	0.27	1.82	1.69
Sports, holidays, hobbies	0.59	0.71	0.67	-	-	0.96	2.29	-	-	1.46	0.98	-
Transport incl. own car	2.02	1.97	1.68	2.55	2.41	-	-	-	1.53	-	-	1.88
Union fees, subscriptions etc.	2.12	1.90	0.10	0.27	2.17	2.24	2.65	0.09	2.22	1.83	-	1.86

*) 1. Higher public servants and salaried employees. The Capital. 2. Lower public servants and salaried employees. The Capital. 3. Skilled workers. Provincial towns. 4. Unskilled workers. The Capital. 5. Higher public servants and salaried employees. Provincial towns. 6. Lower public servants and salaried employees. Provincial towns. 7. Skilled workers. Provincial towns. 8. Unskilled workers. Provincial towns. 9. Lower public servants and salaried employees. Rural districts. 10. Skilled workers. Rural districts. 11. Unskilled workers. Rural districts. 12. Agricultural workers. Rural districts.

Table V,7. Number of significant test results among 12 groups of wage and salary earners. Significance level 95% (χ^2 , F², l) and 5% (N, d.)

	$\log y = a + b (\log x - \bar{\log x})$					$\log y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$					$y = a + b (\log x - \bar{\log x})$					$y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$					$\log k + \log \Phi (a + \log x)$				
	χ^2	F	N	l	d	χ^2	F	N	l	d	χ^2	F	N	l	d	χ^2	F	N	l	d	χ^2	F	N	l	d
Dwelling.....	0	1	0	0	0	0	5	1	1	2	5	5	2	3	2	6 ³	6 ³	8	9	8	1	2	1	1	2
Fuel & light.....	5	1	3	2	5	2	3	4	4	2	5	2	4	4	1	6	4	4	5	4	5	2	2	2	5
Food.....	1	2	1	2	1	1	5	2	1	5	1	3	2	0	3	4	12	9	11	10	1 ²	2 ²	0 ²	0 ²	1 ²
Tobacco.....	4	6	1	0	0	3	9	1	0	1	0 ¹	1 ¹	0	0	1	3 ⁴	5 ⁴	2	4	6	3 ¹	7 ¹	2 ¹	1 ¹	2 ¹
Clothing.....	1	3	0	0	0	1	4	2	1	2	2	5	2	1	2	3 ³	6 ³	7	8	6	1 ³	4 ³	1 ³	1 ³	1 ³
Footwear.....	1	0	2	0	0	1	0	0	0	0	3	0	0	0	0	2	4	1	2	2	1	1	0	0	0
Washing & cleaning.....	1	2	2	0	1	1	6	3	2	8	5	7	3	1	8	10	11	8	7	11	0	2	0	0	3
Durables excl. vehicles.....	1	10	0	2	0	2	10	0	2	0	7 ¹	9 ¹	2	4	1	7 ³	8 ³	6	5	2	1 ¹	9 ¹	0 ¹	0 ¹	1 ¹
Personal hygiene.....	2	3	0	1	0	2	3	1	2	3	6	3	1	2	3	8 ¹	7 ¹	9	8	8	1	2	0	0	1
Books, newspapers etc.....	0	4	0	0	0	1	5	2	2	1	6 ¹	5 ¹	3	2	2	6 ³	8 ³	6	6	7	2 ¹	4 ¹	1 ¹	1 ¹	1 ¹
Sports, holidays, hobbies....	1	3	0	0	1	0	6	3	3	3	3 ⁶	3 ⁶	5	7	6	6 ⁶	6 ⁶	10	10	8	2 ⁵	6 ⁵	5 ⁵	5 ⁵	6 ⁵
Transport incl. own car.....	3	11	1	0	0	4	12	0	1	1	8 ²	10 ²	8	9	3	6 ⁵	7 ⁵	10	10	4	3 ⁵	6 ⁵	0 ⁵	0 ⁵	1 ⁵
Union fees, subscriptions etc..	1	3	1	0	0	1	3	3	2	4	1	3	5	2	5	7 ⁴	7 ⁴	6	7	7	3 ¹	5 ¹	3	3	3 ¹

¹ only 11 test results.

² only 10 test results.

³ only 9 test results.

⁴ only 8 test results.

⁵ only 7 test results.

⁶ only 6 test results.

where t_k is the deviation of the observed from the calculated expenditure on a given expenditure item for the k 'th group of observations, where the observations are arranged by increasing values of income.

As in the χ^2 -test it is always the weighted residuals, which are used in the tests for the two relations in which the untransformed value of the expenditure, y , is the dependent variable.

In their article⁵⁾ Durbin and Watson have examined the distribution function for d and have calculated significance zones for this quantity under alternative assumptions as regards number of observations (L) and number of parameters in the function on the basis of which the residuals have been calculated (in this case 2). Durbin and Watson's tables do not allow of any precise delimitation of the level of significance since d -values *within* the calculated significance zones do not permit any conclusion as regards rejection or acceptance of the test hypothesis.

It will be seen from table V,6 and table V,7 that the d -test permits a more precise conclusion than the run tests. While both function (I,1) and (I,2) pass the N - and I -tests fairly well, the double-logarithmic function getting on the whole the best marks, the d -test reveals the difference between these two functions more clearly.

In a number of cases the semi-logarithmic function gives d -values which are clearly beyond the significance zone presumably because the size of the residuals in the first and in the last run is in certain cases larger than permissible—even where the *number* of elements in these runs (as determined by the shifts of sign) may not be significant.

5. Summary of test results.

In table V,7 a summary of the tests has been given. For each item of expenditure the number of significant test results among the twelve social groups has been counted separately for each of the five Engel functions. Bearing in mind the reservations mentioned above as regards the validity of the different tests, this summary clearly emphasizes the conclusion which has gradually emerged in the course of the discussion of the individual tests. The double-logarithmic Engel function gives the best goodness of fit among the five functions tested. This does not, of course, mean that the "true" Engel curve function have thus been found, but the selection of the double-logarithmic function as the "best" may, nevertheless, justify a further analysis of the result of the parameter estimation for this function.

Vc. Analysis of estimates of the parameters.

1. Regression analysis versus two-way cross-tabulation.

Table V,8 shows estimates of the standard deviation in the distribution of $\log y$ for all 13 expenditure items, separately for each of the 12 social groups. To make it possible to assess the effect of the introduction of the disposable income x as explanatory variable, estimates of the standard deviation in the distribution of the residuals from the calculated

⁵⁾ Cf. Durbin and Watson (4).

double-logarithmic Engel function have been shown too. It will be seen that the unexplained part of the variation in expenditures, is reduced by 10 to 60 per cent as one goes from the simple mean value description to the Engel function. The table shows that the gain is fairly constant from one social group to another for the same expenditure item—whereas there are great variations among the expenditure items. The size of the gain depends partly on the slope of the Engel curve and partly on the size of the variance within groups. If this variance is small, the gain from the regression analysis will, *ceteris paribus*, be relatively great, and if the regression line is steep, the gain will, *ceteris paribus*, also be great.

The food group has a small variance, and even if the slope of the regression line is moderate a considerable gain is nevertheless achieved through the regression. The item sports, holidays, hobbies etc., has a relatively high variance, but since the regression lines are rather steep, the gain is also in this case considerable. Expenditures on fuel, light and footwear rise slowly with income and the variance is substantial; the regression gain is insignificant. This is also the case of the item of durable goods, where the extremely high variance almost completely counteracts the effect of the fairly steep regression line.

Another measure of this regression gain is obtained by calculating the ratio of the estimates of the slopes to their estimated standard errors. This is at the same time a test for the hypothesis $\beta = 0$, since this hypothesis can be tested by a t-test. The quantity $t = \frac{b - 0}{s_b}$ follows the t-distribution with $N-2$ degrees of freedom, where N is the number of group averages included in the calculation of b . The t-values thus calculated show that the hypothesis $\beta = 0$, i.e. that the slope of the regression line is zero, cannot be accepted in one single case.

It thus seems justified to conclude that the description of the consumption behaviour of the household has gained considerably in precision by the inclusion of the disposable household income as explanatory variable.

By arranging the observations of expenditures in cross-tables where each household is placed in a cell according to its disposable income, it is often attempted to include disposable income as an explanatory variable. The frequent use of this type of tables in publications dealing with household-surveys is often explained by the wish to present the material in a clear manner without adopting a definite hypothesis concerning the *form* of the relationship between the two variable expenditure and income.

The method used here for the description of the expenditure-income relationships, however, has obvious advantages over this frequently used grouping method. As shown by Amundsen⁶⁾, information is lost to a considerable extent when the basic material is split up into the many cells of such a table since only the observations of the individual cell is used in the calculation of the average expenditure figure of this cell.

The parameter estimates have thus made it possible to give a more precise description of the observations than the usual description by averages—be they overall averages or “cell averages” in a cross table.

⁶⁾ Amundsen (2).

Table V,8. Gain of regression. Standard errors in the distribution of expenditures and in the distribution of deviations from the double logarithmic Engel function. $\log y = a + b (\log x - \overline{\log x})$.

Group of wage and salary earners		Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear	Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscription
The capital														
Higher public servants and salaried employees	$S_{\log y} \dots$	0.23	0.18	0.12	0.29	0.21	0.15	0.20	0.31	0.19	0.24	0.30	0.40	0.17
	$S_{\log y} x \dots$	0.14	0.12	0.069	0.25	0.13	0.13	0.13	0.29	0.11	0.14	0.13	0.36	0.13
Lower public servants and salaried employees	$S_{\log y} \dots$	0.22	0.20	0.12	0.32	0.25	0.17	0.22	0.36	0.23	0.25	0.31	0.37	0.19
	$S_{\log y} x \dots$	0.13	0.17	0.067	0.27	0.14	0.12	0.13	0.32	0.12	0.17	0.17	0.31	0.14
Skilled workers	$S_{\log y} \dots$	0.22	0.14	0.14	0.25	0.24	0.14	0.19	0.35	0.17	0.20	0.32	0.43	0.20
	$S_{\log y} x \dots$	0.13	0.12	0.050	0.18	0.13	0.10	0.14	0.26	0.11	0.13	0.13	0.30	0.086
Unskilled workers	$S_{\log y} \dots$	0.23	0.18	0.15	0.29	0.26	0.17	0.21	0.35	0.21	0.28	0.34	0.38	0.25
	$S_{\log y} x \dots$	0.14	0.17	0.062	0.20	0.16	0.13	0.14	0.29	0.13	0.18	0.19	0.27	0.13
Provincial towns.														
Higher public servants and salaried employees	$S_{\log y} \dots$	0.19	0.16	0.12	0.27	0.22	0.14	0.22	0.37	0.18	0.27	0.33	0.54	0.17
	$S_{\log y} x \dots$	0.11	0.12	0.056	0.22	0.12	0.11	0.15	0.32	0.097	0.17	0.14	0.43	0.093
Lower public servants and salaried employees	$S_{\log y} \dots$	0.23	0.18	0.15	0.32	0.25	0.17	0.27	0.41	0.23	0.29	0.38	0.46	0.21
	$S_{\log y} x \dots$	0.14	0.16	0.072	0.25	0.13	0.13	0.17	0.34	0.14	0.17	0.18	0.35	0.12
Skilled workers	$S_{\log y} \dots$	0.17	0.14	0.13	0.27	0.25	0.16	0.22	0.44	0.17	0.21	0.34	0.34	0.17
	$S_{\log y} x \dots$	0.10	0.14	0.067	0.19	0.14	0.096	0.15	0.34	0.090	0.14	0.14	0.34	0.079
Unskilled workers	$S_{\log y} \dots$	0.20	0.14	0.13	0.28	0.25	0.18	0.21	0.36	0.21	0.24	0.34	0.44	0.19
	$S_{\log y} x \dots$	0.14	0.11	0.060	0.17	0.17	0.13	0.15	0.25	0.10	0.13	0.18	0.32	0.086
Rural districts														
Lower public servants and salaried employees	$S_{\log y} \dots$	0.22	0.19	0.14	0.33	0.26	0.18	0.25	0.39	0.23	0.27	0.38	0.48	0.20
	$S_{\log y} x \dots$	0.14	0.16	0.074	0.25	0.13	0.11	0.16	0.32	0.12	0.19	0.20	0.36	0.12
Skilled workers	$S_{\log y} \dots$	0.21	0.13	0.13	0.23	0.23	0.17	0.21	0.39	0.20	0.22	0.36	0.50	0.20
	$S_{\log y} x \dots$	0.12	0.12	0.052	0.17	0.13	0.12	0.13	0.33	0.13	0.15	0.20	0.35	0.096
Unskilled workers	$S_{\log y} \dots$	0.25	0.18	0.15	0.32	0.25	0.16	0.23	0.32	0.21	0.25	0.38	0.45	0.21
	$S_{\log y} x \dots$	0.15	0.12	0.052	0.19	0.14	0.12	0.13	0.27	0.10	0.14	0.19	0.29	0.076
Agricultural workers	$S_{\log y} \dots$	0.27	0.19	0.14	0.31	0.29	0.14	0.25	0.31	0.18	0.29	0.37	0.36	0.21
	$S_{\log y} x \dots$	0.19	0.12	0.068	0.22	0.17	0.11	0.19	0.26	0.11	0.19	0.24	0.31	0.13

2. Interpretation of main results.

The next stage of the analysis, tackles the central problem: How can the parameter estimates be interpreted on the basis of economic theory?

Also in this connection it is the double-logarithmic Engel function which will be selected for treatment; primarily because this function emerged as the best among the five types tested for goodness of fit, but also because the parameter estimate b of the slope β is at the same time an estimate of the income elasticity in the demand for a given expenditure item.

The estimate a of the parameter α indicates the average value of the logarithmically transformed expenditure observations. This estimate, together with $\overline{\log x}$, the average value of the transformed income observations, determines the coordinates of the centre of gravity, the mean, of the observations and can accordingly be interpreted as an estimate of the level of expenditures within the social group in question.

With the modifications which result from the use of transformations other than the logarithmic one, this interpretation can be extended to cover all four linear regression models: the estimate a in conjunction with the average transformed or untransformed income observations indicates the *level* of the expenditure in the social group in question. In the case of the cumulative log-normal distribution function, the situation is different.

The estimates k and a in this function, $\log y = \log k + \log \Phi (a + \log x)$, can here be interpreted as regulators of the unit of measurement in terms of which the expenditure and the disposable income are to be measured. It is postulated that one and the same Engel curve can describe the relationship between the disposable income and any expenditure item, although for any given expenditure item an adjustment must be made of the two units of measurement on the x - and y -axis determined by the values of the parameters α and κ^7). The estimate k may be interpreted as an estimate of the saturation expenditure, κ , on the given item for the given social group, i.e. the total expenditure which households of that group would spend on that item, if the income tended towards infinity.

Also the functions

$$\log \eta = a + \beta (v^{-1} - \bar{v}^{-1})$$

and

$$\eta = a + \beta (v^{-1} - \bar{v}^{-1})$$

have a saturation expenditure, namely

$$\text{antilog} (a - \beta \bar{v}^{-1})$$

and

$$a - \beta \bar{v}^{-1}$$

respectively.

The estimates of these two expressions and of κ are widely different; this is only a reflection of the fact that the different Engel functions deviate considerably from one another as soon as we move outside the range of the observation cf. fig. V,3; however,

⁷⁾ Cf. Aitchison and Brown (1), p. 131.

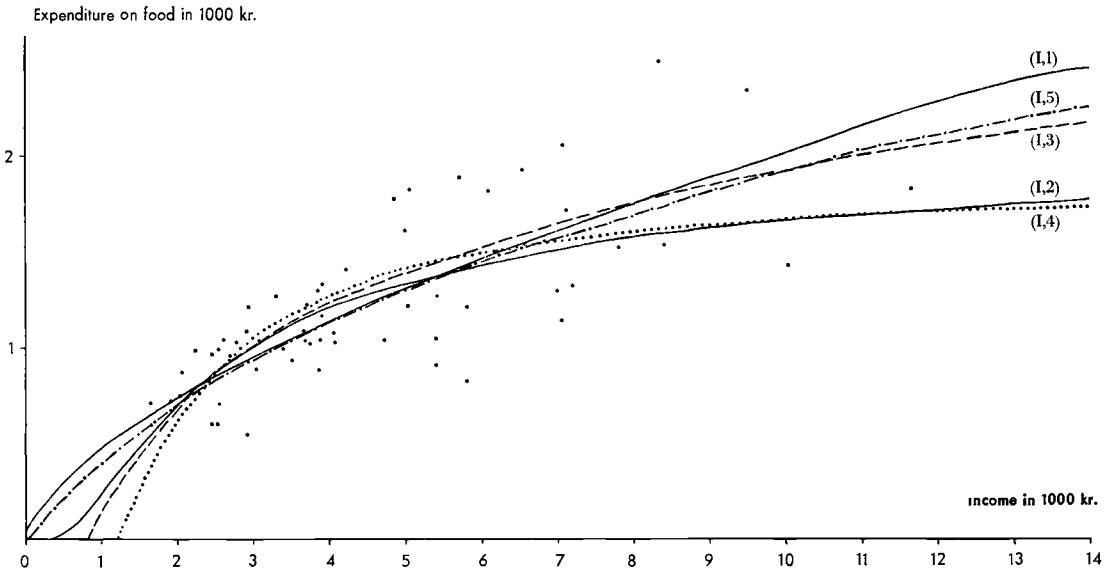


Fig. V, 3. Five Engelfunctions. Income and expenditure on food. Skilled workers in the provincial towns.

what determines the suitability of an Engel function in the description of the data is primarily the goodness of fit of this function and not, for instance, whether such a conceptually vague quantity as the saturation expenditure deviates more or less from some preconceived ideas about it.

The tables of results in appendix A, pp. 126–173, show that the expenditure level varies systematically from one social group to another for all expenditure items, as was in fact to be expected since the social grouping is at the same time to a high degree a grouping by income. Since the expenditure is rising with rising income for all 13 expenditure items, the parameter estimates a , which illustrate the level of expenditures, therefore show the highest values for the social groups of higher public servants and salaried employees and the lowest for the groups of unskilled workers and farm workers.

3. Interpretation of the estimates of the slope of the regression line.

However, it is of greater interest to examine the parameter estimate b or the two parameter estimates in conjunction, i.e. the whole Engel curve.

It will then be natural to examine first whether the division into twelve groups of wage and salary earners has been appropriate, or in other words, whether the twelve Engel curves for a given expenditure item which are estimated separately for each of the twelve groups of wage and salary earners, are significantly different.

The reason for dividing the observations into these twelve groups was an assumption that differences in geographical and social grouping would be reflected in cha-

racteristic differences in consumption behaviour. To the extent that this geographical and social grouping is at the same time a grouping by income levels, this will naturally influence the level of the consumption expenditure as shown by the estimate a. But the question is whether this grouping is anything more than a grouping by income. May we consider the twelve Engel curves as being generated by the same consumption process, i.e. be considered estimates of the same Engel curve?

This problem can be examined by comparing the twelve regression lines for each expenditure item, a comparison which, like the calculation of the parameter estimates, is a standard feature of the regression analysis⁸⁾.

Such a comparison has been made in the case of the double-logarithmic function for all 13 expenditure items.

First, it is examined whether the estimates of the slope b_1, b_2, \dots, b_{12} calculated for a given expenditure item can be considered as estimates of the same "true" slope β , and if this hypothesis can be upheld, it is examined whether the parallel Engel curves can be considered as estimates of one single Engel curve. The former test is a test for the parallelism of the Engel curves, and the latter a test for their identity.

The test for the parallelism of the Engel curves is performed as an F-test, in which a variance calculated on the basis of the variation among the 12 estimated b-values is compared with an expression of the inner variance:

$$F_{f_2, f_1} \simeq \frac{s_2^2}{s_1^2}; \left[f_1 = \sum_1^{12} (n_k - 1) \text{ and } f_2 = 12 - 1 \right]$$

where

$$s_2^2 = \frac{1}{11} \sum_1^{12} (b_k - \bar{b})^2 (\log x_k - \overline{\log x_k})^2$$

and

$$s_1^2 = \frac{1}{\sum_1^{12} (n_k - 2)} \sum_1^{12} (n_k - 2) s_{\log y_k | x}^2$$

In these relations $\log x_k$ and $\log y_k$ are the coordinates of the mean of the observations of the k'th group of wage and salary earners; n_k is the number of observations in this group.

The average slope \bar{b} for a given expenditure item has been calculated as a weighted average of the twelve individual slopes ⁹⁾

$$\bar{b} = \frac{\sum_1^{12} b_k \sum (\log x_k - \overline{\log x_k})^2}{\sum_1^{12} \sum (\log x_k - \overline{\log x_k})^2}$$

⁸⁾ Cf. for instance, A. Hald (8), pp. 579-584.

⁹⁾ Cf. A. Hald (8), p. 580.

If the hypothesis is correct (concerning the parallelism of the Engel curves) it will be true that \bar{b} is normally distributed around β with the variance

$$\sigma_{\bar{b}}^2 = \frac{\sigma^2 \log y | x}{\sum_1^{12} \sum (\log x_k - \overline{\log x_k})^2}$$

It is this property of \bar{b} which is utilized in forming the estimate s_2^2 .

The F-tests appear from table (V,9).

Table V,9. F-test for parallelism of Engel curves.

Dwelling	1.32
Fuel and light	4.22
Food	3.46
Tobacco	2.38
Clothing	1.72
Footwear	2.40
Washing and cleaning	1.06
Durables	3.72
Personal hygiene	2.89
Books, newspapers etc.	1.45
Sports, holidays, etc.	1.30
Transport	6.09
Union fees etc.	1.89
F _{.95}	1.89

and the 12×13 calculated slopes b have been shown in table V,10.

Table V,9 shows that in 6 out of 13 cases the hypothesis concerning the parallelism of the Engel curves cannot be rejected, whereas it must be rejected in the 7 remaining cases.

Bearing in mind that the estimate b of the slope in the double-logarithmic Engel curve is also an estimate of the income elasticity of the expenditure on a given item, it seems to be a reasonable *a priori* hypothesis that β varies from the higher to the lower social groups because these groups are at different income levels. If y is observed over a sufficiently wide income interval, the income elasticity, i.e. the relative increase in the expenditure in proportion to a given relative increase in income, must be expected to be decreasing with increasing income.

Actually it is rather strange that the double logarithmic function, according to which the income elasticity is assumed to be constant, should, as far as can be seen from the tests made, turn out to be the "best" of the five types of functions *within* each of the twelve social groups, since each social group after all spans an income interval of several thousand kroner. But if we go further and cover the whole scale from agricultural labourers and unskilled workers to higher public servants and salaried employees, a hypothesis of constant income elasticity seems to be contrary to all sensible *a priori* assumptions.

Nevertheless, table V,9 shows, as mentioned, that for the six expenditure items: *dwelling, clothing, washing and cleaning, books and newspapers, etc., sport and holidays*, and finally *union fees, etc.*, such a hypothesis of common slope, i.e. constant income elasticity, cannot be rejected.

It will not be attempted here to explain—or rather to explain away—this phenomenon. As will be remembered, the purpose of the present inquiry has been laid down as an attempt to *describe* the observations of incomes and expenditures, since an attempt to explain the consumption behaviour of the households must for the time being be considered unduly ambitious¹⁰).

It should be mentioned, however, that the very rough grouping of the almost endless number of goods and services into only 13 items is undoubtedly one of the decisive causes of the stability found in the income elasticity between social groups. If, instead, sharply defined individual commodities and services had been considered, lounge suits of a particular quality, flats of a given size and quality, etc. the income elasticity of demand for these goods and services would undoubtedly have been falling with rising disposable income.

However, the six expenditure items with constant income elasticity are not such heterogenous items in which have been included many different types of goods and services. The items *dwelling, washing and cleaning, clothing*, and partly the item *sport and holidays, etc.* correspond to rather well defined parts of the budget of any household, and it is actually very interesting that the income elasticity for these items is so stable as shown by table V,9. The need for shelter, clothing, for entertainment, etc. naturally makes itself felt at all income levels, but the interesting thing is that at any place in the income scale the same relative increase in the expenditure is produced by a given relative rise in income. Not least in the expenditure on the item *sport, holidays, etc.* does the income elasticity seem to be remarkably constant at a very high level around 1.5. Of course the goods and services demanded—restaurants, theatres, cinemas, holiday trips, hobbies and sport—vary widely over the different income classes, social groups and age groups, but for all groups there seems to be a very long, still unfulfilled list of demand in this field.

For the seven items for which the hypothesis of the parallelism of the regression lines had to be rejected table V,10 confirms that as a general rule this is precisely due to the fact that the income elasticity is falling as the income level increases, cf. the items of *food, tobacco and footwear*. However, this is not the whole explanation—indeed, in two cases the explanation seems to be the opposite, namely that the income elasticity rises with the income, cf. the items of *fuel and lighting, and transport* (incl. expenditure on motor vehicles), where *b* in several cases rises as one moves from a lower social group to a higher one within the same geographical area. In the case of these seven items there is also another interesting phenomenon which emerges clearly, viz. the significant influence of the social grouping on the income elasticity. If, e.g., one takes the large item of *food*, table V,10 shows that the wage-earning groups in the capital have a considerably higher income elasticity in their demand for food than the groups of salaried em-

¹⁰) Cf. E. Jørgensen (12).

ployees, 0.69 and 0.70 respectively compared with 0.52 and 0.53. There is also a marked difference in the level of b in the case of *footwear* as we move from wage-earners to salaried employees.

The item of *personal hygiene* also exhibits significant differences in the b -values for the two social groups, but here the groups of salaried employees are at the highest level. The breakdown into social groups, and particularly the distinction between salaried employees and manual workers, thus seems to correspond to a real difference in behaviour in the case of several important items of consumption. The geographical breakdown, on the other hand, seems to be justifiable on the basis of existing differences in expenditure behaviour only as far as the items of *dwelling* and *fuel and lighting* are concerned.

4. Are the Engel curves for different social groups identical?

For six expenditure items the hypothesis of parallel regression lines for the twelve social groups could not be rejected. In these cases it was subsequently examined whether these parallel curves could be considered as identical (test for identity). This identity test is performed in two stages; at the first stage it is examined, by means of an ordinary F-test for linearity, whether the twelve mean points ($\log x_k, a_k$) can be considered as being on a straight line—which, of course, they must if the test hypothesis is correct. If this

Table V,10. Calculated income elasticities for 13 expenditure items for each of 12 groups of wage and salary earners.

	The capital				Provincial towns				Rural districts			
	1	2	3	4	1	2	3	4	2	3	4	5
Dwelling.....	0.96	0.93	0.87	0.94	0.80	0.86	0.75	0.70	0.84	0.93	0.98	0.96
Fuel & light.....	0.73	0.50	0.40	0.31	0.57	0.44	0.28	0.43	0.55	0.32	0.64	0.76
Food.....	0.52	0.53	0.69	0.70	0.55	0.64	0.60	0.62	0.58	0.65	0.67	0.65
Tobacco.....	0.80	0.84	0.93	1.07	0.76	0.90	1.07	1.17	1.06	0.93	1.28	1.13
Clothing.....	0.89	1.12	1.02	1.05	0.91	1.02	1.10	1.01	1.12	1.02	0.99	1.24
Footwear.....	0.43	0.62	0.50	0.61	0.38	0.57	0.67	0.63	0.71	0.62	0.50	0.49
Washing & cleaning.....	0.80	0.85	0.70	0.81	0.82	0.98	0.90	0.77	0.94	0.88	0.89	0.87
Durables excl. vehicles.....	0.57	0.89	1.21	0.98	1.01	1.04	1.50	1.37	1.10	1.07	0.79	0.85
Personal hygiene.....	0.81	1.00	0.68	0.83	0.73	0.87	0.77	0.97	0.95	0.81	0.87	0.73
Books, newspapers etc.....	1.02	0.95	0.77	1.05	1.02	1.11	0.86	1.04	0.93	0.86	0.91	1.10
Sports, holidays, hobbies.....	1.39	1.37	1.51	1.46	1.46	1.56	1.68	1.54	1.59	1.61	1.57	1.43
Transport incl. own car.....	0.86	1.09	1.56	1.34	1.68	1.39	1.58	1.62	1.53	1.96	1.66	0.92
Union fees, subscriptions etc...	0.63	0.66	0.90	1.10	0.76	0.79	0.80	0.91	0.82	0.94	0.94	0.88
$\overline{\log x} \dots$	3.81	3.75	3.71	3.67	3.75	3.66	3.58	3.53	3.49	3.49	3.47	3.36

1. Higher public servants and salaried employees.
4. Unskilled workers.

2. Lower public servants and salaried employees.

3. Skilled workers

5. Agricultural workers.

proves to be the case, it is finally examined, by means of a t-test, whether the slope \hat{b} of the line formed by the twelve mean points is identical with the weighted average of the twelve individual slopes.

It turns out that none of the six expenditure items pass this test for identity. The item of *sport, holidays, etc.* passes the first stage of the test (concerning the linearity of the twelve mean points), but shows significance for the second stage of the test; the other five items show significance already for the linearity test, see fig. V,4a and fig. V,4b, where the twelve mean points have been plotted for the items of books, newspapers, etc. and sport, holidays, etc. together with the twelve individual Engel functions.

Now, what does this result mean? The immediate interpretation is, of course, that we are here confronted with the "layer effect" described by Wold¹¹⁾ and commented on page 25 above. The twelve social groups have the same income elasticity for the six expenditure items, but there is a difference in the level of the actual expenditure among the social groups. This may be due to differences in environment, in upbringing and in habits of life, or it may be due to differences in the accessibility of the goods in question, for instance, owing to differences in distance from places where the goods are available. Thus restaurants, cinemas and other forms of entertainment are more easily available in the towns, cf. the instance of this mentioned on page 26.

If this theory of the layer effect holds good, it may be concluded that the subdivision of the material into geographical and—particularly—social groups corresponds to a real difference in consumption behaviour not only for the seven items with different income elasticities, but also for the six items where the twelve social groups could be considered as having the same income elasticity; in the case of the latter items the existence of the layer effect should then be the explanation of the difference in the expenditure level from social group to social group¹²⁾.

If this interpretation of the results is accepted, the conclusion must be that by subdivision of the data into relevant groups it is possible to derive estimates of the income-expenditure relationships which are less biased than those relationships which could be derived for the total sample

If this subdivision had not been undertaken, the income elasticities for the two items of *books, newspapers, etc.* and *sport, holidays, etc.*, to mention two examples, would have been considerably higher than the average of the elasticities of the twelve subgroups (viz. 1.28 against 0.98 and 2.00 against 1.50), cf. fig. V,4a and fig. V,4b, on page 87.

5. An important reservation.

As regards this interpretation of the results it is, however, necessary to make an important reservation. All the results analysed so far are derived from linear regression analyses. In chapter III and IV the assumptions of the inquiry was discussed and it was found that this form of analysis was a suitable analytical tool if due regard is paid to problems

¹¹⁾ Cf. Wold (19), p. 68.

¹²⁾ Such layer effects may, of course, also be imagined to exist for the seven items for which the Engel curves of the individual social groups are not parallel.

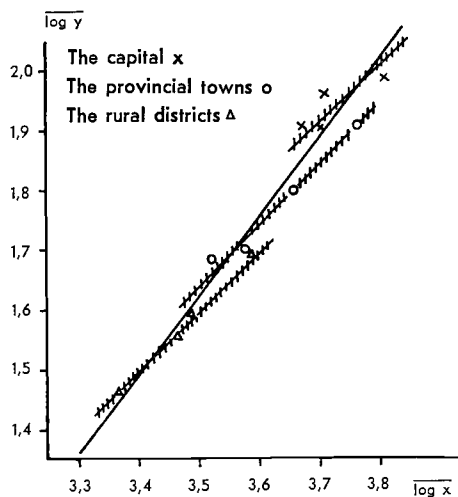


Fig. V, 4a. Mean points for the twelve Engel curves for expenditures on books, newspapers etc. For each of the three parts of the country the capital (x), the provincial towns (o) and the rural districts (Δ) the four Engel curves may be considered identical. The average Engel curve for whole country deviates significantly.

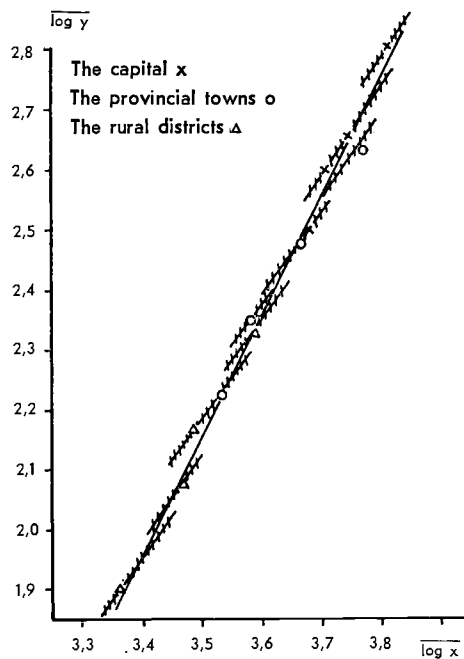


Fig. V, 4b. Mean points for the twelve Engel curves for expenditures on sports, holidays etc. In three cases only one Engel curve may be considered identical to another one. The average Engel curve for the whole country deviates significantly.

concerning zero observations and variance assumptions. However, it will probably be useful to point out that when we choose a given method, we also choose, to some extent, the results. If, for instance, the independent variable of the regression analysis, the disposable income per person (or transformations hereof), cannot be considered fully independent in relation to the dependent variables, there will be bias in the estimates.

Now, considering again fig. V,4b, which shows the Engel curves of the twelve social groups with corresponding mean points for the expenditure item of sport, holidays, etc., it may very well be imagined that the line through the twelve mean points—and not the twelve single Engel curves deviating systematically from this mean point regression line—represented the “true” relationship between $\log x$ and $\log y$. It is worth noting in this connection that the weighted average of the b for the twelve social groups was appreciable lower than the estimate of the slope of the regression line through the twelve mean points in the case of five out of the six expenditure items that had passed the test for parallelism¹³); i.e. displayed the same type of bias as may exist for sports, holidays etc.

On the whole, we are led to conclude that the immediate interpretation of the results of the calculations, namely that the subdivision into special groups seems to correspond to real differences in expenditure behaviour is upheld. This is so especially in the case of the expenditure items where the Engel curves of the social groups showed significant differences both in slope and level; but it must be borne in mind that purely technical factors—here the choice of the regression technique as analytical tool—can exercise some influence which would involve modifications of the conclusions drawn.

6. Conclusions.

If the results for the double-logarithmic Engel function are to be summarized in a single table, the average income elasticity can be shown for each expenditure item. It is true that it was found above that these social groups differed systematically as regards their expenditures, but this applied primarily to the *level* of the calculated Engel curves and *not* to their *slope*. In the case of the six items for which the calculated Engel curves could be considered parallel the calculation of such averages is natural, and although the elasticities differ significantly for the remaining seven items it is nevertheless characteristic that this difference between expenditure items within each social group is more pronounced than the difference between the individual social groups for the same expenditure item, so that also here it does make sense to calculate an average b -value. In table V,11 have been shown the 13 average values for each of the 12 income elasticities calculated as a weighted average of the twelve b -values for each item, the weights being the sum of the squared deviations from the mean income, cf. above p. 82.

¹³) The item of union fees, etc. is influenced by the great expenditures of the wage-earning groups on union fees, so that the line through the mean points of the twelve social groups for this item will have less slope than the average of the slopes of the twelve individual lines.

Table V,11. Average income elasticities for 13 expenditure items.

Expenditure item	Average income elasticity
1 Fuel & light	0.51
2 Footwear	0.56
3 Food	0.61
4 Union fees, subscriptions etc.	0.82
5 Personal hygiene	0.86
6 Washing & cleaning	0.86
7 Dwelling	0.89
8 Books, newspapers etc.	0.98
9 Tobacco	0.98
10 Durables excl. vehicles	0.99
11 Clothing	1.04
12 Transport incl. own car	1.39
13 Sports, holidays, hobbies	1.50

In the table the \bar{b} -values have been arranged by order of magnitude, and it will be seen how the slopes of the Engel curves, i.e. the income elasticities, of the 13 expenditure items fall into three clearly distinguishable groups: 3 items which could be called necessities ranging from 0.51 to 0.61; 8 items which may be labelled neutral goods range from 0.82 to 1.04; and 2 items belonging in the category of luxury goods with 1.39 and 1.50¹⁴). Reverting to table V,10, where all b -values have been shown, it will be seen that this ordering by size of average income elasticities comes very close to the ordering by size within each of the 12 social groups, so that stability in the income elasticity of the main groups of expenditure items seems to be a general feature.

In the discussion of the usefulness of the Engel curve analysis in the description of the relationship between x and y , it was found that some expenditure items were not "explained" much more adequately by the inclusion of the disposable income as an explanatory variable—in particular the items of durable goods and transport (incl. transport by own motor vehicle). Now it is found that the estimated parameter values for these items show wide fluctuations of a random nature from social group to social group. The estimate b range from 0.86 to 1.96 for the item of transport and from 0.57 to 1.50 for durable goods.

However, the remaining items seem to show such a high degree of stability that the calculated average income elasticities should be of value also in a wider context, e.g. in the description of the expenditures of the whole population or groups of the population on these items¹⁵).

¹⁴) Cf. E. Jørgensen (12).

¹⁵) Cf. E. Jørgensen (12), where it has been attempted to utilize the results in such a wider context.

Chapter VI.

FURTHER ANALYSES. THE CONCEPT OF UNIT CONSUMERS, MULTIPLE REGRESSION ANALYSES, ETC.

VIa. The unit-consumer concept.

The two variables of the Engel function the dependent variable y , i.e. the expenditure on a given item, and the explanatory variable x , i.e. the disposable income, have been defined (p. 23, chapter III) as expenditure and income *per person* in the individual households. The argument in favour of adopting this definition of x and y was, of course, that differences in the size of households (number of persons per household) are reflected very clearly in the consumption behaviour of the households. When x and y are measured as income and expenditure per person, most of the effects of this source of variation in the income-expenditure relationships will be eliminated, particularly in the case of such main items as food, clothing and footwear depending primarily on the number of persons in the household. As far as items such as dwelling, durable goods, sports, holidays and hobbies are concerned, it may be doubtful how much the unexplained part of the variation in y according to the Engel curve adopted will be reduced.

It is natural to raise the more general question: Is it possible to set up a model for the Engel curve in which x and y are specified in such a way that the effect from differences not alone in size of household but also in the *type of persons* will be eliminated? Or less ambitiously: Can x and y be specified in a way which provides a better approximation to this ideal than the specification used in this analysis (income and expenditure per person)? This is the approach adopted by Prais and Houthakker in their attempt to calculate unit-consumer scales separately for each expenditure item, where the scale indicates, for each type of person, the weight at which the person in question is to be included in the specification of x and y for the individual households.

In a household consisting of a man, aged 47, a wife aged 43, a girl of 11 and a boy of 8, the specification of x and y according to the method adopted in this inquiry would simply consist in dividing the total disposable income of the household and its total expenditure on the given commodity group by the number of persons in the household. The unit-consumer scale, as set up by Prais and Houthakker, indicates for each commodity or commodity group how these four persons are to be measured to arrive at the divisor which gives the desired specification of x and y . The idea is thus that a standard measure is

introduced, so that for the household mentioned the value on the income scale may be e.g. 1.9 units and on the scale for expenditure on food 3.2 units, etc., the unit chosen being the average income and average food expenditure, respectively, of one adult male. Prais and Houthakker suggest a method for calculating estimates of such scales¹⁾, which may be briefly described as follows:

By means of the tests the best Engel function is selected. This function is now assumed to give the best description of the relationship between income and expenditure (on a given commodity in a given population group).

It is now imagined that all persons in a household are converted into income units (unit e.g. = average income for one married man between 30 and 40 years of age) and consumption units (average consumption for one married man between 30 and 40 years of age), the Engel curve chosen would then apply to the relationship between income per income unit and consumption per consumption unit. If one distinguishes among t types of persons, the following relationship will apply

$$(VI, 1) \quad \frac{y}{\sum k_1 \cdot n_1} = f \left(\frac{x}{\sum k_{01} \cdot n_1} \right)$$

where n_1 is the number of persons in the household of type i , k_1 is this type of person's value on the consumption unit scale for the given expenditure item, and k_{01} this type person's value on the income unit scale. Denoting $\frac{x}{\sum k_{01} \cdot n_1}$ by m

(VI, 1) can be formulated in the following way:

$$(VI, 2) \quad \frac{y}{f(m)} = k_1 \cdot n_1 + k_2 \cdot n_2 + \dots + k_t \cdot n_t \quad .$$

(VI, 2) is a multiple regression equation, and by ordinary regression analysis k_1, k_2, \dots, k_t may be determined if $y/f(m)$ is known. It will be seen from the above that the method is based on an assumption that the Engel function originally selected is the correct one, also after the observed values have been converted into consumption and income units, and also on an assumption that the effects from the "scale values" of the t types of persons enter linearly, since otherwise it will be extremely difficult to solve (VI, 1).

It is easy to point to defects in this approach: the requirement of a *priori* knowledge of the "best" type of function and, be it noted, the best one *after* the consumption unit adjustment, the requirement that the contributions of the individual types of persons to the total income of a given household and total expenditure on a given commodity group enter linearly, etc. And it is difficult to see how these defects could be overcome²⁾. But *if* such scales could be determined, it would not only be possible to reduce the residual variance, but also to have scales by which it would be possible to answer such questions as: How much extra expenditure for a household on a given item would be caused by a ten-year-old boy? etc.

¹⁾ Cf. Houthakker, H. S. and Prais, J. S. (10), p. 133.

²⁾ Cf. Forsyth. (6).

Such a tool would be highly relevant in the design of social welfare policy and taxation.

But even if it may, on the face of it seem extremely interesting to obtain replies to such very general questions as the one mentioned above, it seems that questions of this type are too general; they cannot be answered satisfactorily since the consumer scale values of given types of persons depend very much on their income level and on the household type to which they belong. And if one tries to include these two factors in the consumer scale calculations, these calculations would, for one thing, become enormously complicated and for another the result of the calculations would be very difficult to interpret.

These considerations naturally lead up to an attempt to illustrate the influence of the household type in another way. It seems evident that household type and income level will have to be taken into account if the objective—achieving realistic descriptions—is to be fulfilled.

It seems as if this objective could be more satisfactorily fulfilled by calculating income-expenditure relationships separately for the different household types.

In that case one must abandon the idea of a general model describing the observations. The method suggested is more primitive and moderate in its approach, but through careful comparison between the Engel curves of different household types, it will probably be possible to achieve a more realistic description. This "method" of treating the influence of the household type corresponds completely to the method used in treating the residence and social status effect, i.e., separate calculations for each subgroup and comparison of the results. However, the method presupposes that there is a sufficient number of observations for an adequate description of each of the subgroups to be given. In the present study the material has been divided into 12 groups taking into account differences in residence and social status. A further breakdown of each of these 12 groups into a large number of subgroups according to household type would not leave a sufficient number of observations in each subgroup. It would therefore be necessary to abandon the original grouping and this in turn would necessitate a correction for the observed differences among the 12 residential and social groups. Such comprehensive correction and regrouping have been outside the scope of the present study, and therefore these calculations have not been made.

In order to get some idea of the influence of the household type, tables VI,1a to VI,1n have been set up. The tables show for the thirteen expenditure items included in the survey and for savings the average expenditure per person for certain income brackets for all social groups as a whole, separately for different household types. In the calculation of averages for the whole country of the expenditures of the 12 groups of wage and salary earners the shares of the individual groups in the total population of wage and salary earners have been used as weights. In brackets after each expenditure average has been shown the number of observations on the basis of which the average has been calculated.

In interpreting the tables the weaknesses of such a tabular description must naturally be borne in mind, confer the remarks on this subject in chapter V, p. 78.

Thus the number of observations in many of the cells of the table is so small that the averages are subject to so great inaccuracy that their usefulness is rather limited.

Despite the weaknesses of the table, it is still possible to draw some important con-

Table VI, 1a. Average expenditure per person on dwelling in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man	360 (1)	290	509 (6)	560 (51)	820 (69)	950 (49)	1715 (10)
2 Single woman	361 (4)	336 (25)	645 (42)	734 (99)	961 (69)	1129 (44)	1410 (8)
3 Couples without children	203 (4)	319 (84)	431 (233)	564 (172)	754 (99)	1042 (45)	1075 (12)
4 Couples with 1 child . .	143 (9)	297 (314)	431 (251)	518 (61)	685 (7)	1176 (4)	
5 Couples with 2 children	172 (43)	261 (438)	386 (144)	534 (16)	737 (3)	305 (2)	
6 Couples with 3 children	124 (58)	234 (146)	356 (34)	428 (3)	278 (11)		
7 Couples with 4 or more children	116 (72)	199 (40)					
8 Single man with 1 or more children	216 (2)	109 (2)	356 (5)	690 (3)	525 (2)	160 (1)	
9 Single woman with 1 or more children . . .	195 (7)	343 (36)	420 (33)	440 (7)	515 (3)	1364 (1)	
10 Other types	150 (18)	233 (88)	415 (49)	434 (12)	1076 (5)	918 (1)	

Table VI, 1b. Average expenditure per person on fuel and light in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man	5 (1)	123 (5)	200 (52)	197 (51)	215 (68)	270 (49)	612 (10)
2 Single woman	423 (3)	162 (25)	273 (42)	295 (99)	359 (69)	469 (44)	548 (8)
3 Couples without children	235 (4)	289 (84)	347 (234)	340 (172)	384 (99)	456 (45)	530 (2)
4 Couples with 1 child . .	207 (9)	223 (314)	246 (251)	279 (62)	310 (7)	471 (4)	
5 Couples with 2 children	170 (43)	187 (438)	222 (144)	279 (16)	237 (3)	220 (2)	
6 Couples with 3 children	145 (58)	171 (146)	190 (34)	185 (3)	214 (1)		
7 Couples with 4 or more children	121 (72)	148 (40)					
8 Single man with 1 or more children	93 (2)	207 (2)	188 (5)	432 (3)	278 (2)	313 (1)	
9 Single woman with 1 or more children . . .	160 (7)	233 (36)	259 (33)	343 (71)	243 (3)	630 (1)	
10 Other types	132 (18)	188 (88)	281 (49)	325 (12)	508 (5)	634 (1)	

Table VI,1c. Average expenditure per person on food in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0- 1,999	2,000- 3,999	4,000- 5,999	6,000- 7,999	8,000- 9,999	10,000- 14,999	15,000 and above
1 Single man.....		720 (6)	1392 (52)	1944 (51)	2136 (69)	2515 (50)	2310 (10)
2 Single woman.....	647 (5)	674 (25)	1201 (42)	1687 (99)	1750 (69)	1882 (44)	2027 (8)
3 Couples without children.....	1056 (4)	1208 (84)	1537 (234)	1711 (172)	1894 (99)	2053 (45)	1743 (2)
4 Couples with 1 child..	799 (9)	1050 (324)	1272 (251)	1502 (67)	1588 (7)	1455 (4)	
5 Couples with 2 children	722 (43)	940 (438)	1122 (144)	1322 (16)	1214 (3)	1560 (2)	
6 Couples with 3 children	672 (58)	896 (146)	1156 (34)	1518 (3)	1174 (1)		
7 Couples with 4 or more children.....	599 (72)	849 (40)					
8 Single man with 1 or more children.....	839 (2)	1048 (2)	1622 (5)	2014 (3)	2039 (2)	2100 (1)	
9 Single woman with 1 or more children...	631 (7)	997 (36)	1368 (33)	1773 (7)	1366 (3)	1724 (1)	
10 Other types.....	723 (18)	919 (88)	1292 (49)	1585 (12)	1353 (5)	2840 (1)	

Table VI,1d. Average expenditure per person on tobacco in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0- 1,999	2,000- 3,999	4,000- 5,999	6,000- 7,999	8,000- 9,999	10,000- 14,999	15,000 and above
1 Single man.....	91 (1)	362 (7)	414 (52)	472 (50)	646 (68)	802 (49)	672 (10)
2 Single woman.....	228 (8)	98 (25)	220 (42)	335 (99)	315 (68)	300 (44)	319 (8)
3 Couples without children.....	116 (4)	188 (84)	272 (234)	405 (172)	411 (99)	452 (44)	357 (1)
4 Couples with 1 child..	148 (9)	173 (314)	258 (251)	326 (62)	286 (7)	298 (4)	
5 Couples with 2 children	78 (43)	150 (438)	212 (144)	263 (16)	311 (3)	264 (2)	
6 Couples with 3 children	87 (58)	137 (146)	206 (34)	405 (3)	364 (1)		
7 Couples with 4 or more children.....	64 (71)	144 (40)					
8 Single man with 1 or more children.....	28 (2)	431 (2)	254 (4)	24 (2)	453 (2)	430 (1)	
9 Single woman with 1 or more children...	18 (7)	121 (36)	195 (32)	299 (7)	263 (3)	314 (1)	
10 Other types.....	64 (18)	131 (88)	253 (49)	440 (12)	471 (5)	900 (1)	

Table VI,1e. Average expenditure per person on clothing in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man	131 (1)	453 (7)	433(52)	530 (51)	657 (69)	1005 (50)	1267 (10)
2 Single woman	211 (11)	457 (25)	530 (42)	760 (99)	959 (69)	1152 (44)	1240 (8)
3 Couples without children	274 (4)	215 (84)	403 (234)	572 (172)	753 (99)	937 (45)	2489 (2)
4 Couples with 1 child . .	80 (9)	258 (314)	437 (251)	569 (62)	863 (7)	926 (4)	
5 Couples with 2 children	136 (43)	259 (438)	464 (144)	827 (16)	592 (3)	336 (2)	
6 Couples with 3 children	171 (58)	244 (146)	446 (34)	581 (3)	1064 (1)		
7 Couples with 4 or more children	124 (72)	297 (40)					
8 Single man with 1 or more children	94 (2)	355 (2)	411 (5)	574 (3)	1167 (2)	680 (1)	
9 Single woman with 1 or more children	103 (7)	322 (36)	473 (33)	539 (7)	1052 (3)	636 (1)	
10 Other types	123 (18)	251 (88)	359 (49)	578 (12)	551 (5)	971 (1)	

Table VI,1f. Average expenditure per person on footwear in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man	35 (1)	81 (7)	87 (52)	118 (51)	125 (69)	151 (50)	144 (10)
2 Single woman	85 (7)	117 (25)	125 (42)	159 (99)	184 (69)	193 (44)	202 (8)
3 Couples without children	38 (4)	48 (84)	87 (234)	111 (172)	136 (99)	156 (45)	259 (2)
4 Couples with 1 child . .	36 (9)	68 (314)	97 (251)	127 (62)	109 (7)	104 (4)	
5 Couples with 2 children	43 (43)	74 (438)	107 (144)	165 (16)	144 (3)	111 (2)	
6 Couples with 3 children	52 (58)	73 (146)	118 (34)	104 (3)	110 (1)		
7 Couples with 4 or more children	41 (72)	76 (40)					
8 Single man with 1 or more children	52 (2)	94 (2)	81 (5)	158 (3)	227 (2)	136 (1)	
9 Single woman with 1 or more children	58 (7)	104 (36)	133 (33)	117 (7)	167 (3)	175 (1)	
10 Other types	41 (18)	60 (88)	78 (49)	115 (12)	118 (5)	178 (1)	

Table VI,lg. Average expenditure per person on washing and cleaning in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0- 1,999	2,000- 3,999	4,000- 5,999	6,000- 7,999	8,000- 9,999	10,000- 14,999	15,000 and above
1 Single man.....	(1)	107 (7)	150 (52)	180 (51)	213 (69)	297 (48)	247 (10)
2 Single woman.....	24 (6)	38 (25)	92 (42)	147 (99)	165 (69)	199 (44)	207 (8)
3 Couples without children.....	44 (4)	60 (84)	88 (234)	115 (172)	154 (99)	199 (45)	176 (2)
4 Couples with 1 child..	40 (9)	63 (314)	96 (251)	114 (62)	84 (7)	103 (4)	
5 Couples with 2 children	34 (43)	59 (438)	81 (144)	120 (16)	99 (3)	81 (2)	
6 Couples with 3 children	36 (58)	53 (146)	79 (34)	41 (3)	47 (1)		
7 Couples with 4 or more children.....	32 (72)	49 (40)					
8 Single man with 1 or more children.....	31 (2)	86 (2)	133 (5)	123 (3)	135 (2)	65 (1)	
9 Single woman with 1 or more children.....	59 (7)	74 (35)	101 (33)	124 (7)	95 (3)	256 (1)	
10 Other types.....	29 (18)	53 (88)	80 (49)	150 (12)	176 (5)	89 (1)	

Table VI,h. Average expenditure per person on durables excl. vehicles in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0- 1,999	2,000- 3,999	4,000- 5,999	6,000- 7,999	8,000- 9,999	10,000- 14,999	15,000 and above
1 Single man.....		53 (7)	75 (48)	219 (51)	268 (67)	548 (50)	1179 (10)
2 Single woman.....	22 (9)	99 (25)	335 (42)	406 (99)	682 (69)	653 (44)	344 (8)
3 Couples without children.....	82 (4)	140 (84)	343 (234)	599 (172)	511 (99)	592 (45)	504 (2)
4 Couples with 1 child..	54 (9)	193 (314)	306 (251)	320 (62)	398 (17)	905 (4)	
5 Couples with 2 children	74 (43)	167 (438)	296 (144)	501 (16)	130 (3)	622 (2)	
6 Couples with 3 children	91 (58)	131 (146)	262 (34)	196 (3)	438 (1)		
7 Couples with 4 or more children.....	66 (72)	150 (40)					
8 Single man with 1 or more children.....	32 (2)	80 (2)	138 (5)	244 (3)	502 (2)	222 (1)	
9 Single woman with 1 or more children.....	123 (7)	99 (36)	300 (33)	291 (7)	305 (3)	428 (1)	
10 Other types.....	39 (18)	147 (88)	281 (49)	244 (12)	269 (5)	329 (1)	

Table VI,li. Average expenditure per person on personal hygiene in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0- 1,999	2,000- 3,999	4,000- 5,999	6,000- 7,999	8,000- 9,999	10,000- 14,999	15,000 and above
1 Single man.....	113 (1)	105 (7)	129 (52)	159 (51)	162 (69)	206 (48)	219 (16)
2 Single woman.....	99 (6)	146 (25)	185 (42)	302 (99)	296 (69)	320 (44)	413 (8)
3 Couples without children.....	27 (4)	64 (84)	117 (234)	170 (172)	214 (99)	243 (45)	142 (2)
4 Couples with 1 child..	37 (9)	83 (314)	124 (251)	155 (62)	121 (7)	169 (4)	
5 Couples with 2 children	42 (43)	79 (438)	122 (144)	164 (16)	244 (3)	120 (2)	
6 Couples with 3 children	43 (58)	70 (146)	114 (34)	75 (3)	173 (1)		
7 Couples with 4 or more children.....	32 (72)	66 (40)					
8 Single man with 1 or more children.....	27 (2)	71 (2)	107 (5)	131 (3)	196 (2)	196 (1)	
9 Single woman with 1 or more children.....	46 (7)	99 (35)	148 (33)	186 (7)	306 (3)	363 (1)	
10 Other types.....	33 (18)	66 (88)	120 (49)	153 (12)	133 (5)	207 (1)	

Table VI,lj. Average expenditure per person on books, newspapers, etc. in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0- 1,999	2,000- 3,999	4,000- 5,999	6,000- 7,999	8,000- 9,999	10,000- 14,999	15,000 and above
1 Single man.....	(1)	63 (7)	85 (52)	112 (51)	148 (69)	233 (48)	280 (10)
2 Single woman.....	53 (6)	70 (25)	87 (42)	136 (99)	164 (69)	220 (44)	334 (8)
3 Couples without children.....	40 (4)	55 (84)	84 (234)	111 (172)	142 (99)	160 (45)	71 (2)
4 Couples with 1 child..	24 (9)	46 (314)	76(251)	90 (62)	120 (7)	97 (4)	
5 Couples with 2 children	25 (43)	45 (438)	71 (144)	86 (16)	63 (3)	56 (2)	
6 Couples with 3 children	24 (58)	44 (146)	58 (34)	55 (3)	32 (1)		
7 Couples with 4 or more children.....	18 (72)	44 (40)					
8 Single man with 1 or more children.....	20 (2)	65 (2)	52 (5)	79 (3)	60 (2)	273 (1)	
9 Single woman with 1 or more children.....	18 (7)	41 (35)	81 (32)	130 (7)	248 (3)	297 (1)	
10 Other types.....	24 (18)	40 (88)	74 (49)	83 (12)	119 (15)	240 (1)	

Table VI,1k. Average expenditure per person on sports, holidays, hobbies in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man	851 (1)	305 (7)	574 (52)	730 (51)	1122 (69)	1430 (50)	2774 (10)
2 Single women	74 (11)	172 (25)	311 (42)	602 (99)	880 (69)	1463 (44)	2061 (8)
3 Couples without children	36 (4)	123 (84)	330 (234)	596 (172)	839 (99)	1288 (45)	1969 (2)
4 Couples with 1 child . .	52 (9)	189 (314)	360 (251)	741 (62)	1189 (7)	940 (4)	
5 Couples with 2 children	54 (43)	177 (438)	468 (144)	826 (16)	846 (3)	1068 (2)	
6 Couples with 3 children	62 (58)	165 (146)	397 (34)	760 (3)	1430 (1)		
7 Couples with 4 or more children	47 (72)	204 (40)					
8 Single man with 1 or more children	33 (2)	285 (2)	367 (4)	399 (3)	547 (2)	1303 (1)	
9 Single woman with 1 or more children	64 (7)	153 (36)	421 (33)	646 (7)	955 (3)	987 (1)	
10 Other types	66 (18)	169 (88)	352 (49)	705 (12)	571 (5)	1266 (1)	

Table VI,1l. Average expenditure per person on transport incl. own car in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man	104 (1)	199 (7)	324 (52)	889 (51)	762 (69)	1116 (50)	897 (10)
2 Single woman	61 (10)	126 (25)	219 (42)	379 (99)	432 (69)	518 (44)	520 (8)
3 Couples without children	42 (4)	105 (84)	227 (234)	429 (172)	750 (99)	1446 (45)	674 (2)
4 Couples with 1 child . .	44 (9)	157 (314)	359 (251)	490 (62)	542 (7)	1609 (4)	
5 Couples with 2 children	48 (43)	137 (438)	359 (144)	659 (16)	1359 (3)	3406 (2)	
6 Couples with 3 children	40 (58)	128 (146)	407 (34)	392 (3)	212 (1)		
7 Couples with 4 or more children	54 (72)	206 (40)					
8 Single man with 1 or more children	171 (2)	19 (2)	319 (4)	940 (3)	71 (2)	992 (1)	
9 Single woman with 1 or more children	50 (7)	86 (36)	144 (33)	297 (7)	336 (3)	264 (1)	
10 Other types	64 (18)	164 (88)	247 (49)	866 (12)	161 (5)	16 (1)	

Table VI,lm. Average expenditure per person on union fees, subscriptions etc. in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man.....	(1)	106 (7)	234 (52)	365 (51)	345 (69)	384 (48)	248 (10)
2 Single woman.....	87 (5)	105 (25)	197 (42)	220 (99)	197 (69)	227 (44)	194 (8)
3 Couples without children.....	209 (4)	189 (84)	247 (234)	262 (172)	230 (99)	208 (45)	85 (2)
4 Couples with 1 child..	101 (9)	150 (314)	155 (251)	179 (62)	193 (7)	119 (4)	
5 Couples with 2 children	104 (43)	125 (438)	131 (144)	94 (16)	132 (3)	187 (2)	
6 Couples with 3 children	102 (58)	99 (146)	117 (34)	164 (3)	121 (1)		
7 Couples with 4 or more children.....	67 (72)	87 (40)					
8 Single man with 1 or more children.....	92 (2)	114 (2)	134 (2)	197 (3)	292 (2)	467 (1)	
9 Single woman with 1 or more children.....	44 (7)	83 (35)	111 (32)	238 (7)	168 (3)	124 (1)	
10 Other types.....	65 (18)	107 (88)	146 (49)	179 (12)	112 (5)	78 (6)	

Table VI,ln. Average «expenditure» per person on savings in certain income groups separately for different types of household.

Type of household	Disposable income per person						
	0-1,999	2,000-3,999	4,000-5,999	6,000-7,999	8,000-9,999	10,000-14,999	15,000 and above
1 Single man.....	2 (1)	10 (6)	43 (52)	54 (50)	43 (68)	29 (50)	158 (10)
2 Single woman.....	30 (9)	40 (25)	43 (39)	30 (97)	49 (69)	95 (44)	407 (8)
3 Couples without children.....	52 (4)	25 (84)	34 (234)	29 (172)	72 (99)	77 (45)	181 (2)
4 Couples with 1 child..	6 (7)	7 (314)	13 (251)	20 (62)	62 (7)	31 (4)	
5 Couples with 2 children	4 (38)	4 (438)	14 (144)	85 (16)	4 (3)	47 (2)	
6 Couples with 3 children	2 (14)	12 (34)	40 (3)	1 (1)			
7 Couples with 4 or more children.....	-1 (49)	3 (29)					
8 Single man with 1 or more children.....	-11 (2)	1 (1)	-6 (3)	20 (2)	10 (2)	18 (1)	
9 Single woman with 1 or more children.....	6 (5)	18 (26)	11 (32)	1 (6)	25 (3)	170 (1)	
10 Other types.....	16 (10)	9 (75)	21 (49)	54 (12)	197 (5)	252 (1)	

clusions from the averages shown. In general it may be said that even if the conversion of all expenditure and income figures from amounts per household into amounts per person has undoubtedly eliminated also a substantial part of influence of the household type, there still remain systematic effects on the income-expenditure relationship arising from differences in household type among the observed households. This conclusion leads to the observation that the existing differences between the residential and social groups may to some extent be caused by systematic differences in household types between these groups.

Figures VI,1 and VI,2 show income and expenditure on the items of food and of sports, holidays and hobbies, the most typical necessity item and luxury item, respectively according to the figures in the tables. For the necessity item the diagram shows that the expenditure per person for given income per person falls appreciably with the number of persons (children) in the household. With a given income (per person) the large households spend a smaller share of their income on food than households with few persons. The expenditure items of dwelling and tobacco show the same picture. However, in the case of sports, holidays and hobbies the figure shows an equally appreciable shift in the opposite direction: for a given income per person the expenditure (per person) rises with the size of household. The items of clothing and transport present the same picture, if not quite so markedly.

The conclusion of these results compared with the results of the residential and social grouping effect referred to in chapter V above is that future analyses of consumption survey data should presumably place more emphasis on the household type effect and less on differences in residential and social grouping.

If in the present survey the observations had been divided into only four groups by residence and social grouping (one group of salary earners and one of wage earners in the two areas Copenhagen and the rest of Denmark) it should have been possible to undertake Engel curve analyses separately for three household types within each of these four groups of wage and salary earners and have the same number of observations available in each Engel curve analysis as in the present survey. Judging from the results shown above it seems that such a breakdown of the observations would have given greater homogeneity in the individual subgroups, and thus presumably more stable Engel curves (i.e. less residual variation) and consequently a more precise description of the observed income-expenditure relationships.

VI.b. Multiple Regression Analyses.

As the main tool in the description of the expenditure behaviour of households of wage and salary earners in this analysis the Engel function has been chosen, in which the disposable income of the household has been included as the only explanatory variable. A decisive reason for this choice was, of course, that for several purposes it is of interest to know possible expenditure reactions to changes in disposable income.

The object of the analysis has thus been shifted slightly away from the general one of providing a description of consumption behaviour towards an attempt to show the effect of income on expenditure. Accordingly it has been attempted to isolate this effect

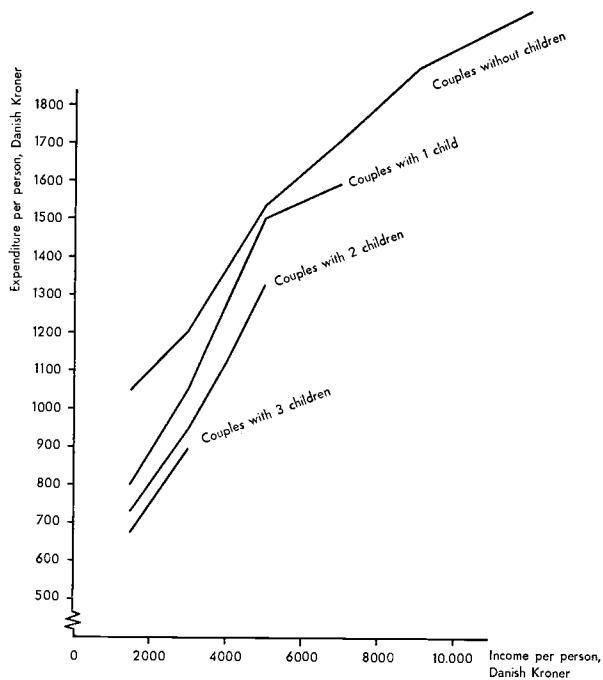


Fig. VI, 1. Income and expenditure on sport, holidays and hobbies per person for four different types of households.

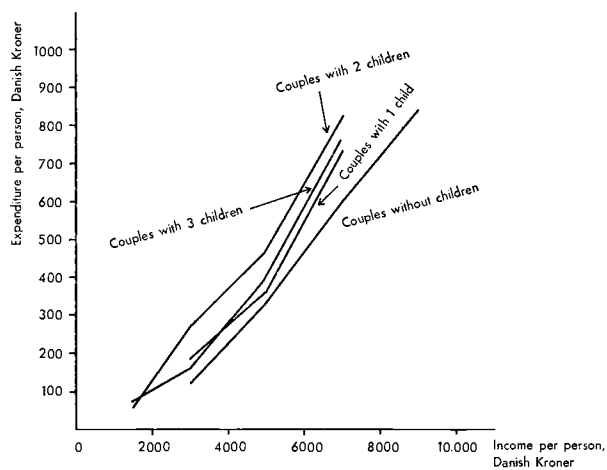


Fig. VI, 2. Income and expenditure on food per person for four different types of households.

by careful specification of the two variables and by a subdivision into residential and social grouping.

However, if in an attempt to arrive at a good description of the expenditure behaviour of the households one does not wish to be constrained by the considerations that led to placing the main emphasis on the influence of income and thereby to the preoccupation with the Engel curve, it seems natural to try to make the description more complete by introducing more explanatory variables in the expenditure functions in addition to disposable income.

Among such variables which may be imagined to influence the expenditures of the households on the different expenditure items there are a great many about which the basic material gives no information. There are the prices of the commodities, but also past or expected changes in these prices as well as variables reflecting plans or expectations in the households.

However, the observations do contain additional information which might be included in the description of household expenditures on the different commodity groups; presumably the residual variation of y could be reduced thereby. The information referred to concerns *expenditures* on one or more other commodity groups, the *personal wealth of the households*, their *saving* during the survey period and finally the *income changes* experienced by the households during the preceding two-year period. Information on the *year of establishment of the households* should be valuable as a supplementary variable in the case of certain commodity groups (dwelling, durable goods).

Appendix C gives data on the expenditures of the individual households on all the 13 items, so that in the analysis of one expenditure item it would be very simple to include information on the expenditure of the individual households on the other items; the appendix moreover contains information which makes it possible to include all the other explanatory variables suggested above.

The present chapter only takes up the expenditure on one of the other commodity groups for further discussion. This is not because it is thought that this supplementary information necessarily gives the greatest contribution to reducing the residual variation of y , but because—as a byproduct of the main analysis—it is possible to ascertain for each expenditure item y_1 the expenditure item y_h which will give the greatest effect if added as supplementary explanatory variable.

The more comprehensive analysis introducing one or more of the data shown in the appendix (income changes, personal wealth, etc.) with a view to providing a more adequate description or explanation of the expenditure behaviour of the households, therefore still remains. In this connection it should be mentioned that in a separate study of the saving pattern of households it was found that particularly income changes seemed to have great effect³). Households with rising incomes recorded considerably higher savings than households who had experienced a fall in income. Moreover, differences in personal wealth were reflected in significant differences in saving (for given income level), households whose personal wealth was around zero saving less than households with considerable positive or negative wealth⁴). It is, perhaps, not unreasonable to expect that these variables would also have an effect in the explanation of the expenditure on some of the commodity groups (particularly durable goods).

In choosing an expenditure item, y_h , as supplementary explanatory variable for the expenditure y_i , the criterion for selecting y_h must be some expression of the correlation between y_i and y_h . However, this expression must naturally be adjusted for the influence of income, on both variables. The problem is this: If a household with a given income, residence, social grouping, household type, etc., has a higher expenditure on item i than expected according to the "best" Engel function, can this be ascribed to any appreciable extent to this household's expenditure on item h ?

It is here necessary to point out that every expenditure item y_i can be described or "explained" exhaustively by means of the income and *all* other $m - 1$ expenditure items including savings on the basis of the budget relation

$$(VI, 3) \quad x = \sum_{i=1}^m y_i .$$

It is obvious that a given household's expenditure on commodity group No. i is uniquely determined if the disposable income and all other uses of that income are included in the explanation. There will then be so many constraints on the variables that there simply are no more degrees of freedom left. According to the budget relation (VI, 3) the following identity exists

$$(VI, 4) \quad y_i = x - \sum_{h=1}^m y_h .$$

The—positive or negative—correlation between the residuals in the relations describing household expenditures on items i and h will determine whether it will be useful to include y_h in the description of y_i . It is evident that such a correlation will be most marked in the case of commodities which are close substitutes or complements in the consumption of the households. Abnormally high consumption of butter will thus, probably, occur at the same time as abnormally low consumption of margarine, high values for expenditure on petrol will often occur together with high values for expenditure on purchases of motor cars, etc.

In the breakdown of expenditure items which has been undertaken in the present analysis it has been attempted to place commodities which, in the opinion of the household are closely related to one another in the same group, cf. chapter IV, p. 38; the 13 expenditure items represent as far as possible 13 "unrelated" commodity categories, and the mentioned—positive or negative—correlation cannot therefore be expected to have any high value.

If now the best of the Engel functions studied is taken and for each of the expenditure items in question are calculated the residuals for the individual households from the expected expenditure, a table of the correlation between these deviations for each item can be set up. Such a correlation table for the group of skilled workers in the capital has been given in table VI,2. In the appendix will be found such tables for all twelve groups of wage

³⁾ Opsparing i lønmodtagerhusstandene 1955, *Statistiske Undersøgelser*, No. 3, Copenhagen 1960, p. 31.

⁴⁾ Same as above, pp. 31–32.

Table VI,2. Correlation between each two of thirteen expenditure items.

Skilled workers. The Capital.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.369	-0.117	0.003	-0.324	-0.283	-0.046	0.054	-0.047	0.155	0.025	0.017	0.162
2.....	-	-	0.110	-0.057	-0.185	-0.146	-0.126	0.015	-0.038	0.205	-0.066	-0.124	0.225
3.....	-	-	-	-0.017	-0.193	-0.037	0.267	-0.165	0.065	0.079	0.048	-0.340	0.197
4.....	-	-	-	-	-0.127	-0.089	0.244	0.085	0.102	0.034	0.171	-0.185	-0.031
5.....	-	-	-	-	-	0.480	-0.176	0.112	0.265	0.061	0.110	0.010	-0.207
6.....	-	-	-	-	-	-	-0.132	0.056	0.120	-0.077	0.074	-0.077	-0.122
7.....	-	-	-	-	-	-	-	-0.055	0.018	0.099	0.204	-0.035	0.086
8.....	-	-	-	-	-	-	-	-	0.076	0.152	0.109	-0.182	-0.039
9.....	-	-	-	-	-	-	-	-	-	0.173	0.116	0.018	-0.055
10.....	-	-	-	-	-	-	-	-	-	-	0.165	-0.110	0.064
11.....	-	-	-	-	-	-	-	-	-	-	-	-0.219	-0.227
12.....	-	-	-	-	-	-	-	-	-	-	-	-	-0.097
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

Unskilled workers. The Capital.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.336	-0.097	0.024	0.161	0.122	0.151	0.067	0.169	0.074	0.034	-0.167	-0.016
2.....	-	-	-0.166	-0.034	-0.171	-0.123	-0.073	0.133	0.079	-0.031	-0.049	0.020	0.127
3.....	-	-	-	0.133	0.025	-0.086	0.275	-0.143	0.014	0.114	0.049	-0.032	0.186
4.....	-	-	-	-	-0.100	-0.030	0.186	-0.104	0.025	0.069	0.024	0.041	0.129
5.....	-	-	-	-	-	0.532	0.172	0.196	0.338	0.126	0.182	-0.187	0.034
6.....	-	-	-	-	-	-	0.265	0.099	0.300	0.053	0.092	-0.032	-0.066
7.....	-	-	-	-	-	-	-	-0.034	0.121	0.180	-0.049	-0.026	-0.029
8.....	-	-	-	-	-	-	-	-	0.111	-0.023	-0.053	-0.161	0.040
9.....	-	-	-	-	-	-	-	-	-	0.110	0.083	-0.012	0.114
10.....	-	-	-	-	-	-	-	-	-	-	0.102	-0.171	0.190
11.....	-	-	-	-	-	-	-	-	-	-	-	-0.164	0.186
12.....	-	-	-	-	-	-	-	-	-	-	-	-	-0.016
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

1. Dwelling. 2. Fuel and light. 3. Food. 4. Tobacco. 5. Clothing. 6. Footwear. 7. Washing and cleaning. 8. Durable goods. 9. Personal hygiene. 10. Books, newspapers etc. 11. Sports, holidays, etc. 12. Transportation. 13. Union fees and subscriptions.

and salary earners, cf. appendix B, p. 174. Table VI,2 shows for each of the 13 expenditure items the correlation between the deviations of the individual households from the expected expenditure on this item and the deviation from the expected expenditure on each of the other 12 expenditure items—where the expected expenditure has been calculated according to the double logarithmic Engel function.

An estimate of the correlation between the deviations of the observed values of two expenditure items from the calculated values according to the double logarithmic function should be based on the logarithmically transformed expenditures. However, since, i.a. for technical reasons, correlations had to be computed on the basis of the individual households and not on the basis of the deviations of expenditures of *groups* of households, the zero observation problem again arose, cf. p. 36 above, and it was therefore decided to base the calculations on the deviations of the untransformed, observed expenditures from the antilogarithm of the calculated value $f(x) = a + b(\log x - \bar{\log x})$. For expenditure items No. i and h the estimate of the correlation coefficient, r , is accordingly obtained in the following way

$$r = \frac{S P D_{i, h}}{\sqrt{S S D_i \cdot S S D_h}}$$

where

$$S P D_{i, h} = \sum_j^N (Y_{i, j} - y_{i, j})(Y_{h, j} - y_{h, j}) - \frac{\sum_j^N (Y_{i, j} - y_{i, j}) \sum_j^N (Y_{h, j} - y_{h, j})}{N}$$

and

$$S S D_i = \sum_j^N (Y_{i, j} - y_{i, j})^2 - \frac{\left(\sum_j^N (Y_{i, j} - y_{i, j}) \right)^2}{N}$$

and

$$S S D_h = \sum_j^N (Y_{h, j} - y_{h, j})^2 - \frac{\left(\sum_j^N (Y_{h, j} - y_{h, j}) \right)^2}{N}$$

where $Y_{i, j} = \text{antilog } f_i(x_j)$ and $Y_{h, j} = \text{antilog } f_h(x_j)$

Since it is expenditures rather than logarithms of expenditures which are used, the expenditure deviations will, in the case of the high income brackets, be included with too much weight a consequence of which is a certain bias. The primary aim in computing the correlations has been to find the sets of expenditure items which were "closest" or "farthest", and not to measure *how* close or *how* far. In other words the computations offer a qualitative rather than a quantitative impression; it has not, accordingly, been attempted to introduce any correction for the bias in the correlation coefficients and the tables must therefore be read with this reservation in mind.

It must also be borne in mind that the budget relation (VI, 3) brings about a slight negative correlation between the expenditure residuals in the equations explaining the in-

Table VI,3. The biggest positive and negative correlation coefficient

Expenditure item	Group of wage				
	1	2	3	4	
1 Dwelling.....	positive	2 0.309	2 0.395	2 0.369	2 0.336
	negative	13 -0.100	4 -0.055	5 -0.324	12 -0.167
2 Fuel and light.....	positive	1 0.309	1 0.395	1 0.369	1 0.336
	negative	11 -0.385	11 -0.144	5 -0.185	5 -0.171
3 Food.....	positive	7 0.365	4 0.262	7 0.267	7 0.275
	negative	8 -0.045	12 -0.261	12 -0.340	2 -0.166
4 Tobacco.....	positive	3 0.288	3 0.262	7 0.244	7 0.186
	negative	13 -0.156	6 -0.064	12 -0.185	8 -0.104
5 Clothing.....	positive	6 0.578	6 0.469	6 0.480	5 0.532
	negative	2 -0.240	2 -0.125	1 -0.324	12 -0.187
6 Footwear.....	positive	5 0.578	5 0.469	5 0.480	5 0.532
	negative	12 -0.112	2 -0.075	1 -0.283	2 -0.132
7 Washing and cleaning.....	positive	3 0.365	9 0.236	3 0.267	3 0.275
	negative	12 -0.027	8 -0.043	5 -0.176	2 -0.073
8 Durable goods.....	positive	1 0.176	6 0.148	10 0.152	5 0.196
	negative	10 -0.139	12 -0.054	12 -0.182	12 -0.161
9 Personal hygiene.....	positive	5 0.485	5 0.365	5 0.265	5 0.338
	negative	2 -0.145	2 -0.139	13 -0.055	12 -0.012
10 Books, newspapers etc.....	positive	9 0.381	13 0.347	2 0.205	13 0.190
	negative	2 -0.181	12 -0.116	12 -0.110	12 -0.171
11 Sports, holidays etc.....	positive	5 0.359	4 0.102	7 0.204	13 0.186
	negative	2 -0.385	10 -0.116	13 -0.227	12 -0.167
12 Transportation.....	positive	13 0.065	4 0.102	9 0.018	4 0.041
	negative	6 -0.114	10 -0.116	3 -0.340	5 -0.187
13 Union fees and subscriptions.....	positive	11 0.141	10 0.347	2 0.225	10 0.190
	negative	2 -0.156	6 -0.006	11 -0.227	6 -0.066

dividual items. A household with an expenditure considerably above the expected expenditure on an item which weighs heavily in the budget must necessarily display negative deviations from the expected expenditure on one or more of the other items. However, this cannot be seen from the correlation tables, partly because the income concept used, disposable income, is not precisely equal to the sum of the 13 expenditure items studied, as some small items (domestic help, gifts and charity, etc.) are left out, which is also the case for the part of the disposable income used for saving, and partly because of the above-mentioned bias in the correlation estimates.

Only very few of these correlation estimates exceed a level as moderate as 0.5, which is some sort of a confirmation of the impression that the commodity classification has been reasonable, cf. above. Even if very high correlation values do not occur, there is still, in the case of some items, a considerable correlation between the deviations of the households from the expected expenditure behaviour. Consequently the description of the expenditure behaviour of the households with regard to these commodity groups will evidently become more satisfactory (resulting in a lower residual variance of the

separately for each group of wage and salary earners.

and salary earners*)

5		6		7		8		9		10		11		12	
2	0.294	2	0.250	2	0.278	2	0.292	2	0.183	2	0.341	2	0.191	2	0.491
12	-0.122	11	-0.160	4	-0.302	11	-0.335	8	-0.199	6	-0.205	3	-0.174	11	-0.233
1	0.294	1	0.250	1	0.278	1	0.292	1	0.183	1	0.341	10	0.296	1	0.491
5	-0.192	9	-0.252	11	-0.274	12	-0.288	11	-0.263	11	-0.224	11	-0.447	11	-0.228
2	0.338	13	0.159	13	0.288	13	0.260	11	0.174	13	0.214	7	0.165	10	0.220
5	-0.235	8	-0.132	8	-0.236	5	-0.388	13	-0.172	8	-0.327	1	-0.174	1	-0.190
9	0.368	7	0.204	7	0.293	10	0.211	11	0.315	2	0.254	9	0.228	7	0.220
8	-0.095	8	-0.094	1	-0.302	8	-0.135	2	-0.099	8	-0.234	2	-0.253	8	-0.124
9	0.507	6	0.510	6	0.596	6	0.590	6	0.428	6	0.496	6	0.447	6	0.458
3	-0.235	2	-0.252	12	-0.396	3	-0.388	2	-0.222	8	-0.292	13	-0.230	12	-0.195
5	0.469	5	0.510	5	0.596	5	0.590	5	0.428	5	0.496	5	0.447	5	0.458
3	-0.200	2	-0.223	12	-0.336	3	-0.327	2	-0.227	1	-0.205	3	-0.101	2	-0.146
13	0.371	4	0.204	11	0.467	9	0.482	9	0.291	5	0.429	13	0.210	11	0.333
12	-0.093	8	-0.044	2	-0.202	12	-0.196	11	-0.152	12	-0.305	1	-0.125	8	-0.214
5	0.130	9	0.052	5	0.191	6	0.088	5	0.251	11	0.054	9	0.240	9	0.290
11	-0.130	3	-0.132	13	-0.242	12	-0.191	1	-0.199	3	-0.327	13	-0.175	11	-0.245
5	0.507	6	0.397	10	0.309	7	0.482	6	0.360	7	0.384	6	0.402	5	0.299
2	-0.182	2	-0.252	12	-0.045	3	-0.326	2	-0.248	8	-0.229	2	-0.146	3	-0.153
11	0.282	11	0.182	9	0.309	4	0.211	7	0.171	9	0.276	2	0.296	7	0.262
12	-0.162	1	-0.130	12	-0.223	8	-0.174	1	-0.112	8	-0.188	11	-0.091	8	-0.115
9	0.323	4	0.191	7	0.467	12	0.411	4	0.315	6	0.216	6	0.254	7	0.333
3	-0.206	2	-0.165	2	-0.274	1	-0.335	2	-0.263	2	-0.224	2	-0.447	8	-0.242
13	0.049	7	0.014	1	0.062	11	0.411	10	0.130	11	0.136	1	0.122	7	0.322
11	-0.182	2	-0.167	5	-0.396	2	-0.288	2	-0.186	7	-0.305	3	-0.173	5	-0.195
7	0.371	3	0.159	3	0.288	3	0.260	7	0.110	3	0.314	7	0.210	10	0.242
8	-0.072	12	-0.139	8	-0.242	9	-0.200	11	-0.180	8	-0.270	5	-0.230	5	-0.162

*) 1. Higher public servants and salaried employees, the Capital; 2. Lower public servants and salaried employees, the Capital; 3. Skilled workers, the Capital; 4. Unskilled workers, the Capital; 5. Higher public servants and salaried employees, the provincial towns; 6. Lower public servants and salaried employees, provincial towns; 7. Skilled workers, provincial towns; 8. Unskilled workers, provincial towns; 9. Lower public servants and salaried employees, rural districts; 10. Skilled workers, rural districts; 11. Unskilled workers, rural districts; 12. Farm workers, rural districts.

dependent variable y) if the expenditure on the commodity group for which the table shows the highest positive or negative correlation is introduced as an explanatory variable in addition to disposable income.

Table VI, 3 shows for each expenditure item the two other items displaying the greatest positive or negative correlation coefficient. Only in the case of three items would the same supplementary determining variable be chosen in all twelve groups of wage and salary earners if this table were used as the criterion. The items of *dwelling* and *fuel & lighting* are so closely related that in the description of one of them the other would everywhere be included as the supplementary variable y_h . This is also true, except for one group of wage and salary earners, for the items of *clothing* and *footwear*. In the case of both sets of expenditures there is positive correlation. For the remaining items the picture changes from one group of wage and salary earners to the other, and especially

Table VI,4. The highest and lowest values of the correlation coefficient of deviations, separately for each social group.

Social groups	Highest		Lowest	
	Expenditure group*)	Coefficients	Expenditure group*)	Coefficient
1 Higher public servants and salaried employees, the Capital	5-6	0.578	2-11	-0.385
	5-9	0.485	2-5	-0.240
	6-9	0.432	2-10	-0.181
	9-10	0.381	2-13	-0.156
	10-11	0.378	4-13	-0.156
2 Lower public servants and salaried employees, the Capital	5-6	0.469	2-11	-0.144
	1-2	0.395	2-9	-0.139
	6-9	0.370	2-5	-0.125
	5-9	0.365	10-12	-0.116
	10-13	0.347	2-6	-0.075
3 Skilled workers, the Capital	5-6	0.480	3-12	-0.340
	1-2	0.369	1-5	-0.324
	3-7	0.267	1-6	-0.283
	5-9	0.265	11-13	-0.227
	4-7	0.244	11-12	-0.219
4 Unskilled workers, the Capital	5-6	0.532	5-12	-0.187
	5-9	0.338	5-2	-0.171
	1-2	0.336	10-12	-0.171
	6-9	0.300	1-12	-0.167
	3-7	0.275	2-3	-0.167
5 Higher public servants and salaried employees, provincial towns	5-9	0.507	3-5	-0.235
	5-6	0.469	3-11	-0.206
	6-9	0.423	3-6	-0.200
	7-13	0.371	2-5	-0.192
	4-9	0.368	2-9	-0.182
6 Lower public servants and salaried employees, provincial towns	5-6	0.510	2-5	-0.252
	6-9	0.397	2-9	-0.252
	5-9	0.390	2-6	-0.223
	1-2	0.250	2-12	-0.167
	4-7	0.204	2-11	-0.165

*) 1. Dwelling. 2. Fuel and light. 3. Food. 4. Tobacco. 5. Clothing. 6. Footwear. 7. Washing and cleaning. 8. Durables excl. own car. 9. Personal hygiene. 10. Books, newspapers etc. 11. Sports, holidays, hobbies, etc. 12. Transport, incl. own car. 13. Union fees, subscriptions etc.

in the case of the negatively correlated items such typically interrelated expenditure sets are lacking.

Table VI,4 gives the five highest coefficients of correlation of each category (positive and negative) for each of the twelve groups of wage and salary earners. Also here speci-

Table VI,4. continued.

Social groups	Highest		Lowest	
	Expenditure group*)	Coefficients	Expenditure group*)	Coefficient
7 Skilled workers, provincial towns	5-6	0.596	5-12	-0.396
	7-11	0.467	6-12	-0.336
	9-10	0.309	2-11	-0.274
	4-7	0.293	8-13	-0.242
	3-13	0.288	3-8	-0.236
8 Unskilled workers, provincial towns	5-6	0.590	3-5	-0.388
	7-9	0.482	1-11	-0.335
	5-7	0.445	3-6	-0.327
	11-12	0.411	3-9	-0.326
	6-9	0.394	2-12	-0.288
9 Lower public servants and salaried employees, rural districts	5-6	0.428	2-11	-0.263
	6-9	0.360	2-9	-0.248
	5-9	0.359	2-6	-0.227
	4-11	0.315	2-5	-0.222
	7-9	0.291	1-8	-0.199
10 Skilled workers, rural districts	5-6	0.496	3-8	-0.237
	5-7	0.429	7-12	-0.305
	7-9	0.384	3-12	-0.301
	1-2	0.341	5-8	-0.292
	3-13	0.314	7-8	-0.274
11 Unskilled workers, rural districts	5-6	0.447	2-11	-0.447
	6-9	0.402	2-4	-0.253
	5-9	0.398	5-13	-0.230
	2-10	0.296	8-13	-0.175
	6-11	0.254	1-3	-0.174
12 Farm workers, rural districts	1-2	0.491	8-11	-0.245
	5-6	0.458	1-11	-0.233
	7-11	0.333	2-11	-0.228
	7-12	0.322	7-8	-0.214
	5-9	0.299	5-12	-0.195

*) 1. Dwelling. 2. Fuel and light. 3. Food. 4. Tobacco. 5. Clothing. 6. Footwear. 7. Washing and cleaning. 8. Durables excl. own car. 9. Personal hygiene. 10. Books, newspapers etc. 11. Sports, holidays, hobbies, etc. 12. Transport, incl. own car. 13. Union fees, subscriptions etc.

ally the negatively correlated items are characterized by great differences in the pattern from one group of wage and salary earners to the other.

How the extra determining variable is to be fitted into the Engel function to yield the maximum reduction in the residual variance of the dependent variable, will not be discussed here.

One technically simple method would be to let y_h or a transformation thereof enter linearly so that the result will be e.g. a function of the following form

$$\log \eta_i = a + \beta (\log v - \overline{\log v}) + \gamma (\log \eta_h - \overline{\log \eta_h})$$

whereby efficient estimates of parameters and of the residual variance on y_i , can be calculated according to the theory of multiple linear regression.

The results of such calculations as regards the case of footwear in the group of skilled workers in the provincial towns are shown in table VI, 5, expenditure on clothing entering as supplementary explanatory variable. The residual variance of $\log y_1$ is reduced by about 35 per cent namely from 0.00928 to 0.00587.

Table VI,5. Unexplained variance in the regression analysis. Skilled workers in provincial towns. $\log y_1 = a + b_1 (\log x - \overline{\log x}) + b_2 (\log y_2 - \overline{\log y_2})$. Expenditure on footwear, y_1 as a function of income, x and expenditure on clothing, y_2 .

$$\begin{aligned} \log y_1 &= 1.8718 + 0.1909 (\log x - 3.5746) + 0.4326 (\log y_2 - 2.4665) \\ s^2_{\log y_1} \mid \log x, \log y_2 &= 0.005873 \\ s_{\log y_1} \mid \log x, \log y_2 &= 0.0766 \\ \log y_1 &= a' + b' (\log y_2 - \overline{\log y_2}) \\ s_{\log y_1} \mid \log y_2 &= 0.0784 \\ \log y_1 &= a'' + b'' (\log x - \overline{\log x}) \\ s_{\log y_1} \mid \log x &= 0.0965 \end{aligned}$$

DANSK RESUMÉ

Undersøgelsen af danske lønmodtagerhusstandes indkomst-, forbrugs- og opsparingsforhold for året 1955, som gennemførtes i begyndelsen af året 1956, er den største og mest detaljerede af de forbrugsundersøgelser, Det statistiske Departement har foretaget, siden man i 1897 påbegyndte denne art af undersøgelser. Forbrugsundersøgelseernes primære formål var oprindeligt at fremskaffe oplysning om »Livsvilkår i de forskellige samfundslag, derunder ernærings- og forbrugsforhold«,¹⁾ men efter at pristalsreguleringen af lønninger og ydelser og tilskud af forskellig art vandt stærkt frem, har de foretagne forbrugsundersøgelser her som i mange andre vesteuropæiske lande i første række skullet tjene som redskab til opstilling af vægte ved prisindeksberegningerne. I de seneste år synes imidlertid det alment beskrivende, som var det primære sigte med de første forbrugsundersøgelser, atter at komme i første række. Dette skyldes bl.a., at man har erkendt, at det grundmateriale, som tilvejebringes ved en omhyggelig planlagt og udført forbrugsundersøgelse – i denne forbindelse må de senere års betydelige fremskridt indenfor undersøgelsesteknikken haves i erindring – rummer oplysninger om væsentlige økonomiske sammenhænge især vedrørende anvendelsen af den indtjente indkomst, der ikke, eller kun mangelfuldt, kan belyses ad anden vej.²⁾

Forbrugsundersøgelsen for året 1955 har da også været genstand for en mere omfattende bearbejdelse end nogen af de foregående undersøgelser.

En almindelig oversigt over 1955-undersøgelsen, dens tilrettelæggelse og dens hovedresultater er givet i Statistiske Efterretninger i 1957.³⁾ Fødevarerforbruget blev særskilt behandlet i en artikel i Statistiske Efterretninger i 1958.⁴⁾ De indhentede oplysninger vedrørende lønmodtagerhusstandenes opsparings- og formueforhold blev gjort til genstand for en særskilt analyse, hvis resultater meddeltes i et hæfte i serien Statistiske Undersøgelser i 1960.⁵⁾ I samme serie behandlede de indhentede oplysninger om lønindkomsternes fordeling og sammensætning⁶⁾.

Hovedparten af de indhentede oplysninger fra de adspurgte lønmodtagerhusstande vedrørte disses husstandes forbrugsudgifter i året 1955, og man besluttede derfor at underkaste lønmodtagernes forbrugsadfærd en mere indgående analyse. Det er resultaterne fra denne analyse, der indeholdes i nærværende publikation.

¹⁾ Lov om Statens statistiske Bureau 1895.

²⁾ Jfr. I. L. O. (11).

³⁾ Statistiske Efterretninger 1957, nr. 83.

⁴⁾ Statistiske Efterretninger 1958, nr. 46.

⁵⁾ Opsparing i lønmodtagerhusstandene 1955, Statistiske Undersøgelser nr. 3, Kbh. 1960.

⁶⁾ Lønmodtagerindkomster, Fordeling og sammensætning, Statistiske Undersøgelser nr. 6, Kbh. 1962.

2. *Analysens hovedresultater.*

Analysen tilsigtede at give en præcis beskrivelse af sammenhængen mellem de danske lønmodtagerhusstandes disponible indkomst og udgiften til nogle væsentlige udgiftsposter i året 1955. Denne sammenhæng mellem disponibel indkomst og udgiften til givne udgiftsposter er utvivlsomt af væsentlig betydning, hvis man vil prøve at forklare forskelle i forbrugsadfærd fra den ene husstand til den anden, selvom naturligvis mange andre forhold spiller ind såsom husstandstype, bopæls- og socialgruppering m.v. Indkomst-udgiftsrelationen er imidlertid tillige af væsentlig betydning, hvis man vil forsøge at foretage skøn over forbrugets sandsynlige udvikling ved givne, alternative indkomstforskydninger, hvad enten dette nu drejer sig om den enkelte husstand eller husstandsgruppe, eller det drejer sig om alle husstande under eet.⁷⁾

Hovedvægten i analysen blev derfor lagt på udledning af de indkomst-udgiftsrelationer, som ifølge det foreliggende grundmateriale gav den bedste beskrivelse af sammenhængen. De nævnte indkomst-udgiftsrelationer går ofte under betegnelsen Engelfunktioner efter den tyske økonom og statistiker, Ernst Engel, og det konkrete analysearbejde har bestået i at beregne skøn over parametrene i fem på forhånd udvalgte funktionstyper og derefter ved et antal test at sammenligne disse funktionstyper for at finde frem til den for hver udgiftspost bedst egnede Engelkurve.

For at eliminere de væsentligste forstyrrende påvirkninger hidrørende fra forskelle i husstandsstørrelse i de undersøgte husstande omregnedes alle udgifts- og indkomstbeløb for hver af de 3100 husstande til beløb pr. person.

Engel funktionens uafhængigt variable fastlagdes som disponibel indkomst (samtlige kontantindtægter minus betalte personlige skatter) pr. person, og der blev udledt Engelfunktioner for følgende 13 udgiftsposter, der tilsammen udgør 85 pct. af totalforbruget for samtlige lønmodtagerhusstande.

1. Bolig.
2. Brændsel og belysning.
3. Fødevarer (incl. regelmæssig fortæring ude og øl, vin og spiritus indenfor det sædvanlige husholdningsforbrug).
4. Tobak.
5. Beklædning.
6. Fodtøj.
7. Vask og rengøring.
8. Varige goder (excl. motorkøretøjer).
9. Personlig pleje.
10. Bøger, aviser m.v.
11. Sport, ferie, fritid m.v. (incl. restaurationsbesøg, teater, biograf og øl, vin og spiritus uden for det sædvanlige husholdningsforbrug).
12. Transport (incl. motorkøretøjer).
13. Kontingent og forsikringer m.v. (excl. livs- og pensionsforsikringer).

Beregningerne udførtes særskilt for 12 lønmodtagergrupper, nemlig 4 socialgrupper indenfor hver af 3 landsdele, jfr. resultattabellen i bilag A. De 5 funktionstyper, hvis

⁷⁾ Jfr. Erling Jørgensen (12), side 54-61.

parametre man dannede skøn over, var følgende, hvor v betegner den disponible indkomst og η udgiften til en given udgiftspost:

$$\begin{aligned} (1) \quad \log \eta &= a + \beta (\log v - \overline{\log v}) \\ (2) \quad \log \eta &= a + \beta \left(\frac{1}{v} - \frac{\bar{1}}{\bar{v}} \right) \\ (3) \quad \eta &= a + \beta (\log v - \overline{\log v}) \\ (4) \quad \eta &= a + \beta \left(\frac{1}{v} - \frac{\bar{1}}{\bar{v}} \right) \\ (5) \quad \log \eta &= \log \kappa + \log [\Phi (a + \log v)] \end{aligned}$$

Væsentlige dele af analyserapporten behandler estimationsproblemer, således at man kan sige, at udredningen af *analysemetoderne* var et andet hovedformål ved analysearbejdet ved siden af selve beregningen af *analyseresultaterne*.

De foretagne testberegninger viste næsten samstemmende, at funktionstype (1), den dobbeltlogaritmiske funktion, for samtlige 13 udgiftsposter gav den bedste fremstilling af Engelrelationen. Dette resultat er i en vis forstand overraskende, fordi det indebærer, at indkomstelasticiteten i de pågældende husstandes efterspørgsel efter de 13 udgiftsposter er konstant (for given socialgruppe, idet beregningerne som nævnt er udført særskilt for 12 lønmodtagergrupper) og altså uafhængig af indkomstniveauet. Dette følger af, at indkomstelasticiteten er identisk med parameteren β i den dobbeltlogaritmiske Engel-funktion. Man måtte vel på forhånd vente, at varegrupper, der i de højeste indkomstgrupper betragtes som nødvendighedsvarer (lav indkomstelasticitet), i de lavere indkomstgrupper ville gå over til at blive betragtet som luksusvarer (høj indkomstelasticitet). Imidlertid viser det sig⁸⁾, at der er en bemærkelsesværdig stabilitet til stede, også når vi går fra lønmodtagergruppe til lønmodtagergruppe for så vidt angår det nævnte parameterskøn over β , skønnet over indkomstelasticiteten. For 6 udgiftsposters vedkommende kan en hypotese om konstant indkomstelasticitet alle 12 lønmodtagergrupper igennem opretholdes, og for de resterende 7 posters vedkommende er afvigelserne omend statistisk signifikante dog ikke særligt store. Konstateringen af denne stabilitet i lønmodtagerhusstandenes indkomstelasticitet i udgiften til de væsentligste udgiftsposter er et af de mest iøjnefaldende resultater af analysearbejdet⁹⁾.

Denne stabilitet gør det forsvarligt at beregne gennemsnit af de 12 lønmodtagergrupperes indkomstelasticiteter for hver de 13 udgiftsposter. Disse gennemsnitselasticiteter er vist i nedenstående oversigt, hvor udgiftsposterne er ordnet efter gennemsnitsindkomstelasticitetens størrelse.

Man ser umiddelbart af denne tabel, at de beregnede gennemsnitselasticiteter falder i tre klart afgrænsede størrelsesgrupper:

1. En gruppe, man kunne kalde nødvendighedsvarer, hvor elasticiteten ligger på godt 0.5, bestående af de tre poster, fødevarer, fodtøj og brændsel og belysning.

⁸⁾ Jfr. kap. V, side 83.

⁹⁾ Dette resultat frister til at postulere, at de fundne indkomstelasticiteter for lønmodtagerbefolkningen har generel gyldighed for alle befolkningsgrupper. Om konsekvenserne heraf se Erling Jørgensen, (12).

Udgiftspost	Indkomst-elasticitet	Gnstl. Indkomst-elasticitet ¹
Nødvendighedsvarer		
Brændsel og belysning.....	0.511	} 0.59
Fodtøj.....	0.562	
Fødevarer.....	0.608	
»Neutrale« varer		
Kontingenter og forsikring.....	0.821	} 0.94
Personlig pleje.....	0.856	
Vask og rengøring.....	0.859	
Bolig.....	0.885	
Bøger, aviser m.v.....	0.977	
Tobak.....	0.980	
Varige goder excl. motorkøretøjer.....	0.989	
Beklædning.....	1.035	
Luksusvarer		
Transport incl. motorkøretøjer.....	1.386	} 1.45
Sport, ferie og fritid.....	1.500	

¹⁾ Ved beregningen af de 3 gennemsnitselasticiteter er de 13 udgiftsposters andel i totalforbruget anvendt som vægte.

2. En anden gruppe, man kunne kalde neutrale varer, hvor elasticiteten ligger på et niveau omkring 1.0, og udgiften derfor stiger med samme procent som indkomsten. I denne gruppe ligger bl.a. de to vigtigste udgiftsposter bolig og beklædning.

3. Endelig er der den tredje gruppe, som man kunne kalde luksusvarer, hvor elasticiteten er ca. 1.5, bestående af de to poster transport (incl. motorkøretøjer) og sport, ferie og fritid.

Analysen gav videre til resultat, at de beregnede 12 Engelkurver for hver udgiftspost – nemlig en for hver af de 12 lønmodtagergrupper, hvori materialet var opdelt – ikke kunne betragtes som sammenfaldende, men at denne opdeling efter bopæl og socialgruppering syntes at modsvare faktisk eksisterende forskelle i forbrugsadfærd de tolv grupper imellem¹⁰⁾.

Hovedformålet med analysen har som nævnt været at formulere en præcis beskrivelse af sammenhængen mellem lønmodtagerhusstandenes indkomst og udgifter til væsentlige udgiftsposter. Den anvendte analysemetode, som overvejende består i lineær regressionsanalyse, synes at give tilfredsstillende resultater for de fleste udgiftsposter, med »pæne« udledte Engelkurvefunktioner til følge. For enkelte poster, især de to poster *varige goder* og *transport (incl. motorkøretøjer)*, er den uforklarede del af udgiftsvariationen fra husstand til husstand imidlertid uforholdsmæssig høj og er kun blevet reduceret ganske lidt ved inddragelsen af husstandenes disponible indkomst i undersøgelsesperioden som uafhængig, forklarende variabel.

Man kan formentlig heraf konkludere, at analysen af husstandenes udgifter til disse poster må gå ad andre veje end den her anvendte, med inddragelse af oplysninger om

¹⁰⁾ Jfr. kapitel V, side 85.

husstandstype og øvrige milieubetingede faktorer samt ikke mindst af oplysninger vedrørende indkomståndringer og tidligere perioders forbrugsadfærd. En sådan dynamisk analyse har imidlertid ligget uden for rammerne af dette arbejde, men det må erkendes, at de her repræsenterede resultater for disse posters vedkommende er utilfredsstillende.

På et par punkter er man gået ud over det ovenfor afgrænsede analyseformål, idet man i et afsluttende kapitel har undersøgt dels, i hvilket omfang de 13 udgiftsposter er korrelerede, d.v.s. om husstande, der giver meget eller lidt ud til een bestemt udgiftspost udviser, en karakterisk udgiftsadfærd med hensyn til een eller flere af de øvrige poster (har husstande med et højt tobaksforbrug et mindre fødevarerforbrug end husstande med lavt forbrug af tobak? etc.); dels har man forsøgt at skitsere, hvilken betydning forskelle i husstandstype (husstandens størrelse og sammensætning) har for husstandenes forbrugsadfærd i de forskellige indkomstklasser.

Hvad det første problem angår – korrelationen mellem de 13 udgiftsposter – viste de foretagne beregninger, at der kun i ringe grad kunne påvises en sådan korrelation. Kun for så vidt angår de to poster *bolig* og *brændsel-belysning* fandtes en stærk (positiv) korrelation. Dette resultat falder godt i tråd med hele oplægget til analysen, idet grupperingen af de mangfoldige varer og tjenester, hvorom oplysninger indhentes, i et beskedent antal hovedudgiftsposter netop sigtede mod en gruppering, hvor der kun var en ringe positiv eller negativ korrelation mellem de enkelte grupper. Herved ville man søge at nå frem til stabile indkomst-udgiftsrelationer, men måtte naturligvis samtidig give afkald på at beskrive husstandenes forbrugsadfærd overfor enkeltvarer og tjenester.

Med hensyn til husstandstypens betydning for husstandenes forbrugsadfærd viste de foretagne undersøgelser, at selve husstandsstørrelsen var den dominerende faktor, og at man ved den foretagne omregning til beløb pr. person fik elimineret størsteparten af denne »forstyrrende« påvirkning. For visse udgiftsposter, bl.a. *bolig* og *fritidsudgifter*, var der imidlertid stadig mærkbare påvirkninger at spore udover personaltaleffekten, og generelt gjaldt det, som man vel også på forhånd ville vente, at der består economies of scale, d.v.s., at udgiften pr. person til en given udgiftspost er faldende med persontallet pr. husstand.

Rapportens kapitel I indeholder en oversigt over arbejdets baggrund og tilrettelæggelse samt over nogle af analysens hovedresultater. Kapitel II er en gennemgang af det anvendte grundmateriale. Denne gennemgang indeholder dels en beskrivelse af forbrugs- og opsparingsundersøgelsens praktiske udførelse, d.v.s. grundmaterialets indsamling og bearbejdelse, og dels en udledning af den statistiske usikkerhed, som de fra undersøgelsesmaterialet udledte tal er behæftet med. I kapitel III afgrænses analyseopgaven, idet forskellige modeller til beskrivelse af de adspurgte husstandes udgiftsadfærd diskuteres, en diskussion der munder ud i en motivering for valget af Engelkurveproblematikken som analysens hovedemne. Kapitel IV indeholder en detaljeret gennemgang af analysemetoderne. Hvilke funktionstyper skal lægges til grund ved udledningen af Engelkurver for de forskellige udgiftsposter? Hvorledes skal de i Engelfunktionen indgående variable nærmere afgrænses? Og ikke mindst, hvorledes skal de anvendte funktionstypers egnethed ved beskrivelsen af indkomst-udgiftsrelationerne afprøves?

Herefter følger i kapitel V rapportens hovedafsnit, nemlig gennemgangen af analyseresultaterne, hvorunder især den dobbeltlogaritmiske Engelkurvefunktion, som efter de

udførte test *for goodness of fit* fandtes at være den »bedste« af de 5 afprøvede funktionstyper, kommenteres.

Endelig er der i det afsluttende kapitel VI anført nogle eksempler på nærliggende videregående beregninger, der skønnes at kunne bidrage til en yderligere præcision i beskrivelsen af husstandenes forbrugsadfærd, end nærværende analyses hovedredskab, Engelkurven, muliggør. Til »forklaring« af de observerede forskelle i husstandenes udgifter til en given udgiftspost fremdrages dels forskelle i husstandenes størrelse og sammensætning og dels husstandenes udgifter til een eller flere andre udgiftsposter.

I bilag til rapporten er anført dels en samlet oversigt over analyseresultaterne, der falder i to afsnit, hovedanalysens resultater, jfr. kap. V og resultater af de videregående beregninger jfr. kap. VI, dels det ved analysen benyttede grundmateriale suppleret med visse yderligere oplysninger, der vil kunne inddrages i eventuelle supplerende analyser.

En liste over den benyttede litteratur er anført side 117–118.

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APPENDICES

Appendix A. Results of regression analysis.

The results comprise the estimates a, k and b for the five Engel functions

$$\begin{aligned}
 1 \quad \log y &= a + b (\log x - \overline{\log x}) \\
 2 \quad \log y &= a + b \left(\frac{1}{x} - \frac{\bar{1}}{\bar{x}} \right) \\
 3 \quad y &= a + b (\log x - \overline{\log x}) \\
 4 \quad y &= a + b \left(\frac{1}{x} - \frac{\bar{1}}{\bar{x}} \right) \\
 5 \quad \log y &= \log k + \log \Phi (a + \log x),
 \end{aligned}$$

x denoting disposable income per person and y denoting expenditure per person on a given item. Also included are the averages of the dependent variable, $\overline{\log x}$ or $\frac{\bar{1}}{\bar{x}}$ and estimates of the standard errors of a and b, s_a and s_b as well as the estimates s_1 and s_2 denoting the square roots of the variance in the distribution of y within groups and of the variance in the distribution of residuals respectively. The tables of result finally contain the results of the following tests: the correlation coefficient, R, between observed and calculated expenditures, N-test for number of runs and l-test for number of elements in the longest run, d-test for size and sign of the residuals, F-test, and χ^2 -test for normality of the residuals.

The limits of significance (5 or 95 per cent) are given in the following table, separately for each of the twelve groups of wage and salary earners¹⁾.

¹⁾ This table do not include the limits of significance for the N-test the test for number of runs, as these limits differ from expenditure item to expenditure item within each group of wage and salary earners, cfr. table V,5.

Limits of significance.

Group of wage and salary earners	Number of groups of 3 households	χ^2 -test		d-test		F-test		t-test t _{.95}
		degrees of freedom	$\chi^2_{.95}$	degrees of freedom	d _{.95}	degrees of freedom	F _{.95}	
1. Higher sal. public servants and empl., the capital.	112	7	14.1	110	1.64	111.224	1.31	11
2. Lower public servants and sal. empl., the capital.	154	9	16.9	512	1.67	153.308	1.26	12
3. Skilled workers, the capital	83	7	14.1	81	1.59	82.166	1.36	10
4. Unskilled workers, the capital.	68	5	11.1	66	1.54	67.136	1.41	10
5. Higher public servants and sal. empl.	70	5	11.1	68	1.55	69.140	1.40	10
6. Lower public servants and sal. empl.	111	7	14.1	109	1.64	110.222	1.31	11
7. Skilled workers provincial towns.	51	3	7.8	49	1.46	50.102	1.48	9
8. Unskilled workers, provincial towns..	70	5	11.1	68	1.55	69.140	1.40	10
9. Lower public servants and sal. empl., rural distr.	102	7	14.1	100	1.63	101.204	1.32	11
10. Skilled workers, rural districts.	51	3	7.8	49	1.46	50.102	1.48	9
11. Unskilled workers, rural districts.	93	7	14.1	91	1.61	92.186	1.34	10
12. Agric. workers, rural distr.	53	3	7.8	51	1.46	52.106	1.47	9

Appendix A. Main results. Higher public servants and salaried

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
1: $\log y = a + b (\log x - \overline{\log x})$						
a.....	2.768	2.452	3.151	2.354	2.723	2.057
b.....	0.960	0.734	0.524	0.799	0.885	0.429
$\overline{\log x}$	3.806	3.806	3.806	3.806	3.806	3.806
s_1	0.117	0.105	0.0548	0.197	0.116	0.116
s_2	0.141	0.120	0.0690	0.247	0.127	0.127
s_a	0.013	0.011	0.0065	0.023	0.012	0.012
s_b	0.070	0.060	0.034	0.12	0.063	0.063
R.....	0.79	0.76	0.82	0.53	0.80	0.54
N.....	49	63	63	47	48	58
l.....	9	7	8	7	7	10
d.....	2.24	2.03	1.85	1.86	1.69	2.07
F.....	1.45	1.30	1.59	1.57	1.20	1.19
$\chi^2 (f)$	9.9 (7)	6.3 (7)	8.9 (7)	21.5 (7)	8.4 (7)	10.0 (7)
2: $\log y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	2.768	2.452	3.151	2.354	2.723	2.057
$b \cdot 10^{-4}$	-0.261	-0.206	-0.151	-0.231	-0.250	-0.123
$\frac{1}{\bar{x}} \cdot 10^{+4}$	1.705	1.705	1.705	1.705	1.705	1.705
s_1	0.117	0.105	0.0548	0.197	0.116	0.116
s_2	0.152	0.123	0.0678	0.246	0.130	0.127
s_a	0.014	0.012	0.0064	0.023	0.012	0.012
$s_b \cdot 10^{-4}$	0.022	0.108	0.0097	0.035	0.019	0.018
R.....	0.75	0.74	0.83	0.53	0.79	0.54
N.....	49	57	52	51	48	62
L.....	9	7	8	7	8	10
d.....	1.89	1.94	1.81	1.87	1.59	2.06
F.....	1.69	1.37	1.53	1.56	1.26	1.19
$\chi^2 (f)$	5.9 (7)	7.4 (7)	8.8 (7)	12.9 (7)	9.2 (7)	6.5 (7)
3: $y = a + b (\log x - \overline{\log x})$						
a.....	500.7	251.7	1322	209.2	450.3	111.9
b.....	1300	525	1880	512	1110	133
$\overline{\log x}$	3.679	3.697	3.724	3.683	3.682	3.735
s_1	0.269	0.242	1.26	0.454	0.267	0.267
s_2	0.344	0.263	0.148	0.445	0.281	0.277
s_a	17	6.6	19	9.4	13	3.0
s_b	110	42	120	61	83	18

employees. The capital. (1.1.) 112 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
2.080	2.448	2.178	1.988	2.800	2.547	2.062
0.803	0.574	0.807	1.015	1.387	0.863	0.634
3.806	3.806	3.806	3.806	3.806	3.806	3.806
0.124	0.195	0.105	0.140	0.138	0.215	0.117
0.130	0.287	0.108	0.138	0.132	0.362	0.126
0.012	0.027	0.010	0.013	0.012	0.034	0.012
0.065	0.14	0.054	0.069	0.066	0.18	0.063
0.76	0.36	0.82	0.81	0.90	0.41	0.69
45	59	57	61	55	49	58
9	10	11	9	6	10	6
1.65	2.21	1.84	2.04	2.14	1.98	2.10
1.10	2.16	1.05	0.97	0.91	2.84	1.17
12.4 (7)	3.9 (7)	5.1 (7)	10.6 (7)	6.0 (7)	25.7 (7)	3.3 (7)
2.080	2.448	2.178	1.988	2.800	2.547	2.062
-0.231	-0.186	-0.226	-0.272	-0.393	-0.274	-0.179
1.705	1.705	1.705	1.705	1.705	1.705	1.705
0.124	0.195	0.105	0.140	0.138	0.215	0.117
0.130	0.281	0.112	0.154	0.137	0.353	0.128
0.012	0.027	0.011	0.015	0.013	0.033	0.012
0.018	0.040	0.016	0.022	0.020	0.050	0.018
0.77	0.40	0.80	0.76	0.89	0.46	0.68
44	62	55	51	49	54	61
12	8	11	11	6	9	6
1.60	2.29	1.70	1.72	1.96	2.09	2.05
1.09	2.08	1.14	1.21	0.98	2.70	1.20
8.6 (7)	11.5 (7)	6.6 (7)	11.6 (7)	4.6 (7)	16.9 (7)	6.9 (7)
104.0	282.8	130.2	82.06	402.1	326.6	107.3
245	666	298	220	1910	1390	186
3.686	3.686	3.689	3.671	3.620	3.635	3.711
0.286	0.449	0.241	0.323	0.319	0.495	0.270
0.285	0.660	0.266	0.429	0.334	1.01	0.302
3.0	19	3.5	3.6	15	36	3.2
19	120	22	24	110	260	20

Appendix A. Main results (continued). Higher public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.73	0.77	0.84	0.62	0.79	0.58
N.....	45	57	56	55	58	60
l.....	11	10	8	6	7	10
d.....	1.95	2.03	1.94	1.75	1.59	2.12
F.....	1.63	1.18	1.38	0.96	1.11	1.07
χ^2 (f)	17.2 (7)	14.8 (7)	13.1 (7)	3.6 (7)	4.3 (7)	8.6 (7)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	508.7	253.9	1328	209.7	456.0	112.1
$b \cdot 10^{-4}$	-290	-119	-448	-116	-244	326
$\frac{\bar{1}}{x} \cdot 10^{+4}$	2.234	2.152	2.021	2.224	2.223	1.977
s_1	0.269	0.242	0.126	0.454	0.267	0.267
s_2	0.388	0.301	0.170	0.472	0.324	0.287
s_a	20	7.6	22	10	15	3.1
$s_b \cdot 10^{-4}$	28	11	31	14	21	4.5
R.....	0.70	0.73	0.81	0.62	0.74	0.57
N.....	43	49	50	61	49	58
l.....	11	10	8	6	12	10
d.....	1.43	1.61	1.50	1.64	1.22	2.03
F.....	2.07	1.55	1.81	1.08	1.48	1.15
χ^2 (f)	27.8 (8)	25.0 (7)	8.0 (7)	15.0 (7)	7.6 (7)	5.8 (7)
5: $\log y = \log k + \log \Phi (a + \log x)$						
a.....	-5.687	-5.075	-4.466	-5.258	-5.490	-4.160
k.....	19870	2813	5646	3132	11650	319.8
s_1	0.117	0.105	0.0548	0.197	0.116	0.116
s_2	0.141	0.119	0.0664	0.245	0.125	0.125
s_a	0.29	0.28	0.16	0.51	0.29	0.36
s_k	12000	1300	1000	2800	6600	110
R.....	0.72	0.76	0.84	0.62	0.79	0.58
N.....	53	61	63	49	50	60
l.....	9	7	8	7	7	10
d.....	2.21	2.05	1.93	1.87	1.74	2.09
F.....	1.45	1.29	1.47	1.54	1.16	1.17
χ^2 (f).....	6.05 (9)	6.35 (9)	13.70 (9)	18.97 (9)	12.26 (9)	13.15 (9)

salaried employees. The capital. (1.1.) 112 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.77	0.46	0.79	0.66	0.85	0.46	0.67
48	64	51	53	51	43	57
12	8	12	9	7	14	10
1.63	2.26	1.70	1.57	1.50	2.20	2.05
1.00	21.6	1.21	1.77	1.10	4.14	1.26
10.4 (7)	39.6 (7)	17.7 (7)	50.5 (7)	8.6 (7)	80.6 (7)	8.6 (7)
104.4	277.9	131.1	85.26	398.8	272.0	109.0
-55.1	-154	-67.0	-45.0	-394	-295	-42.8
2.207	2.228	2.194	2.281	2.586	2.616	2.067
0.286	0.449	0.241	0.323	0.319	0.495	0.270
0.315	0.697	0.305	0.571	0.449	0.956	0.332
3.3	20	4.0	4.8	21	31	3.5
4.6	28	5.6	6.7	29	44	5.0
0.75	0.47	0.75	0.56	0.79	0.54	0.63
46	62	37	49	39	45	51
12	8	15	10	11	14	10
1.34	2.22	1.35	1.16	0.81	2.22	1.86
1.21	2.41	1.60	3.13	1.99	3.73	1.51
7.6 (7)	42.1 (7)	35.7 (7)	68.4 (7)	35.1 (7)	86.4 (7)	23.1 (7)
-5.272	-4.636	-5.278	-5.829	-9.705	-5.440	-4.789
1713	1400	2171	4587	2381·10 ⁸	7003	719.0
0.124	0.195	0.105	0.140	0.138	0.215	0.117
0.128	0.283	0.107	1.139	0.257	0.359	0.125
0.32	0.55	0.27	0.34	0.051	0.54	0.32
960	990	1000	3500	50·10 ⁸	7200	320
0.77	0.43	0.79	0.70	0.64	0.34	0.67
46	61	55	59	33	51	63
9	6	11	7	21	9	6
1.69	2.24	1.86	2.11	0.59	2.02	2.12
1.06	2.11	1.03	0.99	3.48	2.79	1.15
9.13 (9)	5.93 (9)	8.73 (9)	9.41 (9)	19.55 (9)	29.28 (9)	4.47 (9)

Appendix A. Main results (continued). Lower public servants and salaried

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.735	2.368	3.152	2.344	2.689	2.063
b.....	0.931	0.498	0.533	0.838	1.115	0.623
$\overline{\log x}$	3.745	3.745	3.745	3.745	3.745	3.745
s_1	0.116	0.149	0.0613	0.228	0.129	0.123
s_2	0.129	0.171	0.0670	0.273	0.138	0.122
s_a	0.010	0.014	0.0054	0.022	0.011	0.0098
s_b	0.055	0.073	0.028	0.12	0.059	0.052
R.....	0.81	0.49	0.84	0.51	0.84	0.70
N.....	76	67	73	70	81	66
l.....	8	7	7	9	9	9
d.....	1.76	1.97	2.06	1.91	2.12	1.91
F.....	1.24	1.31	1.19	1.43	1.14	0.99
$\chi^2 (f)$	5.2 (9)	25.5 (9)	10.2 (9)	30.4 (9)	8.5 (9)	9.4 (9)
$2: \log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$						
a.....	2.735	2.368	3.152	2.344	2.689	2.063
$b \cdot 10^{-4}$	-0.198	-0.104	-0.118	-0.197	-0.242	-0.132
$\overline{\frac{1}{x}} \cdot 10^{+4}$	1.975	1.975	1.975	1.975	1.975	1.975
s_1	0.116	0.149	0.0613	0.228	0.129	0.123
s_2	0.139	0.174	0.0666	0.267	0.145	0.127
s_a	0.011	0.014	0.0054	0.022	0.012	0.010
$s_b \cdot 10^{-4}$	0.013	0.016	0.0060	0.025	0.014	0.012
R.....	0.78	0.46	0.84	0.54	0.82	0.67
N.....	69	67	70	72	77	70
L.....	11	7	8	9	9	9
d.....	1.52	1.90	2.03	1.98	1.90	1.76
F.....	1.43	1.36	1.18	1.38	1.26	1.07
$\chi^2 (f)$	4.8 (9)	16.7 (9)	10.9 (9)	38.8 (9)	3.2 (9)	5.8 (9)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	453.3	228.0	1312	193.0	348.6	107.7
b.....	1040	311	1765	487	1080	161
$\overline{\log x}$	3.596	3.651	3.652	3.580	3.549	3.642
s_1	0.268	0.344	0.141	0.525	0.298	0.282
s_2	0.340	0.362	0.148	0.478	0.333	0.281
s_a	13	6.9	16	8.2	11	2.5
s_b	72	37	87	44	58	14

employees. The capital. 1.2 154 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
2.047	2.396	2.203	1.961	2.653	2.393	2.115
0.853	0.885	0.999	0.946	1.366	1.087	0.656
3.745	3.745	3.745	3.745	3.745	3.745	3.745
0.133	0.236	0.106	0.138	0.155	0.193	0.124
0.134	0.324	0.124	0.167	0.170	0.305	0.141
0.011	0.026	0.010	0.013	0.014	0.025	0.011
0.057	0.14	0.053	0.071	0.072	0.13	0.060
0.77	0.46	0.84	0.74	0.84	0.56	0.66
81	81	95	70	76	80	72
6	5	5	8	5	7	9
1.84	2.07	2.27	1.91	1.87	1.96	1.85
1.02	1.89	1.37	1.46	1.20	2.50	1.29
13.8 (9)	8.2 (9)	9.1 (9)	9.9 (9)	3.9 (9)	40.3 (9)	10.2 (9)
2.047	2.396	2.203	1.961	2.653	2.393	2.115
-0.188	-0.198	-0.221	-0.211	-0.295	-0.241	-0.150
1.975	1.975	1.975	1.975	1.975	1.975	1.975
0.133	0.236	0.106	0.138	0.155	0.193	0.124
0.136	0.323	0.124	0.166	0.180	0.305	0.138
0.011	0.026	0.010	0.013	0.015	0.025	0.011
0.013	0.030	0.012	0.016	0.017	0.029	0.013
0.77	0.47	0.84	0.74	0.82	0.56	0.68
80	82	87	72	65	82	78
6	6	6	8	15	6	6
1.77	2.08	2.23	1.91	1.64	1.95	1.90
1.04	1.88	1.37	1.45	1.35	2.50	1.24
7.8 (9)	9.1 (9)	7.0 (9)	6.7 (9)	5.0 (9)	34.0 (9)	8.2 (9)
94.10	204.4	119.7	74.49	571.8	204.9	117.0
204	686	333	188	1700	722	212
3.601	3.536	3.565	3.582	3.745	3.523	3.623
0.306	0.543	0.244	0.318	-	0.445	0.286
0.321	0.759	0.314	0.379	-	1.028	0.308
2.6	15	3.4	2.5	-	20	72
14	79	18	13	-	110	9

Appendix A. Main results (continued). Lower public servants and salaried

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.76	0.57	0.86	0.67	0.84	0.69
N.....	70	65	76	74	73	70
l.....	18	10	8	9	9	11
d.....	1.51	1.88	2.11	1.81	1.79	1.86
F.....	1.61	1.11	1.10	0.83	1.25	1.00
χ^2 (f).....	18.9 (9)	22.0 (9)	10.7 (9)	11.6 (9)	13.2 (9)	13.8 (9)

$$4: y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$$

a.....	487.1	230.8	131.9	275.4	576.3	109.9
$b10^{-4}$	-167	-54.0	-315	-111	-291	-27.8
$\bar{\frac{1}{x}} 10^{+4}$	2.677	2.449	2.452	1.975	1.975	2.471
s_1	0.268	0.344	0.141	-	-	0.282
s_2	0.414	0.389	0.169	-	-	0.302
s_a	17	7.4	19	-	-	2.8
$s_b \cdot 10^{-4}$	15	7.0	17	-	-	2.6
R.....	0.67	0.53	0.83	0.72	0.68	0.66
N.....	55	61	60	72	61	65
l.....	21	11	20	9	17	16
d.....	1.18	1.75	1.68	2.20	1.91	1.61
F.....	2.39	1.28	1.43	-	-	1.15
χ^2 (f).....	33.4 (9)	34.8 (9)	10.1 (9)	-	-	14.9 (9)

$$5: \log y = \log k + \log \Phi (a + \log x)$$

a.....	-5.496	-4.251	-4.375	-5.257	-	-4.639
k.....	13810	772.5	5447	3440	-	631.9
s_1	0.116	0.149	0.0613	0.228	-	0.123
s_2	0.129	0.171	0.0562	0.270	-	0.122
s_a	0.25	0.38	0.15	0.49	-	0.29
s_k	6800	310	940	3100	-	240
R.....	0.78	0.56	0.85	0.65	-	0.69
N.....	78	67	75	70	-	66
l.....	8	7	7	9	-	9
d.....	1.78	1.96	2.13	1.94	-	1.90
F.....	1.24	1.32	1.13	1.40	-	0.98
χ^2 (f).....	6.66 (11)	20.84 (11)	9.45 (11)	32.74 (11)	-	10.14 (11)

employees. The capital. 1.2 154 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.76	0.58	0.83	0.75	0.45	0.47	0.72
78	80	84	74	63	66	72
6	6	10	8	14	11	9
1.76	2.15	2.16	1.87	1.98	1.95	1.84
1.10	1.96	1.66	1.42	—	5.34	1.16
25.1 (9)	60.0 (9)	36.9 (9)	19.7 (9)	—	229.8 (9)	14.2 (9)
95.79	351.2	122.6	106.6	571.8	374.0	118.7
-32.2	-161	-50.4	-46.7	-339	-185	-34.3
2.795	1.975	3.048	1.975	1.975	1.975	2.639
0.306	—	0.244	—	—	—	0.286
0.360	—	0.375	—	—	—	0.345
3.0	—	4.2	—	—	—	3.5
2.5	—	3.3	—	—	—	3.0
0.72	0.62	0.78	0.70	0.69	0.42	0.67
61	71	57	74	55	58	74
15	9	31	8	14	13	8
1.39	2.07	1.67	2.35	1.99	1.66	1.56
1.38	—	2.36	—	—	—	1.45
29.6 (9)	—	54.5 (9)	—	—	—	31.5 (9)
-5.290	-5.376	-5.678	-5.540	-9.633	-5.906	-4.748
1849	4920	6099	2558	1564·10 ^s	16390	838.4
0.133	0.236	0.106	0.138	0.155	0.193	0.124
0.133	0.322	0.122	0.164	0.287	0.303	0.139
0.29	0.51	0.22	0.29	0.046	0.40	0.29
970	4700	2900	1500	29·10 ^s	15000	340
0.76	0.50	0.83	0.75	0.55	0.43	0.72
82	81	94	70	51	82	76
6	5	5	8	23	7	9
1.86	2.08	2.32	1.95	0.71	1.97	1.90
1.00	1.86	1.32	1.42	3.42	2.47	1.26
12.40 (11)	8.92 (11)	8.28 (11)	8.48 (11)	16.43 (11)	39.25 (11)	11.95 (11)

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.616	2.333	3.166	2.450	2.586	1.999
b.....	0.868	0.402	0.685	0.931	1.020	0.502
$\overline{\log x}$	3.705	3.705	3.705	3.705	3.705	3.705
s_1	0.132	0.131	0.0603	0.156	0.116	0.103
s_2	0.135	0.116	0.0495	0.176	0.129	0.101
s_a	0.016	0.014	0.0060	0.021	0.016	0.012
s_b	0.084	0.072	0.031	0.11	0.080	0.063
R.....	0.79	0.57	0.94	0.72	0.84	0.70
N.....	36	33	35	37	40	33
l.....	4	4	4	5	5	6
d.....	2.03	1.86	2.11	1.70	2.04	1.94
F.....	1.05	0.78	0.67	1.27	1.23	0.96
$\chi^2 (f)$	4.5 (5)	10.8 (5)	1.4 (5)	6.1 (5)	8.7 (5)	9.8 (5)
$2: \log y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	2.616	2.333	3.166	2.450	2.586	1.999
$b \cdot 10^{-4}$	-0.172	-0.091	-0.138	-0.183	-0.213	-0.106
$\frac{1}{\bar{x}} \cdot 10^{+4}$	2.171	2.171	2.171	2.171	2.171	2.171
s_1	0.132	0.131	0.0603	0.156	0.116	0.103
s_2	0.142	0.110	0.0563	0.184	0.124	0.0973
s_a	0.017	0.013	0.0068	0.022	0.015	0.012
$s_b \cdot 10^{-4}$	0.018	0.014	0.0072	0.023	0.016	0.012
R.....	0.76	0.62	0.92	0.69	0.86	0.72
N.....	35	33	31	37	33	34
l.....	6	4	6	5	6	6
d.....	1.84	2.07	1.62	1.58	2.23	2.06
F.....	1.16	0.70	0.87	1.38	1.14	0.90
$\chi^2 (f)$	3.3 (5)	4.5 (5)	0.9 (5)	1.1 (5)	6.6 (5)	3.6 (5)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	341.5	207.3	1276	230.9	279.2	94.47
b.....	794	243	2220	593	850	119
$\overline{\log x}$	3.555	3.624	3.587	3.540	3.521	3.617
s_1	0.304	0.302	0.139	0.360	0.267	0.237
s_2	0.309	0.249	0.120	0.408	0.278	0.240
s_a	14	6.4	20	13	11	2.8
s_b	76	34	110	70	60	15

The capital. (1.3.) 68 groups og 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
2.023	2.376	2.105	1.900	2.597	2.370	2.337
0.702	1.214	0.679	0.772	1.507	1.561	0.895
3.705	3.705	3.705	3.705	3.705	3.705	3.705
0.120	0.198	0.101	0.135	0.143	0.209	0.0972
0.135	0.263	0.112	0.129	0.132	0.299	0.0855
0.016	0.032	0.014	0.016	0.016	0.036	0.010
0.084	0.16	0.070	0.080	0.082	0.19	0.053
0.72	0.67	0.77	0.76	0.91	0.72	0.90
37	36	35	34	34	36	36
5	5	6	6	6	4	7
1.69	2.07	2.06	2.05	2.26	2.34	2.04
1.27	1.76	1.25	0.91	0.86	2.04	0.77
5.0 (5)	12.8 (5)	6.6 (5)	0.9 (5)	4.5 (5)	3.8 (5)	5.0 (5)
2.023	2.376	2.105	1.900	2.597	2.370	2.337
-0.133	-0.241	-0.142	-0.152	-0.298	-0.308	-0.178
2.171	2.171	2.171	2.171	2.171	2.171	2.171
0.120	0.198	0.101	0.135	0.143	0.209	0.0972
0.145	0.270	0.109	0.136	0.154	0.311	0.0962
0.018	0.033	0.013	0.016	0.019	0.038	0.012
0.019	0.035	0.014	0.017	0.020	0.040	0.012
0.66	0.65	0.78	0.73	0.88	0.69	0.87
33	35	37	33	29	34	33
4	5	5	11	13	5	6
1.48	1.97	2.18	1.86	1.69	2.18	1.63
1.47	1.86	1.18	1.01	1.16	2.20	0.98
5.0 (5)	5.5 (5)	8.4 (5)	3.0 (5)	4.0 (5)	5.1 (5)	4.2 (5)
95.11	199.1	112.2	69.39	217.2	104.6	177.7
172	622	204	138	1080	831	396
3.583	3.528	3.582	3.573	3.459	3.407	3.559
0.276	0.456	0.232	0.311	0.329	0.482	0.151
0.330	0.596	0.257	0.330	0.418	0.762	0.205
4.0	16	3.7	3.0	14	14	4.8
22	92	20	16	87	100	26

Appendix A. Main results (continued). Skilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.79	0.66	0.93	0.72	0.87	0.70
N.....	37	31	33	37	33	35
l.....	6	4	6	6	6	8
d.....	1.88	1.95	1.88	1.62	2.16	2.10
F.....	1.03	0.68	0.75	1.29	1.09	1.03
χ^2 (f).....	2.7 (5)	8.0 (5)	7.1 (5)	6.9 (5)	2.4 (5)	18.9 (5)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	436.1	204.9	1285	237.4	286.7	93.00
$b10^{-4}$	-126	-44.3	-362	-89.5	-129	-21.8
$\frac{\bar{1}}{x} 10^{+4}$	3.062	2.660	2.850	3.162	3.281	2.712
s_1	0.304	0.302	0.139	0.360	0.267	0.237
s_2	0.355	0.247	0.165	0.473	0.319	0.242
s_a	16	6.3	27	15	13	2.8
$s_b \cdot 10^{-4}$	13	5.7	23	12	10	2.5
R.....	0.76	0.69	0.89	0.67	0.84	0.73
N.....	27	36	19	31	29	34
l.....	11	4	13	9	7	8
d.....	1.53	2.02	1.02	1.32	1.67	2.06
F.....	1.37	0.67	1.42	1.73	1.44	1.04
χ^2 (f).....	6.8 (5)	9.6 (5)	1.7 (5)	12.8 (5)	1.0 (5)	13.3 (5)
5: $\log y = \log k + \log \Phi (a + \log x)$						
a.....	-5.290	-3.934	-4.787	-5.457	-5.697	-4.246
k.....	7424	532.8	10640	7180	16910	343.7
s_1	0.132	0.131	0.0603	0.156	0.116	0.103
s_2	0.135	0.113	0.0486	0.176	0.125	0.0311
s_a	0.42	0.52	0.20	0.49	0.35	0.38
s_k	5700	250	3200	6900	13000	140
R.....	0.78	0.64	0.94	0.72	0.84	0.69
N.....	37	33	34	37	40	33
l.....	4	4	6	5	5	6
d.....	2.01	1.95	2.15	1.71	2.13	2.10
F.....	1.04	0.74	0.65	1.27	1.16	0.91
χ^2 (f).....	7.51 (7)	8.49 (7)	2.46 (7)	4.95 (7)	4.48 (7)	9.49 (7)

The capital. (1.3.) 68 groups og 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.70	0.63	0.78	0.73	0.84	0.71	0.88
31	35	34	33	25	31	31
9	5	5	11	14	7	6
1.33	1.56	2.19	1.82	1.00	1.87	1.56
1.43	1.71	1.23	1.12	1.62	2.50	0.84
13.5 (5)	3.9 (5)	1.8 (5)	8.1 (5)	14.2 (5)	44.3 (5)	2.7 (5)
95.79	321.6	111.7	71.21	205.7	100.8	182.8
-28.2	-165	-33.8	-22.1	-161	-124	-60.7
2.890	2.171	2.900	2.900	3.892	4.197	3.014
0.276	-	0.232	0.311	0.329	0.482	0.151
0.387	-	0.264	0.374	0.556	0.888	0.268
4.8	-	3.8	3.4	19	18	6.4
4.0	-	3.2	2.9	16	17	5.3
0.65	0.60	0.79	0.68	0.77	0.67	0.81
31	31	35	25	15	29	23
9	8	6	11	20	7	15
1.01	2.10	1.99	1.55	0.57	1.49	0.87
1.96	-	1.30	1.44	2.86	3.40	1.44
16.9 (5)	-	6.9 (5)	13.7 (5)	20.7 (5)	56.7 (5)	3.3 (5)
-4.825	-6.192	-4.772	-5.028	-9.572	-9.546	-9.733
815.5	37430	903.6	867.6	1263·10 ⁸	670.2·10 ⁸	1178·10 ⁸
0.120	0.198	0.101	0.135	0.143	0.209	0.0972
0.136	0.260	0.110	0.129	0.242	0.351	0.410
0.40	0.59	0.34	0.44	0.068	0.11	0.031
450	57000	440	630	36·10 ⁸	32·10 ⁸	12·10 ⁸
0.71	0.66	0.76	0.73	0.67	0.42	0.49
37	36	36	34	29	37	15
5	5	6	6	13	5	17
1.64	2.11	2.12	2.06	0.67	1.68	0.10
1.29	1.73	1.19	0.91	2.86	2.82	17.78
6.19 (7)	4.88 (7)	7.29 (7)	2.31 (7)	6.36 (7)	3.64 (7)	29.44 (7)

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
1 : $\log y = a + b (\log x - \overline{\log x})$						
a.....	2.602	2.328	3.157	2.449	2.551	1.968
b.....	0.940	0.312	0.702	1.074	1.047	0.613
$\overline{\log x}$	3.672	3.672	3.672	3.672	3.672	3.672
s_1	0.133	0.124	0.0615	0.178	0.152	0.131
s_2	0.142	0.172	0.0620	0.204	0.156	0.125
s_a	0.016	0.019	0.0068	0.022	0.017	0.014
s_b	0.080	0.096	0.035	0.11	0.088	0.070
R.....	0.80	0.34	0.91	0.72	0.80	0.69
N.....	38	38	40	50	46	43
l.....	5	7	5	6	6	7
d.....	1.83	1.43	1.76	2.06	2.08	2.30
F.....	1.14	1.92	1.02	1.32	1.06	0.92
χ^2 (f).....	4.1 (7)	8.0 (7)	8.5 (8)	14.6 (7)	13.3 (7)	7.3 (7)
2: $\log y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{x} \right)$						
a.....	2.602	2.328	3.157	2.449	2.551	1.968
$b \cdot 10^{-4}$	-0.158	-0.0644	-0.115	-0.170	-0.176	-0.103
$\frac{\bar{1}}{x} \cdot 10^{+4}$	2.359	2.359	2.359	2.359	2.359	2.359
s_1	0.133	0.124	0.0615	0.178	0.152	0.131
s_2	0.146	0.167	0.0749	0.221	0.161	0.128
s_a	0.016	0.018	0.0082	0.024	0.018	0.014
$s_b \cdot 10^{-4}$	0.014	0.016	0.0072	0.021	0.016	0.012
R.....	0.78	0.41	0.87	0.66	0.78	0.68
N.....	39	42	33	43	43	42
l.....	6	7	11	6	5	6
d.....	1.75	1.52	1.23	1.77	1.94	2.19
F.....	1.21	1.81	1.48	1.54	1.13	0.96
χ^2 (f).....	9.9 (7)	13.7 (7)	7.3 (7)	14.6 (7)	19.7 (7)	7.0 (7)
3: $y = a + b (\log x - \overline{\log x})$						
a.....	308.1	211.2	1237	223.7	239.1	85.06
b.....	707	222	2010	551	696000	122
$\overline{\log x}$	3.473	3.586	3.535	3.467	3.431	3.552
s_1	0.306	0.286	0.142	0.409	0.350	0.301
s_2	0.349	0.310	0.157	0.459	0.341	0.271
s_a	13	7.4	23	13	11	2.6
s_b	60	36	110	58	48	12

The capital. (1.4.) 83 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.993	2.301	2.108	1.907	2.492	2.288	2.295
0.810	0.984	0.828	1.052	1.463	1.342	1.102
3.672	3.672	3.672	3.672	3.672	3.672	3.672
0.134	0.217	0.115	0.155	0.177	0.232	0.119
0.142	0.291	0.132	0.182	0.189	0.270	0.135
0.016	0.032	0.014	0.020	0.021	0.030	0.015
0.079	0.16	0.074	0.10	0.11	0.15	0.075
0.75	0.56	0.78	0.75	0.84	0.70	0.85
33	49	41	40	49	51	31
7	7	6	6	4	4	9
1.89	1.95	1.83	1.88	1.97	2.54	1.59
1.11	1.80	1.30	1.39	1.14	1.36	1.28
9.0 (7)	9.7 (7)	5.0 (7)	3.4 (7)	3.3 (7)	5.1 (7)	8.0 (7)
1.993	2.301	2.108	1.907	2.492	2.288	2.295
-0.125	-0.176	-0.145	-0.170	-0.248	-0.220	-0.188
2.359	2.359	2.359	2.359	2.359	2.359	2.359
0.134	0.217	0.115	0.155	0.177	0.232	0.119
0.158	0.285	0.127	0.195	0.195	0.281	0.137
0.017	0.031	0.014	0.021	0.021	0.031	0.015
0.015	0.027	0.012	0.019	0.019	0.027	0.013
0.67	0.58	0.80	0.71	0.83	0.67	0.85
33	48	44	35	45	43	30
7	7	6	7	6	7	10
1.53	2.02	1.91	1.64	1.84	2.32	1.52
1.38	1.73	1.22	1.59	1.21	1.47	1.32
9.8 (7)	15.0 (7)	8.8 (7)	9.0 (7)	2.2 (7)	7.6 (7)	10.2 (7)
85.51	142.5	94.64	55.44	120.8	89.25	119.5
155	449	231	162	725	509	392
3.520	3.413	3.465	3.426	3.310	3.316	3.405
0.309	0.499	0.266	0.356	0.407	0.534	0.274
0.338	0.557	0.293	0.445	0.479	0.800	0.316
3.4	11	3.5	3.3	9.8	12	5.1
16	48	16	15	50	60	23

Appendix A. Main results (continued). Unskilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.79	0.57	0.90	0.73	0.85	0.74
N.....	37	42	37	49	41	47
l.....	6	7	8	6	6	5
d.....	1.63	1.91	1.44	1.80	1.92	2.20
F.....	1.30	1.18	1.23	1.26	0.95	0.81
χ^2 (f).....	12.9 (7)	8.8 (7)	3.5 (7)	10.7 (7)	9.0 (7)	2.2 (7)

$$4: y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$$

a.....	457,2	213.8	1276	347.2	234.9	84.23
$b \cdot 10^{-4}$	-140	-29.8	-253	-122	-82.7	-16.5
$\bar{\frac{1}{x}} \cdot 10^{+4}$	2.359	2.875	3.266	2.359	4.395	3.276
s_1	-	0.286	0.142	-	0.350	0.301
s_2	-	0.329	0.216	-	0.405	0.287
s_a	-	7.9	32	-	13	2.8
$s_b \cdot 10^{-4}$	-	5.3	19	-	6.3	1.6
R.....	0.78	0.53	0.84	0.60	0.83	0.75
N.....	37	36	19	37	33	39
l.....	6	10	17	10	10	7
d.....	1.77	1.86	0.82	1.91	1.47	1.92
F.....	-	1.32	2.33	-	1.34	0.91
χ^2 (f).....	-	10.9 (7)	9.7 (7)	-	9.4 (7)	8.3 (7)

$$5: \log y = \log k + \log \Phi (a + \log x)$$

a.....	-5.414	-3.575	-	-	-5.695	-4.504
k.....	9976	399.3	-	-	16810	465.2
s_1	0.133	0.124	-	-	0.152	0.131
s_2	0.140	0.169	-	-	0.155	0.124
s_a	0.37	0.49	-	-	0.42	0.41
s_k	7300	130	-	-	15000	240
R.....	0.79	0.55	-	-	0.82	0.73
N.....	38	38	-	-	48	41
l.....	5	7	-	-	5	7
d.....	1.87	1.48	-	-	2.11	2.33
F.....	1.11	1.85	-	-	1.04	0.90
χ^2 (f).....	7.91 (9)	8.89 (9)	-	-	12.53 (9)	9.17 (9)

The capital. (1.4.) 83 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.74	0.72	0.86	0.78	0.85	0.70	0.88
33	48	39	37	43	33	28
7	6	6	7	8	12	11
1.58	2.26	1.87	1.36	1.46	2.12	1.37
1.19	1.24	1.21	1.64	1.39	2.24	1.33
14.7 (7)	5.4 (7)	13.9 (7)	18.2 (7)	9.7 (7)	60.7 (7)	11.7 (7)
87.41	150.8	81.23	55.60	121.9	85.23	230.8
-18.6	-52.6	-28.7	-19.0	-84.3	-59.8	-81.0
3.511	4.341	4.420	4.440	5.467	5.483	2.359
0.309	0.499	0.266	0.356	0.407	0.534	-
0.402	0.602	0.313	0.564	0.577	0.918	-
4.1	12	3.4	4.2	12	14	-
2.3	6.0	1.7	2.1	6.8	7.7	-
0.67	0.71	0.88	0.70	0.81	0.66	0.86
29	42	35	37	31	29	26
15	9	9	7	11	14	11
1.18	2.02	1.62	0.98	1.05	1.84	1.86
1.69	1.45	1.39	2.51	2.01	2.96	-
16.7 (7)	9.8 (7)	8.6 (7)	32.6 (7)	16.8 (7)	77.4 (7)	-
-5.057	-5.537	-5.118	-5.703	-	-6.443	-9.618
1204	6530	1758	3877	-	70590	836.8·10 ⁸
0.134	0.217	0.115	0.155	-	0.232	0.119
0.144	0.287	0.128	0.182	-	0.269	0.339
0.39	0.60	0.33	0.42	-	0.61	0.039
800	8200	1000	3600	-	120000	130·10 ⁸
0.76	0.68	0.81	0.79	-	0.66	0.61
31	49	41	38	-	51	25
10	7	6	7	-	4	18
1.84	1.98	2.06	1.85	-	2.55	0.27
1.15	1.75	1.25	1.38	-	1.34	8.14
11.60 (9)	10.42 (9)	2.71 (9)	3.67 (9)	-	6.06 (9)	23.40 (9)

Appendix A. Main results (continued). Higher public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.697	2.488	3.085	2.289	2.706	2.020
b.....	0.800	0.568	0.548	0.763	0.905	0.379
$\overline{\log x}$	3.751	3.751	3.751	3.751	3.751	3.751
s_1	0.124	0.122	0.0633	0.185	0.114	0.0984
s_2	0.109	0.120	0.0559	0.223	0.118	0.114
s_a	0.013	0.014	0.0067	0.027	0.014	0.014
s_b	0.065	0.072	0.034	0.13	0.071	0.069
R.....	0.83	0.69	0.89	0.57	0.84	0.56
N.....	35	39	36	43	36	34
l.....	6	5	8	4	4	5
d.....	2.15	2.26	1.97	2.35	1.94	1.86
F.....	0.73	0.96	0.78	1.45	1.07	1.34
$\chi^2 (f)$	5.4 (5)	3.8 (5)	7.1 (5)	9.0 (5)	6.4 (5)	5.0 (5)
$2: \log y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	2.697	2.488	3.085	2.289	2.706	2.020
$b \cdot 10^{-4}$	-0.184	-0.130	-0.124	-0.173	-0.206	-0.0863
$\frac{1}{\bar{x}} \cdot 10^{+4}$	1.957	1.957	1.957	1.957	1.957	1.957
s_1	0.124	0.122	0.0633	0.185	0.114	0.0984
s_2	0.111	0.122	0.0611	0.225	0.123	0.115
s_a	0.013	0.015	0.0073	0.027	0.015	0.014
$s_b \cdot 10^{-4}$	0.016	0.017	0.0085	0.032	0.017	0.016
R.....	0.82	0.68	0.87	0.56	0.82	0.55
N.....	37	37	28	42	33	33
l.....	6	5	8	4	6	5
d.....	2.04	2.22	1.72	2.29	1.80	1.83
F.....	0.76	0.99	0.93	1.48	1.16	1.36
$\chi^2 (f)$	4.6 (5)	6.0 (5)	14.0 (5)	10.6 (5)	5.4 (5)	3.0 (5)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	400.0	283.3	1116	169.8	398.0	102.8
b.....	898	431	1516	374	964	101
$\overline{\log x}$	3.592	3.643	3.654	3.592	3.580	3.680
s_1	0.294	0.281	0.146	0.426	0.263	0.227
s_2	0.236	0.258	0.140	0.425	0.273	0.260
s_a	12	9.1	19	9.4	14	32
s_b	63	48	100	48	73	5

salaried employees. Provincial towns. (2.1) 70 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.916	2.462	2.099	1.906	2.638	2.434	2.078
0.816	1.008	0.730	1.022	1.459	1.676	0.757
3.751	3.751	3.751	3.751	3.751	3.751	3.751
0.128	0.219	0.103	0.144	0.149	0.251	0.0953
0.145	0.317	0.097	0.175	0.145	0.431	0.0928
0.017	0.038	0.012	0.021	0.017	0.051	0.011
0.087	0.19	0.059	0.11	0.087	0.26	0.056
0.75	0.54	0.83	0.76	0.90	0.62	0.85
36	39	37	36	41	39	35
5	7	5	6	7	6	7
1.56	2.22	1.97	2.00	2.15	2.41	2.13
1.29	2.10	0.89	1.48	0.95	2.97	0.95
3.8 (5)	14.6 (5)	6.8 (5)	7.4 (5)	10.2 (5)	6.7 (5)	4.1 (5)
1.916	2.462	2.099	1.906	2.638	2.434	2.078
-0.175	-0.221	-0.165	-0.228	-0.333	-0.370	-0.172
1.957	1.957	1.957	1.957	1.957	1.957	1.957
0.128	0.219	0.103	0.144	0.149	0.251	0.0953
0.158	0.324	0.103	0.184	0.155	0.443	0.0982
0.019	0.039	0.012	0.022	0.019	0.053	0.012
0.022	0.045	0.014	0.026	0.022	0.062	0.014
0.69	0.51	0.81	0.73	0.88	0.59	0.84
35	39	29	35	31	40	28
7	7	5	6	8	6	8
1.32	2.13	1.76	1.78	1.86	2.28	1.88
1.53	2.19	1.00	1.65	1.09	3.15	1.06
13.5 (5)	10.6 (5)	4.5 (5)	6.7 (5)	3.0 (5)	11.8 (5)	11.9 (5)
69.58	243.6	107.0	56.25	566.7	94.4	99.3
151	682	207	177	1830	1033	205
3.598	3.530	3.615	3.532	3.751	3.360	3.605
0.294	0.504	0.237	0.331	-	0.575	0.220
0.373	0.835	0.236	0.436	-	1.00	0.225
3.4	28	3.2	3.5	-	21	2.9
17	140	17	17	-	150	15

Appendix A. Main results (continued). Higher public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.87	0.74	0.88	0.69	0.85	0.58
N.....	39	35	32	38	33	32
l.....	6	5	8	5	6	5
d.....	2.12	2.22	1.90	2.32	1.78	1.83
F.....	0.64	0.84	0.92	1.00	1.07	1.32
χ^2 (f).....	12.6 (5)	3.6 (5)	12.5 (5)	6.5 (5)	7.1 (5)	3.1 (5)
4: $y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{x} \right)$						
a.....	393.3	290.2	1144	169.7	396.2	102.4
$b \cdot 10^{-4}$	-140	-71.9	-256	-56.0	-143	-19.5
$\frac{\bar{1}}{x} \cdot 10^{+4}$	2.972	2.483	2.400	2.987	3.074	2.336
s_1	0.294	0.281	0.146	0.426	0.263	0.227
s_2	0.286	0.298	0.190	0.496	0.340	0.269
s_a	15	11	26	11	18	3.4
$s_b \cdot 10^{-4}$	11	9.3	24	7.9	13	3.2
R.....	0.85	0.69	0.79	0.63	0.80	0.60
N.....	27	33	20	30	25	35
l.....	12	8	26	10	13	5
d.....	1.50	1.90	1.27	1.89	1.13	1.74
F.....	0.95	1.12	1.71	1.36	1.67	1.41
χ^2 (f).....	4.8 (5)	1.2 (5)	33.4 (5)	11.9 (5)	9.9 (5)	10.7 (5)
5: $\log y = \log k + \log \Phi (a + \log x)$						
a.....	-5.163	-4.500	-4.439	-5.063	-	-3.884
k.....	6390	1375	5023	2081	-	237.2
s_1	0.124	0.122	0.0633	0.185	-	0.094
s_2	0.107	0.119	0.0551	0.221	-	0.114
s_a	0.40	0.42	0.22	0.58	-	0.39
s_k	4400	710	1300	2000	-	77
R.....	0.83	0.73	0.88	0.68	-	0.58
N.....	34	37	34	43	-	34
l.....	6	5	8	4	-	5
d.....	2.22	2.28	2.09	2.35	-	1.86
F.....	0.75	0.95	0.76	1.43	-	1.33
χ^2 (f).....	2.53 (7)	4.12 (7)	9.05 (7)	8.82 (7)	-	2.01 (7)

salaried employees. Provincial towns. (2.1.) 70 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.74	0.51	0.84	0.80	0.75	0.65	0.86
35	29	35	35	37	34	28
7	8	5	6	9	12	8
1.24	2.31	1.92	1.89	2.29	2.12	2.00
1.61	2.75	0.99	1.74	—	3.03	1.05
8.0 (5)	31.9 (5)	13.8 (5)	16.8 (5)	—	64.5 (5)	8.3 (5)
70.1	263.9	106.2	52.19	566.7	158.0	98.4
—24.1	—101	—32.5	—26.4	—383	—219	—32.5
2.884	3.233	2.808	3.541	1.957	3.493	2.862
0.294	0.504	0.237	0.331	—	0.575	0.220
0.458	0.873	0.289	0.512	—	1.05	0.284
4.3	31	4.0	4.0	—	3.2	3.6
3.1	21	3.0	2.7	—	35	2.7
0.70	0.48	0.79	0.76	0.69	0.60	0.83
29	27	27	21	33	28	29
13	13	10	11	9	13	8
0.89	2.16	1.35	1.34	1.73	1.98	1.38
2.42	3.01	1.48	2.40	—	3.34	1.67
17.9 (5)	32.7 (5)	12.4 (5)	25.1 (5)	—	47.2 (5)	23.4 (5)
—5.198	—5.718	—4.967	—5.752	—	—7.399	—5.043
1133	11970	1137	3606	—	2087000	1237
0.128	0.219	0.103	0.144	—	0.251	0.0953
0.147	0.315	0.0972	0.175	—	0.428	0.0919
0.40	0.65	0.33	0.42	—	0.70	0.30
780	17000	590	3300	—	440000	610
0.72	0.48	0.82	0.72	—	0.55	0.84
36	39	39	36	—	39	35
5	7	5	6	—	6	7
1.53	2.22	1.98	2.00	—	2.41	2.17
1.31	2.07	0.89	1.47	—	2.91	0.93
5.84 (7)	12.99 (7)	8.45 (7)	9.89 (7)	—	8.86 (7)	2.13 (7)

Appendix A. Main results (continued). Lower public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.619	2.405	3.071	2.259	2.628	2.002
b.....	0.856	0.440	0.640	0.896	1.017	0.567
$\overline{\log x}$	3.656	3.656	3.656	3.656	3.656	3.656
s_1	0.131	0.157	0.0660	0.203	0.121	0.120
s_2	0.137	0.157	0.0723	0.253	0.127	0.126
s_a	0.013	0.015	0.0069	0.024	0.012	0.012
s_b	0.061	0.070	0.032	0.11	0.057	0.056
R.....	0.80	0.51	0.88	0.60	0.86	0.69
N.....	48	63	48	54	60	57
l.....	7	9	6	6	6	6
d.....	1.77	2.20	1.90	2.14	2.08	2.05
F.....	1.09	1.00	1.20	1.56	1.10	1.10
χ^2 (f).....	9.4 (7)	15.0 (7)	5.8 (7)	32.0 (7)	6.1 (7)	10.9 (7)
$2: \log y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{x} \right)$						
a.....	2.619	2.405	3.071	2.259	2.628	2.002
$b \cdot 10^{-4}$	-0.146	-0.0720	-0.108	-0.151	-0.173	-0.967
$\frac{\bar{1}}{x} \cdot 10^{+4}$	2.484	2.484	2.484	2.484	2.484	2.484
s_1	0.131	0.157	0.0660	0.203	0.121	0.120
s_2	0.140	0.160	0.0781	0.257	0.134	0.128
s_a	0.013	0.015	0.0074	0.024	0.013	0.012
$s_b \cdot 10^{-4}$	0.011	0.012	0.0060	0.020	0.010	0.0099
R.....	0.79	0.49	0.86	0.59	0.85	0.68
N.....	48	61	53	55	55	58
l.....	7	9	9	6	6	6
d.....	1.69	2.13	1.65	2.09	1.88	1.99
F.....	1.15	1.04	1.40	1.60	1.23	1.13
χ^2 (f).....	3.9 (7)	9.6 (7)	4.3 (7)	18.9 (7)	7.7 (7)	15.5 (7)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	311.3	245.3	1021	149.0	294.7	91.32
b.....	755	292	1610	365	813	134
$\overline{\log x}$	3.448	3.550	3.517	3.447	3.424	3.527
s_1	0.302	0.362	0.152	0.466	0.280	0.277
s_2	0.312	0.313	0.167	0.490	0.338	0.290
s_a	10	7.5	17	7.9	11	2.6
s_b	48	35	80	36	50	12

salaried employees. Provincial towns. (2.2.) 111 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.870	2.367	2.071	1.801	2.478	2.148	2.088
0.983	1.042	0.866	1.107	1.564	1.389	0.788
3.656	3.656	3.656	3.656	3.656	3.656	3.656
0.152	0.243	0.115	0.158	0.155	0.236	0.101
0.168	0.340	0.139	0.169	0.175	0.353	0.123
0.016	0.032	0.013	0.016	0.017	0.034	0.012
0.075	0.15	0.062	0.076	0.078	0.16	0.055
0.78	0.55	0.80	0.81	0.89	0.64	0.81
45	47	51	59	62	45	60
10	7	9	6	6	8	6
1.68	1.84	1.81	1.80	2.15	1.87	2.18
1.22	1.97	1.47	1.14	1.28	2.24	1.50
5.6 (7)	12.6 (7)	7.0 (7)	12.6 (7)	8.5 (7)	7.1 (7)	6.4 (7)
1.870	2.367	2.071	1.801	2.478	2.148	2.088
-0.163	-0.185	-0.149	-0.181	-0.261	-0.240	-0.134
2.484	2.484	2.484	2.484	2.484	2.484	2.484
0.152	0.243	0.115	0.158	0.155	0.236	0.101
0.179	0.337	0.140	0.186	0.198	0.354	0.127
0.017	0.032	0.013	0.018	0.019	0.034	0.012
0.014	0.026	0.011	0.014	0.015	0.027	0.0099
0.75	0.56	0.80	0.77	0.85	0.64	0.79
43	57	54	51	41	50	51
10	7	6	8	8	8	12
1.48	1.87	1.79	1.50	1.70	1.85	2.06
1.38	1.92	1.49	1.39	1.63	2.25	1.60
5.8 (7)	17.6 (7)	10.4 (7)	15.2 (7)	2.5 (7)	8.7 (7)	7.0 (7)
50.49	221.3	91.68	79.57	432.5	42.39	99.36
149	553	208	213	1490	443	201
3.411	3.445	3.458	3.656	3.656	3.234	3.477
0.350	0.559	0.264	-	-	0.543	0.232
0.404	0.901	0.381	-	-	1.05	0.293
2.3	21	3.7	-	-	7.9	3.0
11	98	17	-	-	52	14

Appendix A. Main results (continued). Lower public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.84	0.62	0.89	0.70	0.84	0.72
N.....	52	63	53	61	51	60
l.....	7	8	9	7	7	6
d.....	1.66	2.12	1.83	1.98	1.77	2.00
F.....	1.07	0.75	1.21	1.10	1.46	1.09
χ^2 (f).....	13.5 (7)	3.8 (7)	4.0 (7)	6.0 (7)	11.9 (7)	21.5 (7)

$$4: y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{\bar{x}} \right)$$

a.....	298.9	248.6	1039	155.9	287.1	91.01
$b \cdot 10^{-4}$	-89.4	-37.2	-197	-43.1	-93.8	-17.5
$\frac{\bar{1}}{\bar{x}} \cdot 10^{+4}$	4.316	3.198	3.474	4.055	4.523	3.450
s_1	0.302	0.362	0.152	0.466	0.280	0.277
s_2	0.372	0.355	0.219	0.555	0.417	0.317
s_a	12	8.6	23	9.4	14	2.9
$s_b \cdot 10^{-4}$	6.1	5.2	13	4.7	6.7	1.6
R.....	0.81	0.56	0.83	0.67	0.80	0.72
N.....	39	63	35	49	43	55
l.....	11	11	21	11	18	5
d.....	1.18	1.83	1.18	1.57	1.16	1.76
F.....	1.52	0.96	2.08	1.41	2.22	1.31
χ^2 (f).....	14.7 (7)	12.4 (7)	9.0 (7)	12.2 (7)	19.5 (7)	27.3 (7)

$$5: \log y = \log k + \log \Phi (a + \log x)$$

a.....	-5.200	-3.971	-4.595	-5.307	-5.625	-4.379
k.....	6918	685.4	6896	3753	17720	435.3
s_1	0.131	0.157	0.0660	0.203	0.121	0.120
s_2	0.136	0.157	0.0711	0.252	0.126	0.125
s_a	0.30	0.44	0.16	0.46	0.27	0.31
s_k	3800	280	1500	3300	10000	160
R.....	0.81	0.62	0.89	0.68	0.84	0.71
N.....	48	65	53	54	60	58
l.....	7	9	7	6	6	6
d.....	1.83	2.21	1.97	2.15	2.13	2.10
F.....	1.07	1.00	1.16	1.54	1.08	1.09
χ_2^2 (f).....	6.57 (9)	15.03 (9)	10.73 (9)	25.49 (9)	8.41 (9)	14.15 (9)

salaried employees. Provincial towns. (2.2.) 111 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.82	0.47	0.76	0.42	0.74	0.64	0.81
49	35	52	45	37	40	53
10	13	10	9	13	9	12
1.31	1.77	1.78	2.07	2.02	1.86	2.09
1.33	2.60	2.09	-	-	3.72	1.60
14.3 (7)	91.1 (7)	36.9 (7)	-	-	100.5 (7)	8.5 (7)
48.56	199.0	91.60	79.57	432.5	246.0	105.5
-17.1	-66.8	-24.5	-31.7	-218	-107	-23.7
4.683	4.541	4.110	2.484	2.484	2.484	3.689
0.350	0.559	0.264	-	-	-	0.232
0.492	0.941	0.432	-	-	-	0.355
2.8	22	4.3	-	-	-	3.8
1.4	10	2.1	-	-	-	2.1
0.76	0.52	0.73	0.41	0.65	0.58	0.74
35	36	39	31	33	36	47
17	13	18	17	15	9	16
0.88	1.69	1.41	2.02	1.78	1.80	1.60
1.98	2.83	2.69	-	-	-	2.35
37.7 (7)	98.4 (7)	42.7 (7)	-	-	-	18.4 (7)
-5.535	-5.694	-5.230	-5.855	-9.450	-	-5.016
2508	11440	2077	4643	677.4·10 ⁸	-	1437
0.152	0.243	0.115	0.158	0.155	-	0.101
0.167	0.339	0.136	0.159	0.263	-	0.122
0.34	0.53	0.26	0.34	0.69	-	0.23
1800	14000	1000	3700	2000·10 ⁸	-	560
0.79	0.46	0.76	0.79	0.65	-	0.82
46	47	49	55	45	-	62
10	7	5	6	12	-	5
1.67	1.96	1.87	1.80	0.96	-	2.24
1.21	1.94	1.41	1.14	2.89	-	1.45
9.69 (9)	13.51 (9)	7.16 (9)	11.85 (9)	8.00 (9)	-	10.51 (9)

Appendix A. Main results (continued). Skilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.504	2.368	3.045	2.313	2.467	1.872
b.....	0.750	0.276	0.601	1.068	1.103	0.670
$\overline{\log x}$	3.575	3.575	3.575	3.575	3.575	3.575
s_1	0.112	0.123	0.0521	0.181	0.123	0.108
s_2	0.102	0.135	0.0673	0.187	0.137	0.0965
s_a	0.014	0.019	0.0094	0.026	0.019	0.014
s_b	0.077	0.10	0.051	0.14	0.10	0.073
R.....	0.81	0.36	0.86	0.73	0.83	0.79
N.....	24	25	28	18	30	29
l.....	6	5	5	6	4	6
d.....	2.34	1.42	2.46	1.57	2.44	2.10
F.....	0.83	1.21	1.67	1.07	1.25	0.79
$\chi^2 (f)$	7.0 (3)	6.5 (3)	3.9 (3)	5.8 (3)	2.1 (3)	2.8 (3)
$2: \log y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	2.504	2.368	3.045	2.313	2.467	1.872
$b \cdot 10^{-4}$	-0.117	-0.0550	-0.0933	-0.164	-0.173	-0.105
$\frac{1}{\bar{x}} \cdot 10^{+4}$	2.901	2.901	2.901	2.901	2.901	2.901
s_1	0.112	0.123	0.0521	0.181	0.123	0.108
s_2	0.106	0.129	0.0711	0.194	0.139	0.0978
s_a	0.015	0.018	0.0099	0.027	0.020	0.014
$s_b \cdot 10^{-4}$	0.013	0.016	0.0085	0.023	0.017	0.012
R.....	0.80	0.45	0.84	0.71	0.83	0.79
N.....	28	26	28	19	28	28
l.....	4	5	7	6	5	6
d.....	2.14	1.54	2.18	1.50	2.36	2.05
F.....	0.89	1.10	1.86	1.15	1.29	0.82
$\chi^2 (f)$	5.3 (3)	9.6 (3)	0.9 (3)	6.4 (3)	3.2 (3)	5.4 (3)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	282.9	232.1	1016	161.0	222.8	67.45
b.....	545	236	1560	515	693	116
$\overline{\log x}$	3.465	3.512	3.483	3.411	3.414	3.474
s_1	0.259	0.252	0.120	0.417	0.282	0.249
s_2	0.242	0.264	0.162	0.442	0.314	0.218
s_a	10	8.7	24	11	11	2.1
s_b	60	50	140	70	68	13

Provincial towns. (2.3.) 51 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.846	2.230	1.963	1.703	2.345	2.055	2.285
0.898	1.501	0.767	0.858	1.679	1.576	0.799
3.575	3.575	3.575	3.575	3.575	3.575	3.575
0.121	0.203	0.110	0.135	0.151	0.244	0.0977
0.149	0.344	0.0905	0.138	0.141	0.338	0.0794
0.021	0.048	0.013	0.019	0.020	0.047	0.011
0.11	0.26	0.069	0.10	0.11	0.26	0.60
0.75	0.64	0.85	0.76	0.91	0.66	0.88
21	23	21	29	28	23	32
7	5	6	5	8	6	5
1.74	2.20	1.69	2.14	1.90	1.86	2.64
1.52	2.88	0.68	1.04	0.87	1.92	0.66
0.7 (3)	2.2 (3)	3.7 (3)	4.3 (3)	5.0 (3)	0.5 (3)	13.0 (3)
1.846	2.230	1.963	1.703	2.345	2.055	2.285
-0.135	-0.244	-0.123	-0.136	-0.266	-0.243	-0.124
2.901	2.901	2.901	2.901	2.901	2.901	2.901
0.121	0.203	0.110	0.135	0.151	0.244	0.0977
0.159	0.338	0.087	0.138	0.140	0.345	0.0849
0.022	0.047	0.012	0.019	0.020	0.048	0.012
0.019	0.041	0.010	0.017	0.017	0.041	0.010
0.71	0.65	0.86	0.76	0.92	0.64	0.87
25	26	19	25	26	23	29
7	4	6	4	6	6	6
1.55	2.27	1.80	2.13	1.90	1.82	2.35
1.71	2.78	0.63	1.03	0.85	1.99	0.75
1.2 (3)	0.1 (3)	2.7 (3)	4.1 (3)	0.4 (3)	1.2 (3)	7.8 (3)
59.67	61.87	78.30	41.84	67.65	72.2	163.8
147	695	169	107	731	414	353
3.441	3.287	3.454	3.436	3.290	3.332	3.454
0.279	0.466	0.252	0.312	0.348	0.563	0.225
0.367	0.748	0.205	0.354	0.380	0.813	0.176
3.3	11	2.4	2.2	5.8	10	4.3
20	92	14	14	50	74	26

Appendix A. Main results (continued). Skilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.79	0.56	0.85	0.73	0.82	0.80
N.....	26	24	28	23	28	31
l.....	7	5	7	6	5	6
d.....	2.16	1.53	2.29	1.39	2.18	2.12
F.....	0.87	0.87	1.81	1.12	1.24	0.76
$\chi^2(f)$	2.5 (3)	6.9 (3)	1.4 (3)	6.9 (3)	2.6 (3)	2.7 (3)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{x_0} \right)$						
a.....	290.4	229.8	1026	159.2	232.7	67.47
$b \cdot 10^{-4}$	-67.8	-34.7	-202	-61.1	-83.1	-15.2
$\frac{1}{x} \cdot 10^4$	3.650	3.373	3.535	4.265	4.085	3.645
s_1	0.259	0.252	0.120	0.417	0.282	0.249
s_2	0.283	0.261	0.191	0.515	0.371	0.236
s_a	12	8.6	28	13	14	2.3
$s_b \cdot 10^{-4}$	8.9	6.6	26	9.1	9.5	1.7
R.....	0.73	0.60	0.80	0.69	0.78	0.78
N.....	24	24	21	23	21	29
l.....	7	6	16	6	7	6
d.....	1.66	1.64	1.71	1.18	1.63	1.88
F.....	1.20	0.85	2.53	1.53	1.73	0.90
$\chi^2(f)$	8.2 (3)	10.3 (3)	3.1 (3)	5.5 (3)	1.1 (3)	1.3 (3)
5: $\log y = \log k + \log \Phi(a + \log x)$						
a.....	-4.855	-3.399	-4.428	-5.701	-5.797	-4.629
k.....	3229	414.1	5711	12470	22650	517.4
s_1	0.112	0.123	0.0521	0.181	0.123	0.108
s_2	0.100	0.131	0.0661	0.185	0.135	0.0948
s_a	0.45	0.69	0.22	0.67	0.45	0.44
s_k	2300	180	1600	19000	24000	330
R.....	0.80	0.53	0.85	0.70	0.82	0.79
N.....	26	25	28	18	30	29
l.....	4	5	5	6	4	6
d.....	2.45	1.47	2.48	1.57	2.49	2.17
F.....	0.80	1.14	1.61	1.05	1.20	0.77
$\chi^2(f)$	3.56 (5)	2.39 (5)	0.61 (5)	5.64 (5)	1.55 (5)	5.57 (5)

Provincial towns. (2.3.) 51 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.73	0.74	0.86	0.75	0.91	0.62	0.89
23	25	21	26	21	17	31
7	6	6	4	8	10	5
1.64	2.14	1.83	2.11	1.12	1.34	2.48
1.73	2.57	0.66	1.29	1.19	2.09	0.61
1.3 (3)	9.3 (3)	4.8 (3)	11.0 (3)	2.8 (3)	7.8 (3)	5.4 (3)
61.01	41.89	77.42	41.42	57.15	193.4	164.0
-18.3	-79.4	-21.6	-13.2	-83.0	-113	-44.3
3.888	5.608	3.847	4.014	5.491	2.901	3.827
0.279	0.466	0.252	0.312	0.348	-	0.225
0.414	0.836	0.224	0.394	0.550	-	0.226
3.9	9.8	2.6	2.5	7.9	-	5.5
2.8	11	1.9	1.8	7.7	-	4.0
0.69	0.73	0.85	0.73	0.84	0.24	0.84
19	23	17	23	19	17	27
7	7	7	5	8	10	6
1.31	2.02	1.58	1.83	0.53	2.24	1.67
2.20	3.21	0.79	1.59	2.50	-	1.01
5.5 (3)	11.8 (3)	8.1 (3)	9.2 (3)	13.6 (3)	-	10.3 (3)
-5.255	-	-4.906	-5.154	-7.231	-	-4.991
1531	-	1016	895.9	1758000	-	2492
0.121	-	0.110	0.135	0.151	-	0.0977
0.149	-	0.0873	0.136	0.137	-	0.0782
0.46	-	0.43	0.52	0.54	-	0.38
1400	-	730	870	3400000	-	1600
0.71	-	0.84	0.74	0.89	-	0.89
21	-	23	29	28	-	34
7	-	6	5	8	-	5
1.72	-	1.82	2.19	2.29	-	2.65
1.52	-	0.63	1.02	0.82	-	0.64
0.56 (5)	-	3.48 (5)	7.86 (5)	3.77 (5)	-	7.93 (5)

Appendix A. Main results (continued). Unskilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.457	2.347	3.034	2.215	2.453	1.857
b.....	0.704	0.430	0.617	1.168	1.014	0.626
$\overline{\log x}$	3.526	3.526	3.526	3.526	3.526	3.526
s_1	0.122	0.103	0.0656	0.183	0.123	0.109
s_2	0.144	0.109	0.0597	0.170	0.167	0.132
s_a	0.017	0.013	0.0071	0.020	0.020	0.016
s_b	0.091	0.069	0.038	0.11	0.11	0.083
R.....	0.68	0.60	0.89	0.80	0.76	0.67
N.....	34	32	32	43	35	45
l.....	6	7	13	5	7	6
d.....	1.77	1.64	1.80	2.44	2.32	2.11
F.....	1.38	1.12	0.83	0.86	1.84	1.46
$\chi^2 (f)$	4.6 (5)	3.1 (5)	2.8 (5)	4.9 (5)	6.3 (5)	1.6 (5)
$2: \log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$						
a.....	2.457	2.347	3.034	2.215	2.453	1.857
$b \cdot 10^{-4}$	-0.0899	-0.0527	-0.0821	-0.156	-0.140	-0.0876
$\overline{\frac{1}{x}} \cdot 10^{+4}$	3.261	3.261	3.261	3.261	3.261	3.261
s_1	0.122	0.103	0.0656	0.183	0.123	0.109
s_2	0.152	0.115	0.0667	0.177	0.166	0.130
s_a	0.018	0.014	0.0080	0.021	0.020	0.016
$s_b \cdot 10^{-4}$	0.013	0.010	0.0053	0.015	0.014	0.011
R.....	0.64	0.54	0.86	0.78	0.76	0.69
N.....	30	28	29	41	36	45
l.....	6	10	9	5	7	6
d.....	1.58	1.49	1.47	2.24	2.27	2.10
F.....	1.55	1.25	1.03	0.94	1.82	1.42
$\chi^2 (f)$	3.2 (5)	3.2 (5)	1.8 (5)	9.7 (5)	9.4 (5)	1.9 (5)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	266.0	219.9	984.0	99.26	164.2	65.59
b.....	418	204	1450	365	633	109
$\overline{\log x}$	3.411	3.460	3.428	3.272	3.266	3.414
s_1	0.282	0.238	0.151	0.422	0.284	0.251
s_2	0.363	0.281	0.147	0.398	0.374	0.295
s_a	12	7.5	18	5.9	9.4	2.4
s_b	64	41	96	29	45	13

Provincial towns. (2.4.) 70 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.753	2.169	1.916	1.688	2.240	1.976	2.275
0.771	1.368	0.969	1.041	1.538	1.617	0.913
3.526	3.526	3.526	3.526	3.526	3.526	3.526
0.135	0.220	0.102	0.137	0.167	0.233	0.0908
0.148	0.254	0.102	0.129	0.183	0.317	0.0865
0.018	0.030	0.012	0.015	0.022	0.038	0.010
0.094	0.16	0.065	0.081	0.12	0.20	0.055
0.71	0.72	0.88	0.84	0.85	0.70	0.90
35	33	33	35	36	39	34
7	10	5	5	7	7	8
1.90	1.99	1.62	2.06	2.28	1.79	1.57
1.21	1.33	1.01	0.90	1.20	1.85	0.91
7.8 (5)	3.2 (5)	7.0 (5)	2.8 (5)	14.0 (5)	4.6 (5)	4.9 (5)
1.753	2.169	1.916	1.688	2.240	1.976	2.275
-0.103	-0.190	-0.131	-0.141	-0.207	-0.201	-0.116
3.261	3.261	3.261	3.261	3.261	3.261	3.261
0.135	0.220	0.102	0.137	0.167	0.233	0.0908
0.153	0.252	0.107	0.133	0.193	0.344	0.110
0.018	0.030	0.013	0.016	0.023	0.041	0.013
0.013	0.022	0.0093	0.012	0.017	0.030	0.0096
0.68	0.73	0.86	0.83	0.83	0.63	0.83
33	34	34	31	37	33	25
7	10	5	6	5	7	7
1.72	1.97	1.43	1.89	2.08	1.57	1.06
1.29	1.30	1.10	0.95	1.34	2.18	1.47
1.4 (5)	3.0 (5)	2.9 (5)	3.3 (5)	4.2 (5)	3.4 (5)	3.1 (5)
47.26	51.56	59.02	32.13	239.3	56.01	162.9
105	391	163	102	897	384	321
3.371	3.169	3.331	3.304	3.526	3.289	3.399
0.311	0.509	0.235	0.314	-	0.537	0.209
0.407	0.586	0.259	0.295	-	1.12	0.248
2,5	6.0	2.1	1.4	-	11	25
13	34	10	6,6	-	71	8

Appendix A. Main results (continued). Unskilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.63	0.52	0.88	0.84	0.87	0.72
N.....	36	30	29	41	34	45
l.....	6	10	9	6	9	5
d.....	1.71	1.53	1.61	2.03	2.21	2.04
F.....	1.65	1.40	0.94	0.89	1.74	1.38
χ^2 (f).....	12.9 (5)	12.0 (5)	1.5 (5)	3.8 (5)	9.0 (5)	4.7 (5)

$$4: y = a + b \left(\frac{1}{x} - \bar{\frac{1}{x}} \right)$$

a.....	318.8	225.0	1014	98.77	134.5	60.73
$b \cdot 10^{-4}$	-70.2	-16.9	-140	-31.6	-55.5	-11.4
$\bar{\frac{1}{x}} \cdot 10^{+4}$	3.261	3.721	4.023	6.179	6.658	4.719
s_1	-	0.238	0.151	0.422	0.284	0.251
s_2	-	0.310	0.199	0.501	0.438	0.308
s_a	-	8.5	25	7.6	10	2.4
$s_b \cdot 10^{-4}$	-	5.1	13	3.0	4.1	1.1
R.....	0.50	0.42	0.79	0.78	0.86	0.79
N.....	28	21	27	31	31	37
l.....	7	16	10	8	11	6
d.....	1.57	1.37	1.00	1.35	1.86	1.94
F.....	-	1.70	1.73	1.41	2.38	1.50
χ^2 (f).....	-	11.8 (5)	4.6 (5)	18.3 (5)	8.0 (5)	5.8 (5)

$$5: \log y = \log k + \log \Phi (a + \log x)$$

a.....	-4.667	-3.831	-4.422	-5.905	-5.513	-4.451
k.....	2290	592.6	5920	19230	12270	410.7
s_1	0.122	0.103	0.0656	0.183	0.123	0.109
s_2	0.143	0.109	0.0587	0.169	0.165	0.130
s_a	0.41	0.41	0.23	0.56	0.38	0.38
s_k	1400	230	1800	27000	10000	210
R.....	0.65	0.53	0.89	0.79	0.76	0.68
N.....	32	32	34	44	38	45
l.....	6	7	9	5	7	6
d.....	1.80	1.82	1.88	2.45	2.57	2.29
F.....	1.38	1.12	0.80	0.85	1.79	1.42
χ^2 (f).....	4.55 (7)	4.19 (7)	4.87 (7)	8.51 (7)	5.25 (7)	1.81 (7)

Provincial towns. (2.4.) 70 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.73	0.82	0.90	0.91	0.78	0.55	0.82
33	31	32	35	29	27	25
6	10	5	6	12	11	8
1.78	1.74	1.31	1.82	1.95	1.25	1.05
1.72	1.33	1.22	0.88	—	4.39	1.40
11.1 (5)	10.4 (5)	7.9 (5)	3.9 (5)	—	52.5 (5)	1.8 (5)
43.22	49.65	57.45	30.38	113.6	44.56	208.6
-9.90	-33.9	-14.3	-8.91	-57.8	-45.4	-56.7
5.271	7.420	5.527	5.928	4.991	5.665	3.261
0.311	0.509	0.235	0.314	0.385	0.537	—
0.489	0.694	0.349	0.377	0.649	1.36	—
2.9	7.2	2.8	1.7	11	12	—
1.2	3.4	1.1	0.67	7.1	9.3	—
0.71	0.77	0.84	0.85	0.71	0.52	0.78
32	21	21	27	27	23	21
10	11	17	10	10	11	14
1.49	1.50	0.79	1.13	1.36	1.10	1.94
2.48	1.86	2.21	1.44	2.84	6.38	—
31.9 (5)	19.6 (5)	12.2 (5)	6.6 (5)	25.2 (5)	88.7 (5)	—
-4.858	-6.414	-5.394	—	—	—	-9.571
628.2	77220	2714	—	—	—	1080·10 ⁸
0.135	0.220	0.102	—	—	—	0.0908
0.148	0.251	0.101	—	—	—	0.414
0.45	0.66	0.32	—	—	—	0.028
460	150000	1800	—	—	—	100·10 ⁸
0.69	0.74	0.88	—	—	—	0.62
35	33	33	—	—	—	19
7	10	5	—	—	—	20
1.96	2.05	1.69	—	—	—	0.09
1.20	1.30	0.98	—	—	—	20.71
7.45 (7)	2.23 (7)	1.94 (7)	—	—	—	34.51 (7)

Appendix A. Main results (continued). Lower public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b(\log x - \overline{\log x})$						
a.....	2.477	2.411	3.018	2.197	2.512	1.911
b.....	0.835	0.550	0.577	1.061	1.117	0.712
$\overline{\log x}$	3.585	3.585	3.585	3.585	3.585	3.585
s_1	0.146	0.143	0.0702	0.198	0.136	0.118
s_2	0.140	0.160	0.0743	0.250	0.133	0.105
s_a	0.014	0.016	0.0074	0.025	0.013	0.010
s_b	0.069	0.079	0.036	0.12	0.065	0.051
R.....	0.77	0.57	0.85	0.65	0.86	0.81
N.....	44	42	42	47	55	51
l.....	10	9	11	8	12	5
d.....	1.82	1.59	1.57	2.04	2.21	2.19
F.....	0.92	1.25	1.12	1.60	0.95	0.79
$\chi^2(f)$	5.0 (7)	23.4 (7)	2.8 (7)	11.1 (7)	5.5 (7)	6.7 (7)
$2: \log y = a + b\left(\frac{1}{x} - \frac{\bar{1}}{\bar{x}}\right)$						
a.....	2.477	2.411	3.018	2.197	2.512	1.911
$b \cdot 10^{-4}$	-0.118	-0.0855	-0.0834	-0.162	-0.153	-0.0994
$\frac{\bar{1}}{\bar{x}} \cdot 10^{+4}$	2.891	2.891	2.891	2.891	2.891	2.891
s_1	0.146	0.143	0.0702	0.198	0.136	0.118
s_2	0.148	0.156	0.0776	0.244	0.156	0.115
s_a	0.015	0.015	0.0077	0.024	0.015	0.011
$s_b \cdot 10^{-4}$	0.011	0.011	0.0056	0.018	0.0011	0.0083
R.....	0.74	0.61	0.83	0.68	0.81	0.77
N.....	45	43	43	55	39	49
l.....	8	9	9	6	15	10
d.....	1.66	1.69	1.45	2.18	1.58	1.85
F.....	1.02	1.18	1.22	1.51	1.32	0.95
$\chi^2(f)$	7.7 (7)	20.7 (7)	2.4 (7)	13.4 (7)	4.5 (7)	9.0 (7)
$3: y = a + b(\log x - \overline{\log x})$						
a.....	257.6	229.3	937.2	89.4	221.3	70.82
b.....	484	380	1320	352	620	117
$\overline{\log x}$	3.429	3.444	3.467	3.280	3.342	3.442
s_1	0.336	0.330	0.162	0.456	0.313	0.272
s_2	0.343	0.284	0.173	0.502	0.370	0.263
s_a	9.4	6.8	17	5.9	9.6	2.0
s_b	43	32	79	27	42	9.1

salaried employees. Rural districts. (3.2.) 102 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.818	2.314	1.943	1.698	2.340	2.260	2.009
0.936	1.104	0.954	0.925	1.590	1.530	0.819
3.585	3.585	3.585	3.585	3.585	3.585	3.585
0.154	0.232	0.119	0.163	0.165	0.252	0.102
0.160	0.316	0.115	0.191	0.205	0.361	0.120
0.016	0.031	0.011	0.019	0.020	0.036	0.012
0.078	0.15	0.056	0.093	0.10	0.18	0.059
0.77	0.58	0.86	0.70	0.85	0.65	0.81
48	50	64	45	58	50	51
7	6	7	6	5	8	6
1.79	1.92	2.36	1.85	2.34	1.97	2.21
1.08	1.85	0.94	1.37	1.55	2.05	1.38
3.7 (7)	6.8 (7)	14.9 (7)	8.3 (7)	4.6 (7)	5.7 (7)	8.1 (7)
1.818	2.314	1.943	1.698	2.340	2.260	2.009
-0.124	-0.167	-0.134	-0.131	-0.223	-0.207	-0.114
2.891	2.891	2.891	2.891	2.891	2.891	2.891
0.154	0.232	0.119	0.163	0.165	0.252	0.102
0.180	0.311	0.129	0.197	0.228	0.382	0.132
0.018	0.031	0.013	0.020	0.023	0.038	0.013
0.013	0.022	0.0093	0.014	0.016	0.028	0.0095
0.69	0.60	0.82	0.68	0.81	0.60	0.77
43	50	43	37	49	49	51
7	5	8	7	12	9	7
1.40	1.97	1.89	1.75	1.91	1.76	1.82
1.38	1.80	1.18	1.47	1.92	2.30	1.69
3.5 (7)	2.9 (7)	6.2 (7)	8.1 (7)	6.7 (7)	5.5 (7)	10.8 (7)
53.37	59.98	65.21	37.32	314.2	358.7	84.73
115	551	157	96.8	1107	1522	165
3.393	3.166	3.378	3.356	3.585	3.585	3.419
0.354	0.534	0.273	0.374	-	-	0.234
0.429	0.747	0.319	0.577	-	-	0.311
2.5	8.2	2,3	2.5	-	-	2.8
11	49	10	11	-	-	13

Appendix A. Main results (continued). Lower public servants and

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.74	0.77	0.86	0.81	0.83	0.79
N.....	43	47	42	52	29	49
l.....	8	9	9	6	15	10
d.....	1.60	1.60	1.57	2.08	1.46	1.96
F.....	1.04	0.74	1.15	1.21	1.39	0.94
χ^2 (f).....	9.5 (7)	8.1 (7)	8.9 (7)	12.5 (7)	19.4 (7)	13.9 (7)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	275.0	230.9	962.7	204.0	226.1	74.86
$b10^{-4}$	-46.5	-38.3	-136	-64.1	-59.2	-11.9
$\frac{1}{\bar{x}} 10^{+4}$	4.109	4.179	3.797	2.891	5.379	3.943
s_1	0.336	0.330	0.162	-	0.313	0.272
s_2	0.407	0.330	0.216	-	0.473	0.324
s_a	12	8.0	21	-	13	2.5
$s_b \cdot 10^{-4}$	5.3	3.6	11	-	5.0	1.2
R.....	0.66	0.73	0.79	0.74	0.76	0.71
N.....	35	33	33	53	23	43
l.....	14	19	23	8	24	14
d.....	1.22	1.41	1.08	2.14	0.91	1.41
F.....	1.47	1.00	1.79	-	2.28	1.42
χ^2 (f).....	14.1 (7)	14.7 (7)	14.8 (7)	-	47.1 (7)	22.1 (7)
5: $\log y = \log k + \log \Phi (a + \log x)$						
a.....	-5.075	-4.274	-4.346	-5.679	-9.573	-4.734
k.....	4481	1067	4738	8851	$1573 \cdot 10^8$	660.3
s_1	0.146	0.143	0.0702	0.198	0.136	0.118
s_2	0.139	0.157	0.0726	0.247	0.369	0.110
s_a	0.37	0.40	0.19	0.47	0.038	0.31
s_k	2900	510	1100	9500	$220 \cdot 10^8$	310
R.....	0.76	0.74	0.86	0.76	0.61	0.81
N.....	44	42	44	51	31	49
l.....	10	9	10	8	17	9
d.....	1.90	1.63	1.65	2.09	0.30	2.20
F.....	0.91	1.21	1.07	1.55	7.35	0.79
χ^2 (f).....	2.89 (9)	20.85 (9)	8.98 (9)	11.00 (9)	34.47 (9)	4.92 (9)

salaried employees. Rural districts. (3.2.) 102 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.72	0.76	0.84	0.68	0.77	0.45	0.79
41	44	37	35	39	37	51
7	11	15	12	16	14	7
1.42	1.94	1.79	1.84	1.91	2.18	1.95
1.47	1.96	1.36	2.38	-	-	1.76
14.0 (7)	38.8 (7)	18.6 (7)	43.2 (7)	-	-	11.5 (7)
56.40	33.37	100.6	61.7	314.2	358.7	114.2
-11.5	-51.3	-28.9	-18.7	-137	-174	-28.7
4.578	7.814	2.891	2.891	2.891	2.891	2.891
0.354	0.534	-	-	-	-	-
0.519	0.780	-	-	-	-	-
3.2	6.3	-	-	-	-	-
1.3	4.3	-	-	-	-	-
0.66	0.76	0.79	0.67	0.74	0.46	0.86
37	41	33	35	33	27	39
10	11	15	13	15	14	14
0.99	1.82	1.45	2.01	1.23	1.78	1.54
2.15	2.14	-	-	-	-	-
36.8 (7)	47.0 (7)	-	-	-	-	-
-5.340	-5.790	-5.395	-5.317	-	-9.377	-5.030
1688	15290	2538	1219	-	409.8·10 ⁸	1397
0.154	0.232	0.119	0.163	-	0.252	0.102
0.162	0.313	0.115	0.190	-	0.410	0.120
0.38	0.55	0.29	0.40	-	0.13	0.26
1300	20000	1500	960	-	230·10 ⁸	620
0.73	0.58	0.86	0.71	-	0.30	0.80
47	52	62	42	-	42	53
7	5	7	6	-	11	6
1.72	2.05	2.36	1.86	-	1.52	2.22
1.10	1.82	0.93	1.36	-	2.64	1.38
8.81 (9)	7.93 (9)	13.83 (9)	8.91 (9)	-	3.78 (9)	11.22 (9)

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
1: $\log y = a + b (\log x - \overline{\log x})$						
a.....	2.409	2.336	2.982	2.168	2.366	1.769
b.....	0.926	0.322	0.649	0.933	1.021	0.617
$\log x$	3.488	3.488	3.488	3.488	3.488	3.488
s_1	0.126	0.105	0.0481	0.185	0.134	0.129
s_2	0.123	0.120	0.0520	0.166	0.134	0.122
s_a	0.017	0.017	0.0073	0.023	0.019	0.017
s_b	0.095	0.091	0.040	0.14	0.10	0.093
R.....	0.81	0.45	0.92	0.70	0.82	0.69
N.....	25	19	29	21	22	28
l.....	7	10	6	6	5	5
d.....	1.65	1.43	2.03	2.12	1.95	1.94
F.....	0.95	1.30	1.17	0.81	0.99	0.89
$\chi^2 (f)$	0.6 (3)	2.3 (3)	9.4 (3)	5.9 (3)	3.0 (3)	8.6 (3)
2: $\log y = a + b \left(\frac{1}{x} - \overline{\frac{1}{x}} \right)$						
a.....	2.409	2.336	2.982	2.168	2.366	1.769
$b \cdot 10^{-4}$	-0.126	-0.0565	-0.0870	-0.118	-0.141	-0.0843
$\overline{\frac{1}{x}} \cdot 10^{+4}$	3.523	3.523	3.523	3.523	3.523	3.523
s_1	0.126	0.105	0.0481	0.185	0.134	0.129
s_2	0.120	0.111	0.0618	0.173	0.137	0.124
s_a	0.017	0.016	0.0087	0.024	0.019	0.017
$s_b \cdot 10^{-4}$	0.013	0.012	0.0066	0.019	0.015	0.013
R.....	0.82	0.57	0.88	0.67	0.81	0.68
N.....	25	20	25	25	25	28
l.....	5	10	6	5	4	5
d.....	1.72	1.67	1.32	1.95	1.84	1.86
F.....	0.91	1.11	1.66	0.87	1.05	0.92
$\chi^2 (f)$	4.1 (3)	2.9 (3)	2.2 (3)	3.7 (3)	6.2 (3)	3.7 (3)
3: $y = a + b (\log x - \overline{\log x})$						
a.....	207.1	211.2	870.1	128.8	187.5	55.23
b.....	555	254	1450	331	526	88.6
$\overline{\log x}$	3.349	3.426	3.404	3.364	3.354	3.407
s_1	0.291	0.243	0.111	0.425	0.309	0.298
s_2	0.293	0.247	0.128	0.420	0.283	0.263
s_a	9.3	7.5	16	8.3	8.1	2.1
s_b	57	45	100	53	51	13

Rural districts. (3.3.) 51 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.747	2.114	1.826	1.598	2.169	2.247	2.182
0.875	1.068	0.806	0.862	1.606	1.964	0.938
3.488	3.488	3.488	3.488	3.488	3.488	3.488
0.132	0.215	0.106	0.136	0.164	0.252	0.0943
0.132	0.333	0.128	0.149	0.201	0.347	0.0964
0.018	0.047	0.018	0.021	0.028	0.049	0.014
0.10	0.25	0.098	0.11	0.15	0.26	0.073
0.78	0.52	0.76	0.74	0.83	0.73	0.88
30	23	21	28	29	30	20
4	9	7	4	5	5	6
2.30	1.50	1.47	1.55	2.37	2.23	1.87
0.99	2.41	1.48	1.20	1.51	1.91	1.05
1.1 (3)	3.7 (3)	11.5 (3)	1.5 (3)	7.1 (3)	4.9 (3)	6.6 (3)
1.747	2.114	1.826	1.598	2.169	2.247	2.182
-0.116	-0.152	-0.108	-0.120	-0.208	-0.266	-0.123
3.523	3.523	3.523	3.523	3.523	3.523	3.523
0.132	0.215	0.106	0.136	0.164	0.252	0.0943
0.141	0.331	0.135	0.150	0.231	0.358	0.114
0.020	0.046	0.019	0.021	0.032	0.050	0.016
0.015	0.035	0.014	0.016	0.024	0.038	0.012
0.74	0.52	0.73	0.73	0.77	0.71	0.82
30	23	22	26	25	29	21
4	9	5	4	7	5	6
1.96	1.56	1.31	1.52	1.84	2.11	1.31
1.13	2.38	1.64	1.21	1.99	2.03	1.47
3.2 (3)	2.6 (3)	10.6 (3)	1.3 (3)	2.3 (3)	2.0 (3)	2.2 (3)
47.21	208.9	58.53	34.82	224.3	49.40	128.6
117	928	131	81.3	1170	751	310
3.364	3.494	3.378	3.375	3.494	3.181	3.372
0.305	-	0.243	0.314	-	0.579	0.217
0.357	-	0.282	0.369	-	0.812	0.255
2.5	-	2.5	1.9	-	10	4.9
16	-	15	12	-	100	31

Appendix A. Main results (continued). Skilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.81	0.63	0.90	0.67	0.83	0.70
N.....	23	20	26	23	23	28
l.....	7	10	6	8	4	5
d.....	1.70	1.66	1.60	2.08	1.71	1.90
F.....	1.02	1.03	1.35	0.98	0.84	0.78
χ^2 (f).....	5.4 (3)	3.3 (3)	3.4 (3)	5.4 (3)	0.5 (3)	4.0 (3)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	206.4	209.4	874.2	132.3	196.6	55.00
$b \cdot 10^{-4}$	-52.2	-29.7	-15.5	-32.7	-52.0	-9.81
$\frac{1}{\bar{x}} \cdot 10^4$	4.935	4.070	4.268	4.609	4.675	4.263
s_1	0.291	0.243	0.111	0.425	0.309	0.298
s_2	0.326	0.245	0.178	0.470	0.351	0.274
s_a	10	7.3	23	9.6	11	2.2
$s_b \cdot 10^{-4}$	5.7	4.9	14	5.8	6.2	1.4
R.....	0.79	0.66	0.84	0.64	0.77	0.71
N.....	17	24	17	25	19	25
l.....	12	9	13	9	9	6
d.....	1.31	1.82	0.81	1.81	1.19	1.70
F.....	1.26	1.02	2.60	1.22	1.29	0.85
χ^2 (f).....	8.4 (3)	3.3 (3)	5.9 (3)	3.9 (3)	8.2 (3)	3.8 (3)
5: $\log y = \log k + \log \Phi (a + \log x)$						
a.....	-9.484	-3.527	-	-5.272	-	-4.431
k.....	$1340 \cdot 10^8$	449.9	-	4045	-	341.6
s_1	0.126	0.105	-	0.185	-	0.129
s_2	0.358	0.115	-	0.164	-	0.120
s_a	0.053	0.55	-	0.77	-	0.54
s_k	$260 \cdot 10^2$	190	-	6300	-	2500
R.....	0.27	0.59	-	0.66	-	0.68
N.....	11	19	-	25	-	28
l.....	24	10	-	5	-	5
d.....	0.29	1.51	-	2.10	-	1.96
F.....	8.07	1.21	-	0.79	-	0.87
χ^2 (f).....	25.20 (5)	1.42 (5)	-	2.80 (5)	-	4.22 (5)

Rural districts. (3.3.) 51 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.73	0.21	0.78	0.69	0.41	0.73	0.82
26	17	22	24	13	23	19
4	9	5	8	16	9	6
1.94	2.54	1.24	1.53	2.35	1.45	1.22
1.37	—	1.34	1.39	—	1.96	1.38
7.0 (3)	—	4.5 (3)	4.8 (3)	—	4.1 (3)	1.5 (3)
50.09	109.5	59.15	35.22	224.3	52.37	129.9
—12.1	—37.1	—13.5	—7.60	—133	—72.4	—29.6
4.496	5.628	4.530	4.622	3.479	6.782	4.675
0.305	0.494	0.243	0.314	—	0.579	0.217
0.452	1.21	0.343	0.401	—	1.02	0.351
3.5	20	3.0	2.1	—	15	6.8
2.1	11	1.8	1.2	—	12	4.0
0.65	0.42	0.73	0.66	0.50	0.65	0.72
17	15	21	24	15	21	15
7	11	6	8	16	12	15
1.51	1.17	0.85	1.35	2.19	0.91	0.72
2.20	6.03	1.99	1.63	—	3.10	2.61
19.3 (3)	52.0 (3)	6.4 (3)	10.0 (3)	—	5.5 (3)	11.0 (3)
—5.151	—5.664	—4.963	—9.510	—9.384	—	—5.318
1161	8798	959.2	220.3 · 10 ⁸	548 · 10 ⁸	—	4529
0.132	0.215	0.106	0.136	0.164	—	0.0943
0.131	0.327	0.127	0.394	0.263	—	0.0971
0.51	0.80	0.41	0.054	0.099	—	0.36
1100	16000	700	42 · 10 ⁸	230 · 10 ⁸	—	3300
0.76	0.59	0.78	0.21	0.56	—	0.86
30	23	21	15	27	—	22
4	9	7	14	7	—	6
2.29	1.52	1.44	0.27	1.46	—	1.83
0.99	2.33	1.44	8.37	2.58	—	1.06
2.51 (5)	3.16 (5)	5.95 (5)	23.25 (5)	8.84 (5)	—	2.36 (5)

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
1 : $\log y = a + b (\log x - \overline{\log x})$						
a.....	2.321	2.336	2.984	2.121	2.366	1.743
b.....	0.981	0.642	0.665	1.275	0.993	0.498
$\overline{\log x}$	3.468	3.468	3.468	3.468	3.468	3.468
s_1	0.133	0.114	0.0565	0.169	0.112	0.112
s_2	0.152	0.116	0.0520	0.187	0.136	0.125
s_a	0.016	0.012	0.0054	0.019	0.014	0.013
s_b	0.076	0.058	0.026	0.093	0.068	0.063
R.....	0.80	0.76	0.94	0.82	0.84	0.64
N.....	44	39	46	49	46	47
l.....	5	10	6	6	6	8
d.....	1.91	1.54	1.74	2.28	2.43	2.17
F.....	1.31	1.04	0.85	1.22	1.47	1.25
χ^2 (f).....	3.8 (7)	17.7 (7)	9.0 (7)	11.7 (7)	3.6 (7)	8.9 (7)
2 : $\log y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{\bar{x}} \right)$						
a.....	2.321	2.336	2.984	2.121	2.366	1.743
$b \cdot 10^{-4}$	-0.112	-0.0795	-0.0768	-0.145	-0.111	-0.0565
$\frac{\bar{1}}{\bar{x}} \cdot 10^{+4}$	3.788	3.788	3.788	3.788	3.788	3.788
s_1	0.133	0.114	0.0565	0.169	0.112	0.112
s_2	0.164	0.110	0.0619	0.202	0.153	0.129
s_a	0.017	0.011	0.0064	0.021	0.016	0.013
$s_b \cdot 10^{-4}$	0.0097	0.0065	0.0037	0.012	0.0091	0.0077
R.....	0.77	0.79	0.91	0.78	0.79	0.61
N.....	35	40	35	51	45	47
l.....	11	14	9	8	7	8
d.....	1.66	1.71	1.23	1.95	1.91	2.03
F.....	1.51	0.93	1.20	1.43	1.85	1.33
χ^2 (f).....	7.4 (7)	8.2 (7)	2.9 (7)	10.6 (7)	1.5 (7)	11.7 (7)
3 : $y = a + b (\log x - \overline{\log x})$						
a.....	163.1	184.7	827.9	172.9	173.7	52.66
b.....	416	350	1420	531	461	65.7
$\overline{\log x}$	3.281	3.324	3.336	3.468	3.269	3.370
s_1	0.306	0.262	0.130	-	0.259	0.258
s_2	0.385	0.249	0.126	-	0.367	0.260
s_a	7.3	5.1	11	-	7.5	1.5
s_b	36	26	58	-	37	7.4

Rural districts. (3.4.) 93 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.696	2.068	1.775	1.556	2.073	2.057	2.180
0.888	0.791	0.872	0.914	1.571	1.659	0.935
3.468	3.468	3.468	3.468	3.468	3.468	3.468
0.136	0.207	0.108	0.146	0.173	0.236	0.0857
0.129	0.275	0.104	0.143	0.192	0.294	0.0760
0.013	0.028	0.011	0.015	0.020	0.030	0.0079
0.065	0.14	0.052	0.071	0.096	0.15	0.038
0.82	0.52	0.87	0.80	0.86	0.76	0.93
46	45	51	47	51	49	36
9	6	8	6	6	9	6
1.93	1.76	1.96	1.79	2.02	1.85	1.86
0.91	1.75	0.94	0.96	1.23	1.54	0.79
9.0 (7)	7.3 (7)	3.8 (7)	5.2 (7)	5.2 (7)	4.0 (7)	10.6 (7)
1.696	2.068	1.775	1.556	2.073	2.057	2.180
-0.0993	-0.104	-0.0990	-0.107	-0.179	-0.174	-0.108
3.788	3.788	3.788	3.788	3.788	3.788	3.788
0.136	0.207	0.108	0.146	0.173	0.236	0.0857
0.144	0.264	0.118	0.146	0.214	0.336	0.0911
0.015	0.027	0.012	0.015	0.022	0.035	0.0095
0.0086	0.016	0.0070	0.0087	0.013	0.020	0.0054
0.77	0.57	0.83	0.79	0.83	0.67	0.90
39	44	40	40	45	39	37
9	7	10	6	6	10	6
1.55	1.90	1.49	1.69	1.60	1.42	1.34
1.13	1.61	1.20	1.01	1.53	2.02	1.13
10.5 (7)	7.8 (7)	14.1 (7)	8.8 (7)	4.9 (7)	4.5 (7)	9.4 (7)
39.87	89.28	46.16	27.16	33.27	219.7	114.5
92.5	279	110	71.5	322	1060	282
3.295	3.246	3.290	3.274	3.090	3.468	3.287
0.313	0.477	0.248	0.335	0.398	-	0.197
0.344	0.636	0.268	0.358	0.634	-	0.218
1.6	7.0	1.4	1.1	3.8	-	2.9
7.8	35	7.1	5.7	26	-	14

Appendix A. Main results (continued). Unskilled workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.77	0.82	0.93	0.68	0.79	0.68
N.....	35	44	43	39	49	49
l.....	11	14	6	8	8	5
d.....	1.67	1.69	1.49	2.23	1.65	2.08
F.....	1.58	0.91	0.94	—	2.02	1.02
χ^2 (f)	11.3 (7)	17.7 (7)	5.9 (7)	—	15.2 (7)	5.1 (7)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{x_0} \right)$						
a.....	248.0	182.8	834.5	172.9	276.3	52.94
$b \cdot 10^{-4}$	-60.3	-29.7	-124	-54.0	-72.5	-6.10
$\frac{1}{x} \cdot 10^{+4}$	3.788	5.460	5.206	3.788	3.788	4.771
s_1	—	0.262	0.130	—	—	0.258
s_2	—	0.287	0.194	—	—	0.284
s_a	—	5.9	18	—	—	1.6
$s_b \cdot 10^{-4}$	—	2.3	7.3	—	—	0.72
R.....	0.67	0.80	0.87	0.73	0.69	0.66
N.....	31	40	27	39	37	45
l.....	11	15	17	10	9	9
d.....	1.73	1.35	0.70	1.37	2.31	1.78
F.....	—	1.20	2.22	—	—	1.21
χ^2 (f)	—	20.7 (7)	33.7 (7)	—	—	3.6 (7)
5: $\log y = \log k + \log \Phi (a + \log x)$						
a.....	-5.369	-4.449	-4.504	-9.373	-5.397	-3.998
k.....	7442	1350	6526	$484.7 \cdot 10^8$	8801	188.6
s_1	0.133	0.114	0.0565	0.169	0.112	0.112
s_2	0.152	0.112	0.0518	0.330	0.137	0.125
s_a	0.33	0.31	0.15	0.58	0.28	0.33
s_k	5200	590	1400	$11 \cdot 10^8$	5200	68
R.....	0.75	0.80	0.94	0.47	0.82	0.68
N.....	44	41	46	33	46	47
l.....	5	11	6	18	6	8
d.....	1.92	1.64	1.74	0.75	2.38	2.18
F.....	1.31	0.97	0.84	3.83	1.49	1.24
χ^2 (f).....	4.29 (9)	14.49 (9)	14.75 (9)	13.96 (9)	4.29 (9)	9.18 (9)

Rural districts. (3.4.) 93 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.78	0.66	0.86	0.82	0.81	0.16	0.90
35	42	44	42	31	17	35
9	7	8	7	14	20	6
1.58	1.97	1.51	1.64	0.97	1.62	1.29
1.21	1.77	1.17	1.14	2.53	—	1.22
31.4 (7)	26.2 (7)	27.4 (7)	13.9 (7)	35.7 (7)	—	12.7 (7)
41.69	152.7	45.17	26.93	175.9	219.7	169.9
−7.63	−33.7	−9.03	−5.62	−68.7	−99.9	−40.1
5.619	3.788	6.025	6.204	3.788	3.788	3.788
0.313	—	0.248	0.335	—	—	—
0.422	—	0.358	0.436	—	—	—
2.0	—	1.9	1.4	—	—	—
0.78	—	0.71	0.51	—	—	—
0.73	0.68	0.80	0.75	0.52	0.45	0.88
31	42	31	35	27	19	29
13	7	12	14	14	22	15
1.07	1.78	0.90	1.28	1.38	2.27	1.66
1.82	—	2.09	1.70	—	—	—
42.1 (7)	—	30.6 (7)	28.1 (7)	—	—	—
−5.118	−4.876	−5.077	−5.195	−9.306	—	—
1022	1497	1127	868.9	327.7·10 ⁸	—	—
0.136	0.207	0.108	0.146	0.173	—	—
0.130	0.270	0.105	0.141	0.275	—	—
0.34	0.54	0.27	0.37	0.075	—	—
670	1400	580	620	110·10 ⁸	—	—
0.79	0.59	0.86	0.82	0.58	—	—
48	45	47	49	33	—	—
9	6	8	6	19	—	—
1.88	1.80	1.95	1.82	0.98	—	—
0.91	1.70	0.94	0.94	2.52	—	—
8.19 (9)	7.93 (9)	5.01 (9)	11.25 (9)	11.29 (9)	—	—

Appendix A. Main results (continued). Agricultural workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
$1: \log y = a + b (\log x - \overline{\log x})$						
a.....	2.173	2.276	2.937	1.997	2.195	1.609
b.....	0.962	0.764	0.651	1.132	1.236	0.487
$\overline{\log x}$	3.361	3.361	3.361	3.361	3.361	3.361
s_1	0.170	0.105	0.0618	0.177	0.133	0.117
s_2	0.194	0.123	0.0680	0.221	0.165	0.109
s_a	0.027	0.017	0.0093	0.030	0.023	0.015
s_b	0.14	0.087	0.048	0.16	0.12	0.077
R.....	0.70	0.78	0.88	0.71	0.83	0.66
N.....	21	27	27	26	28	22
l.....	6	6	7	8	5	5
d.....	1.57	1.95	1.58	2.26	2.05	1.83
F.....	1.31	1.36	1.21	1.56	1.54	0.87
$\chi^2 (f)$	6.6 (3)	8.6 (3)	5.8 (3)	1.6 (3)	3.2 (3)	2.4 (3)
$2: \log y = a + b \left(\frac{1}{x} - \frac{\bar{1}}{x} \right)$						
a.....	2.173	2.276	2.937	1.997	2.195	1.609
$b \cdot 10^{-4}$	-0.0816	-0.0655	-0.0592	-0.0969	-0.112	-0.0439
$\frac{\bar{1}}{x} \cdot 10^{+4}$	4.803	4.803	4.803	4.803	4.803	4.803
s_1	0.170	0.105	0.0618	0.177	0.133	0.117
s_2	0.206	0.132	0.0670	0.233	0.166	0.109
s_a	0.028	0.018	0.0092	0.032	0.023	0.015
$s_b \cdot 10^{-4}$	0.013	0.0085	0.0043	0.015	0.011	0.0070
R.....	0.65	0.74	0.89	0.67	0.83	0.66
N.....	21	19	29	25	29	23
l.....	6	14	5	6	4	6
d.....	1.40	1.67	1.65	2.02	2.04	1.83
F.....	1.47	1.58	1.17	1.74	1.55	0.88
$\chi^2 (f)$	2.6 (3)	4.7 (3)	1.8 (3)	0.6 (3)	6.3 (3)	2.9 (3)
$3: y = a + b (\log x - \overline{\log x})$						
a.....	130.3	168.0	756.6	75.22	103.4	38.77
b.....	291	298	1270	242	360	46.8
$\overline{\log x}$	3.205	3.234	3.240	3.158	3.135	3.273
s_1	0.391	0.242	0.142	0.407	0.306	0.268
s_2	0.500	0.317	0.148	0.534	0.409	0.256
s_a	9.7	7.7	16	6.5	6.9	1.4
s_b	51	40	84	34	38	7.3

Rural districts. (3.5.) 53 groups of 3 households.

groups						
Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
1.660	1.953	1.655	1.464	1.900	1.846	2.080
0.865	0.847	0.727	1.096	1.429	0.922	0.875
3.361	3.361	3.361	3.361	3.361	3.361	3.361
0.132	0.220	0.109	0.153	0.191	0.208	0.115
0.189	0.260	0.105	0.191	0.237	0.309	0.130
0.026	0.036	0.014	0.026	0.033	0.042	0.018
0.13	0.18	0.075	0.13	0.17	0.22	0.092
0.67	0.54	0.81	0.75	0.77	0.51	0.80
23	30	32	31	22	25	22
8	5	6	4	7	6	6
1.43	2.33	1.78	1.52	1.34	1.83	1.85
2.07	1.39	0.94	1.54	1.54	2.20	1.27
8.8 (3)	1.3 (3)	5.6 (3)	6.1 (3)	1.8 (3)	9.7 (3)	3.5 (3)
1.660	1.953	1.655	1.464	1.900	1.846	2.080
-0.0634	-0.0823	-0.0656	-0.106	-0.118	-0.0677	-0.0782
4.803	4.803	4.803	4.803	4.803	4.803	4.803
0.132	0.220	0.109	0.153	0.191	0.208	0.115
0.214	0.252	0.106	0.172	0.265	0.327	0.133
0.029	0.035	0.015	0.024	0.036	0.045	0.018
0.014	0.016	0.0068	0.011	0.017	0.021	0.0085
0.54	0.58	0.80	0.80	0.70	0.41	0.79
23	35	24	28	24	23	25
6	4	7	6	7	8	4
1.15	2.50	1.72	1.90	1.07	1.66	1.75
2.65	1.31	0.95	1.25	1.91	2.47	1.33
1.7(3)	1.1 (3)	5.3 (3)	3.3 (3)	6.6 (3)	8.1 (3)	3.4 (3)
43.69	72.85	38.63	14.84	37.47	76.44	93.58
83.5	210	74.6	74.7	244	136	237
3.237	3.169	3.224	3.092	3.062	3.214	3.188
0.303	0.507	0.251	0.354	0.441	0.479	0.265
0.546	0.564	0.251	0.380	0.776	0.894	0.300
3.5	6.4	1.4	1.0	5.7	9.7	4.3
18	34	7.4	6.0	36	51	22

Appendix A. Main results (continued). Agricultural workers.

Parameter estimates and test results	Expenditure					
	Dwelling	Fuel & light	Food	Tobacco	Clothing	Footwear
R.....	0.63	0.72	0.90	0.71	0.80	0.67
N.....	23	21	31	25	27	22
l.....	7	19	5	6	7	6
d.....	1.60	1.82	1.68	2.02	1.90	1.88
F.....	1.63	1.72	1.09	1.72	1.78	0.91
χ^2 (f).....	4.9 (3)	20.2 (3)	0.8 (3)	3.2 (3)	2.1 (3)	8.5 (3)
4: $y = a + b \left(\frac{1}{x} - \frac{1}{\bar{x}} \right)$						
a.....	128.8	169.3	759.5	75.24	192.6	39.22
$b \cdot 10^{-4}$	-20.6	-21.3	-90.3	-15.6	-41.8	-3.39
$\frac{1}{\bar{x}} \cdot 10^{+4}$	7.083	6.497	6.400	7.905	4.803	5.840
s_1	0.391	0.242	0.142	0.407	-	0.268
s_2	0.529	0.344	0.180	0.611	-	0.269
s_a	10	8.5	20	7.4	-	1.5
$s_b \cdot 10^{-4}$	3.5	3.0	7.1	2.5	-	0.57
R.....	0.64	0.71	0.87	0.66	0.72	0.64
N.....	23	17	23	21	23	21
l.....	9	19	10	10	7	6
d.....	1.39	1.47	1.20	1.65	1.94	1.73
F.....	1.83	2.02	1.60	2.25	-	1.00
χ^2 (f).....	3.3 (3)	3.2 (3)	8.3 (3)	2.9 (3)	-	7.0 (3)
5: $\log y = \log k + \log \Phi (a + \overline{\log x})$						
a.....	-5.193	-4.657	-4.323	-5.635	-5.903	-3.849
k.....	4530	1965	5381	8802	28920	131.7
s_1	0.170	0.105	0.0618	0.177	0.133	0.117
s_2	0.193	0.122	0.0651	0.220	0.162	0.108
s_a	0.60	0.39	0.24	0.60	0.45	0.49
s_k	5600	1200	1800	13000	34000	68
R.....	0.59	0.71	0.90	0.70	0.82	0.67
N.....	21	27	29	28	28	22
l.....	6	6	5	4	5	5
d.....	1.56	1.94	1.69	2.25	2.15	1.89
F.....	12.9	1.35	1.11	1.54	1.49	0.85
χ^2 (f).....	2.90 (5)	10.64 (5)	3.69 (5)	0.05 (5)	3.16 (5)	4.00 (5)

Rural districts. (3.5.) 53 groups of 3 households.

groups

Washing & cleaning	Durables excl. vehicles	Personal hygiene	Books, newspapers etc.	Sports, holidays, hobbies	Transport incl. own car	Union fees, subscriptions etc.
0.53	0.66	0.82	0.90	0.71	0.37	0.83
23	31	26	28	18	21	25
8	4	7	8	7	9	4
1.17	2.23	1.73	1.99	0.76	1.67	1.81
3.24	1.24	1.00	1.15	3.10	3.49	1.28
3.2 (3)	2.7 (3)	3.6 (3)	4.0 (3)	29.5 (3)	34.4 (3)	3.2 (3)
45.74	72.83	39.47	12.30	28.90	84.90	94.73
-579	-14.0	-5.15	4.84	-16.6	-10.3	-16.1
6.339	7.524	6.558	9.269	9.861	6.176	7.203
0.303	0.507	0.251	0.354	0.441	0.479	0.265
0.646	0.580	0.299	0.400	0.986	0.998	0.366
4.3	6.7	1.7	1.0	6.6	12	5.3
1.6	2.2	0.61	0.37	2.8	4.6	1.8
0.46	0.66	0.76	0.88	0.64	0.29	0.78
21	33	16	27	17	17	23
8	4	19	8	7	12	8
1.00	2.12	1.37	1.85	0.49	1.47	1.44
4.53	1.31	1.43	1.28	4.99	4.34	1.90
10.0 (3)	5.1 (3)	9.3 (3)	2.4 (3)	45.4 (3)	44.3 (3)	13.1 (3)
-4.918	-4.898	-4.562	-5.549	-	-5.072	-4.964
778.2	1467	399.4	2066	-	1638	2244
0.132	0.220	0.109	0.153	-	0.208	0.115
0.193	0.254	0.103	0.180	-	0.309	0.128
0.47	0.79	0.41	0.53	-	0.74	0.41
680	2100	250	2500	-	2400	1700
0.52	0.60	0.83	0.84	-	0.34	0.82
23	30	32	31	-	25	24
8	5	6	4	-	6	4
1.37	23.7	1.83	1.69	-	1.88	1.86
2.14	1.33	0.90	1.39	-	2.21	1.24
4.91 (5)	1.98 (5)	13.66 (5)	6.66 (5)	-	10.29 (5)	3.03 (5)

Appendix B.

The tables show the correlation coefficient¹⁾ between the residuals of the different expenditure items, where the calculated expenditures are calculated from the function $\log y = a + b (\log x - \overline{\log x})$. The tables are shown separately for each of the twelve groups of wage and salary earners cf. the heads of the tables.

The numbers 1–13 in the head and the front column of the tables denote the different expenditure items according to the following code:

1. Dwelling.
2. Full and light.
3. Food.
4. Tobacco.
5. Clothing.
6. Footwear.
7. Washing and cleaning.
8. Durables (excl. own car).
9. Personal hygiene.
10. Books, newspapers etc.
11. Sports, holidays, hobbies.
12. Transport incl. own car.
13. Union fees, subscriptions etc.

¹⁾ Cf. chapter VI, p. 105.

1.1 Higher public servants and salaried employees. The capital.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.309	0.076	0.045	-0.093	-0.043	0.161	0.176	0.067	-0.096	0.023	0.057	-0.100
2.....	-	-	0.017	-0.053	-0.240	-0.081	-0.002	0.099	-0.145	-0.181	-0.385	-0.060	-0.156
3.....	-	-	-	0.288	0.290	0.102	0.365	-0.045	0.145	-0.032	0.176	0.004	0.033
4.....	-	-	-	-	0.163	0.055	0.176	-0.116	0.233	0.160	0.122	0.032	-0.156
5.....	-	-	-	-	-	0.578	0.319	-0.052	0.485	0.141	0.359	0.003	0.100
6.....	-	-	-	-	-	-	0.191	0.060	0.432	0.116	0.127	-0.114	-0.022
7.....	-	-	-	-	-	-	-	-0.011	0.338	0.020	0.258	-0.027	0.077
8.....	-	-	-	-	-	-	-	-	0.017	-0.139	-0.030	-0.043	0.041
9.....	-	-	-	-	-	-	-	-	-	0.381	0.342	-0.028	0.044
10.....	-	-	-	-	-	-	-	-	-	-	0.378	0.053	0.081
11.....	-	-	-	-	-	-	-	-	-	-	-	0.065	0.141
12.....	-	-	-	-	-	-	-	-	-	-	-	-	0.080
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

1.2 Lower public servants and salaried employees. The capital.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.395	0.082	-0.055	-0.046	0.122	0.154	0.039	0.021	0.077	0.074	-0.028	0.021
2.....	-	-	0.076	-0.055	-0.125	-0.075	0.073	-0.016	-0.139	0.014	-0.144	-0.070	0.006
3.....	-	-	-	0.262	0.013	0.004	0.204	0.048	0.053	0.166	0.073	-0.061	0.152
4.....	-	-	-	-	0.013	-0.064	0.128	-0.011	0.042	0.158	0.067	0.102	0.187
5.....	-	-	-	-	-	0.469	0.199	0.099	0.365	0.124	0.302	-0.046	0.021
6.....	-	-	-	-	-	-	0.132	0.148	0.370	0.078	0.172	-0.073	-0.006
7.....	-	-	-	-	-	-	-	0.043	0.235	0.114	0.185	0.095	0.055
8.....	-	-	-	-	-	-	-	-	0.094	0.146	-0.024	-0.054	0.122
9.....	-	-	-	-	-	-	-	-	-	0.222	0.207	-0.041	0.061
10.....	-	-	-	-	-	-	-	-	-	-	0.109	-0.116	0.347
11.....	-	-	-	-	-	-	-	-	-	-	-	0.051	0.004
12.....	-	-	-	-	-	-	-	-	-	-	-	-	0.082
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

2.1 Higher public servants and salaried employees. Provincial towns.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.294	0.129	0.076	-0.052	-0.102	0.119	0.115	0.054	0.114	-0.053	-0.122	0.240
2.....	-	-	0.338	-0.026	-0.192	-0.131	0.050	-0.075	-0.182	0.044	-0.078	-0.086	0.047
3.....	-	-	-	0.016	-0.235	-0.200	0.087	-0.108	-0.071	-0.039	-0.206	-0.093	0.135
4.....	-	-	-	-	0.282	0.088	0.307	-0.095	0.368	0.072	0.288	-0.070	0.243
5.....	-	-	-	-	-	0.469	0.252	0.130	0.507	0.121	0.237	-0.062	0.087
6.....	-	-	-	-	-	-	0.060	0.001	0.423	0.078	0.268	-0.115	-0.050
7.....	-	-	-	-	-	-	-	-0.048	0.366	0.214	0.032	-0.093	0.371
8.....	-	-	-	-	-	-	-	-	-0.024	-0.096	-0.130	-0.122	-0.072
9.....	-	-	-	-	-	-	-	-	-	0.210	0.323	-0.114	0.311
10.....	-	-	-	-	-	-	-	-	-	-	0.282	-0.162	0.066
11.....	-	-	-	-	-	-	-	-	-	-	-	-0.182	0.066
12.....	-	-	-	-	-	-	-	-	-	-	-	-	-0.049
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

2.2 Lower public servants and salaried employees. Provincial towns.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.250	0.001	0.023	-0.066	-0.109	0.124	-0.010	-0.101	-0.130	-0.160	-0.054	-0.007
2.....	-	-	0.127	-0.040	-0.252	-0.223	0.029	-0.075	-0.252	-0.045	-0.165	-0.167	0.113
3.....	-	-	-	0.009	-0.045	0.073	0.028	-0.132	-0.090	-0.076	-0.117	-0.112	0.159
4.....	-	-	-	-	0.018	-0.053	0.204	-0.094	-0.026	0.084	0.191	-0.055	0.129
5.....	-	-	-	-	-	0.510	0.080	0.020	0.390	0.065	0.111	-0.075	0.026
6.....	-	-	-	-	-	-	0.024	-0.008	0.397	0.167	0.138	-0.042	0.028
7.....	-	-	-	-	-	-	-	-0.044	0.098	0.157	0.104	0.014	0.019
8.....	-	-	-	-	-	-	-	-	0.052	0.002	-0.116	-0.042	-0.107
9.....	-	-	-	-	-	-	-	-	-	0.162	0.172	-0.029	-0.050
10.....	-	-	-	-	-	-	-	-	-	-	0.182	-0.032	0.078
11.....	-	-	-	-	-	-	-	-	-	-	-	-0.148	0.058
12.....	-	-	-	-	-	-	-	-	-	-	-	-	-0.139
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

3.2 Lower public servants and salaried employees. Rural districts.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.183	0.036	-0.005	-0.112	-0.079	-0.044	-0.199	-0.037	-0.112	-0.114	-0.061	-0.011
2.....	-	-	-0.024	-0.099	-0.222	-0.227	-0.123	-0.087	-0.248	0.011	-0.263	-0.186	-0.024
3.....	-	-	-	0.046	0.001	-0.042	-0.016	-0.014	-0.047	0.085	0.124	0.037	-0.172
4.....	-	-	-	-	0.157	0.028	0.195	0.010	0.192	0.017	0.315	-0.066	0.060
5.....	-	-	-	-	-	0.428	0.152	0.251	0.359	0.042	0.090	-0.068	0.069
6.....	-	-	-	-	-	-	0.220	0.092	0.360	0.041	0.032	-0.032	-0.056
7.....	-	-	-	-	-	-	-	-0.044	0.291	0.171	-0.152	0.038	0.110
8.....	-	-	-	-	-	-	-	-	0.097	-0.076	-0.004	-0.149	-0.011
9.....	-	-	-	-	-	-	-	-	-	0.085	0.178	-0.071	-0.111
10.....	-	-	-	-	-	-	-	-	-	-	-0.081	0.130	0.052
11.....	-	-	-	-	-	-	-	-	-	-	-	-0.022	-0.180
12.....	-	-	-	-	-	-	-	-	-	-	-	-	-0.021
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

3.3 Skilled workers. Rural districts.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1.....	-	0.341	0.005	0.021	-0.097	-0.205	0.109	-0.102	-0.004	0.062	-0.149	-0.167	0.035
2.....	-	-	0.093	0.254	0.036	-0.144	0.087	-0.197	-0.154	0.251	-0.224	0.002	0.014
3.....	-	-	-	0.231	0.145	-0.118	0.196	-0.327	-0.035	0.009	-0.096	-0.301	0.314
4.....	-	-	-	-	0.092	0.040	0.251	-0.234	-0.015	0.230	-0.010	-0.207	0.165
5.....	-	-	-	-	-	0.496	0.429	-0.292	0.082	0.057	0.084	-0.124	-0.025
6.....	-	-	-	-	-	-	0.052	-0.098	0.255	0.104	0.216	0.068	-0.025
7.....	-	-	-	-	-	-	-	-0.274	0.384	0.058	0.198	-0.305	0.027
8.....	-	-	-	-	-	-	-	-	-0.229	-0.188	-0.054	-0.265	-0.270
9.....	-	-	-	-	-	-	-	-	-	0.276	0.199	0.024	0.103
10.....	-	-	-	-	-	-	-	-	-	-	0.087	-0.017	0.076
11.....	-	-	-	-	-	-	-	-	-	-	-	0.136	0.041
12.....	-	-	-	-	-	-	-	-	-	-	-	-	-0.095
13.....	-	-	-	-	-	-	-	-	-	-	-	-	-

Appendix C. The basic material.

Disposable income, expenditures on 13 items, savings, assets and some other information separately for each of 3098 households of wage and salary earners in the year 1955.

All amounts are in Danish kroner, (respectively 100 Danish kroner). The amounts given in columns 3–16 are in kroner *per person*, all other informations are per household. The columns 1–22 contains the following information separately for each household.

Column no.	Information
1	Household number within the socialgroup in question.
2	Size of household measured in number of persons.
3	Disposable income per person of the household (i.e. total money income less paid personal taxes).
4	Expenditure per person on dwelling.
5	Expenditure per person on fuel and light.
6	Expenditure per person on food.
7	Expenditure per person on tobacco.
8	Expenditure per person on clothing.
9	Expenditure per person on footwear.
10	Expenditure per person on washing and cleaning.
11	Expenditure per person on durables (excl. own car).
12	Expenditure per person on personal hygiene.
13	Expenditure per person on books, newspapers, etc.
14	Expenditure per person on sports, holidays, hobbies.
15	Expenditure per person on transport (incl. own car).
16	Expenditure per person on union fees, subscriptions, etc.
17	Savings (net changes in assests and debts).
18	Income changes in the period 1953–55 according to the following code: <ol style="list-style-type: none"> 1. Rising through the whole period 1953 ↗ 1954 ↗ 1955. 2. Constant - - - - 1953 = 1954 = 1955. 3. Decreasing - - - - 1953 ↘ 1954 ↘ 1955. 4. Unknown 1953–1954, rising 1954–1955. 5. Unknown 1953–1954, constant 1954–1955. 6. Unknown 1953–1954, decreasing 1954–1955. 9. No information.
19	Type of household according to the following code: <ol style="list-style-type: none"> 1. Single men. 2. Single women.

3. Couples without children.
 4. Couples with one child.
 5. Couples with two children.
 6. Couples with three children.
 7. Couples with four or more children.
 8. Single men with one child or more.
 9. Single women with one child or more.
 0. Other types of household.
- 20 Net assets in 100 kr.
 - 21 Wage- and salary income in per cent of total income.
 - 22 Year of establishment of the household (60 denotes no information).

1.1 Higher public servants and salaried employees. The capital.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
301	1	11643	880	552	2400	150	1275	264	198	140	202	174	585	549	113	2430	4	2	81	99	38
302	2	11678	935	236	2400	0	1818	282	244	890	218	115	1195	115	100	2300	1	3	12	99	31
303	4	11713	533	310	1430	275	657	113	179	1296	139	25	150	179	99	21410	1	5	320	64	46
304	2	11884	1147	350	1800	45	619	251	233	189	277	142	386	463	134	7040	9	3	114	97	54
305	1	11948	1208	390	2347	106	996	187	98	1730	364	93	1118	174	122	130	1	2	19	99	19
306	2	12053	1574	305	1225	235	822	181	255	719	362	219	2050	2305	184	600	4	3	57	95	31
307	1	12108	1260	255	2000	729	1463	190	84	82	288	613	1844	2165	199	2260	1	1	-55	100	60
308	2	12136	1842	772	3120	328	413	221	246	468	212	210	1652	478	120	-2900	9	3	1175	51	49
309	1	12331	780	172	2550	700	1828	286	147	118	439	105	1355	390	91	1100	1	2	208	62	50
310	2	12343	857	384	1088	990	893	270	240	2605	329	82	700	232	58	440	1	3	5	97	54
311	1	12384	900	163	2567	100	1102	320	120	867	227	176	1501	1898	238	1510	9	1	10	100	55
312	1	12441	1653	441	1700	610	1126	186	182	525	538	164	3468	176	64	80	1	2	569	87	60
313	3	12844	1531	496	1467	42	1135	114	151	90	197	56	906	1237	127	18050	1	4	-257	99	49
314	2	12865	1541	495	2311	15	411	28	276	1518	244	82	973	305	163	3140	4	3	243	96	51
315	1	12898	1000	200	2405	50	1215	126	336	410	302	539	2912	1000	262	1220	1	1	40	99	47
316	2	13390	1150	688	4060	900	1350	150	85	34	175	190	907	2093	137	280	4	3	465	99	21
317	1	13698	2110	750	3580	865	1732	146	453	956	234	274	1799	647	252	-1460	1	1	85	97	43
318	2	13835	1128	396	1761	233	633	72	282	158	175	232	1022	3991	239	5530	1	3	330	94	52
319	1	13852	1384	388	3675	1200	954	78	598	20	170	160	2750	260	98	-170	1	1	69	97	60
320	3	13986	2077	515	1905	455	1011	113	134	1185	211	129	1122	2278	67	8090	1	4	318	91	44
321	2	14103	1715	750	2150	503	400	32	395	661	149	97	2653	1966	80	5120	1	3	-148	95	54
322	2	14331	1306	504	1800	710	1541	322	313	497	491	173	903	175	268	4490	4	3	233	90	48
323	1	14773	622	578	1685	50	2255	285	284	200	453	110	2110	656	527	2620	1	2	161	99	40
324	1	15053	1130	868	2777	0	1753	340	362	135	228	154	2145	253	393	1020	1	2	76	99	35
325	1	15500	859	358	1950	30	1335	70	280	145	255	400	1555	875	257	600	9	1	72	95	40
326	1	15846	1665	249	3120	1200	1897	157	250	120	524	265	1885	482	88	-2210	1	2	833	73	45
327	1	15992	1794	560	2036	50	1252	166	59	118	212	197	2837	157	212	4400	1	2	343	91	43
328	1	16013	600	730	3193	910	1195	167	330	240	273	347	2555	574	116	2640	4	1	24	100	39
329	1	16275	2723	1238	2108	11	774	141	135	6463	177	53	1930	1239	267	-2820	6	1	-66	100	38
330	1	16396	1800	0	3048	0	1016	0	392	1348	195	50	2816	1050	694	2940	1	1	17	99	54
331	1	16891	2193	835	1850	350	773	252	274	215	433	158	1877	250	303	5520	1	2	174	93	40
332	1	17613	3073	712	2640	1513	664	36	316	23	255	260	3252	115	164	3840	1	1	-45	94	60
333	1	19226	1650	230	2640	840	1706	388	190	1274	216	350	3019	80	118	2770	4	1	-19	99	36
334	1	20015	450	123	1232	0	398	38	75	231	77	784	4404	1560	335	8030	9	1	47	97	55
335	1	29048	1871	653	1680	1200	2090	354	268	39	1145	1906	6517	1302	324	1390	6	2	269	97	44
336	1	47617	3463	2310	1370	562	640	91	80	150	44	105	885	300	211	34490	9	1	1565	54	60

1.2 Lower public servants and salaried employees. The capital.

Table with 22 columns and multiple rows of numerical data. Headers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22. Rows include data for various categories and individuals, such as 301, 302, 303, etc., with associated numerical values.

1.2 Lower public servants and salaried employees. The capital.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
421	1	9593	1080	3	2480	0	295	117	108	22	107	0	185	180	134	4490	1	1	212	93	50
422	1	9603	1065	418	1709	313	1408	198	151	1088	317	120	727	271	125	370	1	2	2	98	55
423	1	9608	1280	140	3120	600	635	127	81	1989	201	109	480	50	169	240	4	2	0	100	54
424	1	9724	1220	201	1440	10	975	184	82	556	153	35	600	275	125	2410	1	2	259	88	26
425	1	9729	1374	792	1800	1005	1154	274	308	418	208	140	697	340	151	40	4	2	0	86	48
426	1	9776	675	25	2254	350	540	163	234	200	122	372	1667	94	175	2560	1	1	60	81	47
427	1	9789	1431	469	2224	365	798	160	88	1620	220	373	305	300	169	270	1	2	-30	99	21
428	2	9838	1015	305	1500	481	1148	140	198	532	250	209	1244	393	192	1710	9	3	67	99	49
429	1	9838	1018	433	1440	182	1986	192	222	12	288	448	1780	477	237	-230	0	2	103	99	47
430	1	9844	960	547	2400	360	1227	97	174	1205	196	180	1189	264	113	-120	1	2	5	98	40
431	3	9847	492	232	1405	429	1101	191	44	430	326	217	1301	503	127	5550	1	9	107	83	28
432	1	9870	2723	152	2597	700	863	321	316	0	105	105	1492	278	212	210	1	1	58	99	41
433	2	9928	1573	390	1445	47	1139	195	121	83	375	81	643	133	139	4900	1	3	145	96	49
434	1	9964	1266	444	1360	200	1907	253	306	254	643	124	432	220	156	1000	4	2	31	88	47
435	1	9991	18	41	1520	911	1008	233	156	1665	558	252	1535	592	175	170	1	2	12	93	60
436	1	10085	720	294	2265	581	856	142	387	12	328	190	792	438	364	170	1	1	5	99	54
437	1	10100	1713	446	2545	675	1148	250	345	6141	226	190	710	305	300	-5620	4	1	76	99	50
438	1	10136	1500	185	2012	222	589	199	309	130	190	276	1609	2330	276	-20	1	1	-18	98	50
439	2	10157	1102	442	1949	420	712	44	67	100	118	87	588	289	230	5010	1	3	509	79	17
440	1	10165	1300	0	2200	750	1001	110	200	0	437	330	1825	625	150	-500	1	2	27	100	42
441	1	10168	820	153	2328	400	1269	204	283	0	292	70	1500	740	246	1370	1	1	22	99	48
442	1	10198	965	496	1500	109	841	145	85	1600	178	211	904	293	197	2170	4	2	10	97	55
443	1	10253	1499	6	1595	190	839	343	54	77	339	145	1497	509	187	1930	9	2	7	99	47
444	1	10396	1020	169	2580	761	1194	202	360	0	299	155	1397	505	402	1200	6	1	2	98	60
445	1	10502	870	406	2300	716	829	83	145	553	252	230	1025	334	121	240	1	2	9	100	43
446	1	10536	1135	390	1856	143	511	71	114	500	267	129	2685	297	192	1070	2	2	49	99	35
447	1	10545	1120	316	1920	480	1162	188	196	1235	369	460	1027	395	221	660	1	2	7	99	42
448	1	10740	3192	2229	2400	0	48	138	390	215	53	104	285	504	284	500	1	2	1	100	50
449	1	10771	1008	282	1925	25	2061	141	106	190	250	162	305	310	247	-170	9	2	79	97	60
450	1	11061	771	466	1818	136	1228	94	94	435	412	146	1322	220	150	2230	1	2	116	97	40
451	1	11231	996	357	1650	21	1945	230	110	475	409	313	1669	260	231	110	1	2	171	95	43
452	1	11311	1152	549	2000	0	1479	165	198	765	313	158	1796	160	101	180	1	2	16	99	34
453	1	11463	555	248	750	32	117	215	175	226	253	300	562	225	92	520	4	2	-23	98	42
454	2	11651	1130	475	2145	820	797	140	237	115	197	165	746	238	123	470	1	3	315	74	42
455	1	11676	1154	466	2400	75	1456	224	243	1266	509	158	585	283	136	1740	1	2	30	99	42
456	1	11976	821	414	1474	50	732	106	255	104	699	112	889	200	219	3780	1	2	5	84	30
457	1	12131	1650	330	2506	900	95	390	361	938	241	180	2681	320	270	200	9	2	3	97	60
458	1	12290	1265	36	2460	10	1533	269	289	108	221	194	1231	284	102	-260	1	2	14	99	46
459	1	12354	1398	535	2400	1112	854	85	248	382	135	347	1160	465	392	720	1	1	-15	95	53
460	1	12660	900	345	1200	1320	2166	256	189	289	329	178	1714	701	520	160	4	1	0	88	28
461	1	13075	1702	148	2300	364	558	87	180	2369	400	840	1340	315	1039	-1930	1	2	-4	75	34
462	2	13508	837	402	1831	500	1187	145	135	305	331	82	2721	805	208	5350	9	3	634	72	40
463	1	13583	1200	534	3890	895	1403	251	500	263	185	138	420	390	328	1530	1	1	22	75	54
464	2	13610	1792	443	1995	762	1662	244	255	805	297	160	655	2863	392	710	6	3	21	99	50
465	1	13953	1543	548	2538	0	2222	162	147	0	253	163	3576	212	140	470	1	2	26	57	42
466	1	14025	770	453	2445	760	1797	203	194	147	660	164	3064	77	177	700	1	2	8	85	30
467	1	15420	1800	1185	2200	50	1521	247	347	1274	512	180	2246	1480	61	120	4	2	1250	50	25
468	2	20192	876	447	1820	357	3693	206	213	92	217	75	2307	35	30	12970	4	3	9	33	40
469	1	36222	756	314	1076	45	847	103	16	543	366	80	463	97	154	29340	1	2	300	27	54

1.3 Skilled workers. The capital.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
181	1	8990	540	147	1685	1062	1208	167	158	376	312	55	1327	880	336	530	1	1	1	100	52
182	2	9047	460	572	2378	1001	440	140	130	332	281	112	592	1190	318	410	6	3	1	94	43
183	1	9064	948	562	2118	1100	190	107	360	20	73	75	755	220	318	1140	1	1	43	95	18
184	2	9120	353	192	3021	973	341	101	138	917	346	170	1084	321	287	430	1	3	-11	99	53
185	2	9280	512	187	1560	210	1330	199	135	565	290	97	1292	3510	294	-2430	1	3	325	93	41
186	2	9350	466	244	1993	659	834	249	66	811	210	108	833	230	379	1990	1	3	72	99	31
187	2	9353	2017	620	1560	457	515	55	47	668	160	87	503	42	593	5970	4	3	182	92	47
188	1	9354	948	560	2466	624	797	122	97	74	161	236	863	370	399	160	3	1	7	99	27
189	2	9361	766	233	1865	372	1288	270	140	672	402	169	817	180	303	510	1	3	40	99	51
190	2	9425	423	228	2780	310	432	127	81	31	120	147	506	272	392	5560	1	3	110	99	38
191	2	9588	522	161	2217	534	642	60	208	114	237	125	976	229	600	4380	9	3	234	97	32
192	2	9874	612	232	2542	786	768	224	121	110	210	78	1008	173	427	520	1	3	33	99	44
193	2	9937	778	230	1998	181	697	77	101	33	294	106	349	3519	336	600	1	3	-12	99	51
194	1	10076	780	120	3032	1266	272	43	230	895	200	146	906	212	377	980	1	1	78	99	46
195	2	10448	984	458	1668	279	675	66	131	1177	125	256	357	3722	273	240	1	3	-43	99	48
196	1	10825	1020	63	2791	1250	1078	152	252	860	157	288	2028	388	466	-1770	4	1	-7	90	45
197	2	11123	710	85	1905	654	757	144	321	1643	155	23	1140	155	337	230	1	3	9	100	55
198	1	11220	375	17	850	480	501	86	190	390	87	72	985	2585	336	1350	1	2	-17	100	54
199	1	11260	800	0	2600	1448	668	85	609	1001	273	0	1039	1330	514	-850	1	1	9	100	54
200	1	11273	660	461	4200	566	367	86	319	12	124	126	1214	210	514	80	1	1	2	99	34
201	1	12262	830	162	2394	321	987	291	142	954	186	120	1420	1112	569	1130	1	1	2	97	60
202	1	12928	1287	348	2695	340	499	115	158	190	62	104	553	43	599	6320	1	1	26	82	35
203	1	13116	906	493	3570	637	751	152	585	579	305	428	3116	178	423	360	4	1	4	99	52
204	1	14178	1512	566	3168	1450	902	140	390	1097	125	216	2624	508	676	-60	4	1	210	92	20
205	1	14220	1320	15	2290	1000	457	115	470	85	154	120	1686	4871	788	-1300	3	1	-65	100	52
206	1	15952	810	53	2340	1235	1508	207	298	1241	379	132	3698	2180	149	1280	1	1	14	100	38

1.4 Unskilled workers. The capital.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
241	2	9431	371	325	1500	409	891	212	93	912	267	150	1332	171	384	2490	1	3	121	98	50
242	1	9435	1080	405	2500	1200	255	45	290	50	100	410	1700	135	382	250	1	1	3	96	38
243	1	9720	960	42	2653	829	1036	171	195	37	268	200	1801	493	497	160	9	1	4	90	48
244	1	9869	1116	608	656	625	1015	156	139	569	84	590	476	110	448	140	1	1	7	98	19
245	1	10407	830	338	3100	941	1058	189	269	680	173	185	1195	1662	590	-660	1	1	16	98	35
246	1	10856	900	84	3106	416	1259	122	362	248	253	207	455	138	481	1730	4	1	-14	100	12
247	4	11400	190	175	1625	259	174	110	31	284	110	73	86	5033	232	10440	1	5	126	27	33
248	1	11703	840	56	3186	1506	749	140	390	130	299	440	1342	170	586	260	1	1	4	94	45
249	1	12061	1456	0	3445	669	1967	217	350	65	340	1058	1270	100	544	380	1	1	1	78	48
250	1	12683	360	55	3120	1874	832	119	300	36	205	107	775	1235	24	40	9	1	0	100	53
251	1	16489	780	63	1884	80	528	110	155	136	140	102	517	240	136	0	9	2	82	94	48

2.1 Higher public servants and salaried employees. Provincial towns.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
181	2	10260	445	268	3253	292	529	63	161	227	278	47	177	1332	161	2970	4	3	501	94	21
182	1	10385	1116	10	1122	85	659	237	60	882	233	292	2030	120	160	2480	1	2	5	99	49
183	2	10438	738	416	1754	858	1655	287	118	704	321	201	1050	279	251	1810	1	3	17	99	48
184	2	10507	721	360	1762	450	599	139	240	334	275	108	1972	322	369	3720	1	3	15	96	49
185	1	10512	765	803	2340	0	1159	82	193	2335	201	72	228	185	193	-210	1	2	38	99	53
186	2	10581	477	404	1968	35	673	133	87	0	167	122	460	7521	140	-6630	1	3	122	97	48
187	1	10633	1406	590	1820	600	760	327	200	110	415	340	1748	400	160	910	1	2	-6	99	49
188	2	10823	864	410	1555	153	573	100	97	1387	123	253	1376	238	144	5910	1	3	-33	98	54
189	2	10979	897	400	2125	632	1203	173	252	307	251	141	1348	5747	205	-6920	1	3	89	99	51
190	2	11026	918	634	2840	900	971	178	89	329	207	240	1266	16	78	-260	1	0	504	87	20
191	2	11229	1855	689	2230	604	626	80	260	769	353	70	827	242	475	4710	9	3	97	95	45
192	1	11234	1058	38	2391	700	284	58	280	300	149	205	1515	350	258	2750	1	1	76	96	44
193	3	11293	513	74	1100	100	841	78	57	2057	124	151	749	2382	173	8080	1	4	-51	99	51
194	1	11354	1550	781	1718	282	1199	132	164	159	143	357	412	151	290	390	1	2	56	97	26
195	2	11435	779	210	1042	450	882	105	50	645	110	57	302	9129	201	-7320	1	3	141	95	55
196	1	11576	1300	580	1200	604	1015	180	107	1453	255	276	3050	550	195	-280	1	2	87	98	34
197	2	11592	967	704	2380	329	809	102	178	1256	180	161	428	171	233	0	1	3	261	97	26
198	3	11639	547	798	1335	600	713	111	70	306	141	53	981	530	111	13210	1	4	363	78	28
199	2	11812	1225	745	2212	167	1159	101	298	1447	152	197	1442	1098	372	1960	1	3	172	98	25
200	1	12051	480	60	1920	300	1145	199	81	63	300	112	1680	525	196	3960	1	1	171	99	42
201	2	12305	851	745	1437	315	962	143	130	37	251	124	1255	6875	249	-3670	1	3	-82	99	42
202	2	12451	937	812	3300	250	804	154	89	2190	187	192	817	1522	227	160	1	3	293	78	23
203	2	12969	750	516	2012	350	1182	167	254	411	125	105	789	1418	235	7940	1	3	539	92	26
204	1	13057	1304	966	2160	1341	1918	178	413	153	366	254	2925	590	362	-2730	1	1	181	94	40
205	1	13276	484	730	1200	720	1245	95	190	665	127	280	1785	80	148	4400	4	1	-20	93	60
206	1	13394	420	260	1685	7	778	85	175	65	82	295	995	2406	212	3880	1	1	121	95	39
207	2	14522	456	369	1800	55	675	105	148	177	202	62	890	129	221	8820	4	3	106	96	53
208	1	14640	1410	66	1568	1750	2299	218	718	664	690	300	1555	1560	530	260	4	2	0	99	39
209	1	14736	550	601	1200	116	1885	292	415	1851	489	270	2199	356	166	3390	1	2	-17	82	34
210	1	14922	1326	629	1800	80	1523	113	400	622	210	820	1290	390	254	4540	1	1	-32	99	52
211	2	17251	1350	645	1637	0	830	333	125	1072	39	65	1504	1555	161	12120	1	3	853	77	54

2.2 Lower public servants and salaried employees. Provincial towns.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
301	2	8614	609	424	1938	567	919	89	123	319	128	98	582	807	169	4400	4	3	732	87	41
302	1	8616	600	400	1540	600	629	58	78	0	130	0	2722	100	72	890	1	1	17	100	48
303	1	8639	560	220	1863	256	755	206	100	0	100	0	1405	408	239	1620	1	1	2	99	54
304	2	8863	893	391	1200	655	295	55	58	1489	238	57	600	55	159	4340	1	3	128	93	54
305	1	8911	0	2	2040	0	257	70	10	0	164	127	1070	217	179	2110	1	2	367	84	34
306	1	9020	780	410	2458	70	584	204	325	115	126	202	1499	50	561	90	1	1	9	99	54
307	2	9056	768	539	1440	167	324	61	64	2277	136	191	254	59	158	2960	4	3	367	95	27
308	1	9062	714	330	1650	650	1419	140	119	218	314	177	2102	240	274	40	1	2	6	100	52
309	1	9105	0	36	600	130	812	240	6	249	321	403	630	273	347	1540	1	2	6	99	60
310	2	9110	750	368	2270	457	540	143	116	645	355	110	282	2576	143	-130	9	3	8	99	52
311	1	9121	948	427	1715	288	649	151	96	2187	247	158	342	270	85	300	1	2	2	99	26
312	2	9145	740	368	1950	300	1152	118	68	1316	221	90	583	159	155	1710	1	3	132	99	33
313	1	9152	0	325	2640	78	1153	198	150	329	271	305	430	137	103	660	1	2	0	99	47
314	1	9160	600	0	1795	230	1174	173	102	480	92	120	580	300	225	3080	9	1	11	99	60
315	1	9174	814	389	1380	150	487	109	49	606	50	120	746	403	294	1400	1	2	104	95	40
316	1	9205	937	585	1437	0	820	162	210	26	114	80	198	30	173	1570	1	2	153	88	52
317	2	9264	607	436	1800	723	1017	169	146	909	422	90	1220	177	258	1440	1	3	0	99	47
318	1	9305	780	97	2100	700	702	130	146	0	190	109	483	334	250	1120	1	2	109	95	54
319	1	9318	860	1001	2087	214	707	106	103	406	81	151	836	218	269	560	1	2	12	98	30
320	1	9463	960	20	2420	1200	1154	222	559	164	264	405	660	289	257	470	1	1	22	99	52
321	1	9555	420	0	1764	0	1179	390	32	800	465	48	1914	45	224	1680	6	2	90	89	35
322	1	9558	720	646	1715	295	1049	201	205	158	228	50	2085	120	156	1770	1	1	220	82	30
323	1	9586	720	2	1530	838	611	141	290	500	252	378	666	3523	130	-1720	1	1	19	99	60
324	2	9618	500	390	1831	500	550	101	169	25	123	79	1157	131	140	5710	1	3	-64	99	45
325	1	9636	480	325	1350	580	385	66	300	65	345	250	1720	300	171	2130	1	1	3	98	50
326	1	9687	360	134	1040	0	435	59	244	1000	92	384	2030	1807	111	1810	2	1	152	99	36
327	1	9688	900	23	1820	1090	1497	119	424	702	137	298	1529	496	427	190	1	1	10	99	54
328	1	10012	1114	622	2760	445	754	184	271	587	160	280	1333	238	296	300	1	1	1	77	32
329	1	10161	1320	870	1820	186	2097	252	126	462	322	157	497	450	434	-1450	1	2	6	93	33
330	1	10212	842	606	960	642	943	126	293	1148	96	12	884	1883	146	1360	1	2	276	99	48
331	1	10541	1332	632	2814	277	1625	436	226	44	310	224	711	682	181	330	1	2	433	81	29
332	1	10908	867	393	2132	65	1303	129	273	1424	433	210	1235	245	135	1090	9	2	124	96	50
333	1	11043	684	233	1464	284	745	330	222	369	295	130	948	3329	157	3020	5	2	-118	97	45
334	1	11130	625	533	2160	80	1689	226	177	807	161	435	1499	173	285	680	1	2	11	99	48
335	1	11659	600	32	1700	0	932	122	0	0	117	36	169	1998	85	9950	1	1	94	68	54
336	1	12335	925	460	2465	554	1095	90	239	589	128	8	583	875	261	1370	1	1	18	83	46
337	1	12651	972	896	1130	280	833	161	209	599	306	452	1906	0	255	-4830	1	2	44	81	25
338	1	13253	852	1142	1804	1277	902	57	178	255	181	210	2389	114	373	810	1	1	50	98	31
339	1	14162	1410	888	3324	131	624	230	219	598	111	188	1460	264	178	1300	1	2	1551	64	53
340	1	20412	2182	646	2575	1370	3376	286	363	633	250	400	3188	355	214	2810	4	1	44	98	31

2.3 Skilled workers. Provincial towns.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
121	2	5222	230	196	1020	62	307	64	32	320	44	30	115	58	156	4710	4	3	415	86	21
122	3	5309	524	345	1213	222	727	92	84	350	109	78	327	371	165	760	1	4	-156	99	49
123	1	5357	300	175	2000	433	540	147	83	0	26	7	187	197	353	670	2	1	67	65	39
124	2	5372	459	553	2052	639	216	70	90	17	319	86	354	52	256	200	1	3	7	94	30
125	4	5390	704	497	908	92	777	0	117	440	145	43	520	383	218	1700	1	5	-147	87	37
126	2	5410	666	447	1040	139	573	122	37	583	135	57	288	67	520	1360	4	3	281	82	27
127	3	5428	335	241	1599	202	639	98	122	444	155	48	414	118	38	1340	9	4	189	97	21
128	2	5429	270	291	1275	296	1067	176	96	263	207	100	361	122	249	450	1	3	42	96	53
129	2	5544	781	374	1560	153	447	52	67	301	101	51	269	53	248	560	1	3	84	98	26
130	2	5714	335	366	1890	76	536	102	107	273	125	104	668	107	366	740	1	3	108	95	60
131	2	5720	350	362	1620	260	335	110	77	492	347	437	279	48	336	320	1	3	-9	99	52
132	5	5738	345	193	1040	420	175	65	37	456	71	23	83	332	48	9040	4	0	19	68	48
133	3	5819	674	308	827	374	343	64	217	1296	173	92	877	37	170	1550	1	0	-52	99	55
134	1	6088	420	55	1820	750	1005	68	200	0	85	0	586	625	273	80	1	1	0	99	60
135	2	6226	300	239	1500	476	589	115	201	568	81	145	1005	327	389	370	9	3	0	77	52
136	2	6520	300	251	1920	616	871	255	96	192	138	81	237	120	227	710	1	3	-14	99	23
137	1	6614	540	0	1820	0	431	83	240	23	293	35	587	2146	408	40	1	1	-19	91	52
138	2	6861	343	474	1200	530	658	145	147	3061	228	86	673	77	349	-3480	1	3	117	97	47
139	2	7001	317	123	1300	182	550	48	12	376	109	0	175	183	280	7770	4	3	10	90	55
140	2	7047	890	440	1147	318	547	130	120	422	149	96	424	61	321	1560	1	3	-92	95	52
141	2	7121	418	412	1714	408	780	147	101	1861	125	30	223	150	238	-570	4	3	10	97	52
142	2	7211	371	267	1326	399	618	111	65	1048	192	113	402	95	321	2720	1	3	23	99	15
143	1	7238	531	501	2940	521	509	83	114	36	72	362	388	40	606	50	1	1	57	97	51
144	2	7451	254	434	1820	6	253	84	71	66	98	85	199	66	258	2940	7	3	279	66	34
145	1	7565	880	424	2496	631	745	127	208	160	162	110	850	0	425	260	1	1	9	99	53
146	2	7778	529	268	1573	473	716	115	63	4012	186	72	245	439	141	-2160	9	3	23	96	55
147	1	8066	600	26	2247	877	778	158	253	0	157	87	1610	568	573	280	1	1	-11	97	60
148	1	8352	900	142	2499	0	1577	255	165	857	142	117	371	245	14	100	1	2	-1	89	34
149	2	8413	1533	468	1530	6	66	53	109	19	103	64	798	2389	344	1340	1	3	-190	99	54
150	2	8506	558	309	1294	770	1259	173	249	462	260	114	1327	160	457	1860	1	3	30	99	50
151	1	9516	600	131	2340	520	404	87	436	0	111	78	1470	330	476	2790	4	1	364	82	21
152	1	10035	540	40	1425	871	755	126	230	350	84	75	1737	262	646	1910	1	1	2	99	45
153	1	11650	480	0	1820	1805	923	238	260	1220	197	149	1879	175	424	1350	1	1	0	98	51
154	1	12821	300	300	1665	1017	154	38	122	213	186	100	1045	3683	426	1710	4	1	32	30	43

2.4 Unskilled workers. Provincial towns.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
181	3	5345	594	302	850	193	341	89	22	366	77	22	600	673	204	1220	1	4	-87	95	50
182	2	5345	538	372	1430	521	474	103	94	778	97	70	313	197	292	380	1	3	-17	91	23
183	1	5390	786	596	2150	200	496	86	162	13	175	138	27	455	398	130	1	2	2	99	50
184	1	5400	480	207	1215	650	518	99	0	725	80	81	158	36	299	90	6	2	-1	99	48
185	2	5525	277	323	895	459	607	116	101	130	195	48	352	739	270	1720	1	3	96	95	33
186	2	5528	408	291	1300	50	677	70	175	254	112	89	373	694	291	500	1	3	31	92	13
187	2	5576	411	293	2470	78	418	123	126	157	127	43	24	91	284	360	1	3	68	96	14
188	3	5645	398	307	1122	411	508	185	98	160	112	159	529	474	289	2230	9	4	9	92	21
189	2	5665	286	332	1022	565	577	79	93	634	139	83	331	949	446	-500	4	3	27	90	31
190	2	5840	385	557	1275	161	569	186	105	92	179	100	1212	38	384	-60	1	3	214	97	22
191	2	5862	754	744	1820	260	820	76	110	101	65	86	65	60	296	340	9	3	200	86	53
192	1	6173	1030	502	1500	530	406	129	173	0	207	110	630	117	118	220	1	2	228	74	20
193	1	6206	420	119	1880	368	585	91	100	251	158	172	272	193	509	260	1	2	-2	99	54
194	2	6247	480	174	1000	228	1422	251	277	506	247	97	378	447	270	200	1	3	4	99	52
195	2	6279	391	510	1857	265	408	90	108	407	94	71	127	526	512	640	1	3	-7	98	48
196	2	6481	241	395	2272	182	663	57	72	354	172	123	246	256	416	360	2	9	0	99	32
197	2	6783	189	361	2550	669	633	102	231	167	120	59	441	100	427	880	4	0	12	95	34
198	1	6951	160	140	1280	640	76	72	22	264	227	48	785	2955	240	-120	4	1	3	98	52
199	2	6981	175	0	937	150	1019	202	20	2487	130	30	363	92	289	-410	1	3	102	97	55
200	2	7029	313	431	1730	268	878	116	68	713	124	77	327	251	315	2080	1	3	5	97	34
201	2	7236	429	350	1633	262	396	119	153	159	164	47	773	930	499	2590	1	3	88	97	18
202	1	7411	600	0	1715	612	45	80	0	0	202	39	285	1748	422	1400	1	1	-37	100	40
203	2	7470	834	375	1980	650	471	52	175	55	246	150	990	789	275	440	1	3	14	96	49
204	1	7651	1399	179	2028	4	53	68	287	475	198	103	348	573	652	1090	1	1	152	95	27
205	2	7837	509	528	1950	1056	730	138	165	145	168	211	430	78	318	990	1	3	9	99	37
206	1	7963	480	218	1680	767	295	141	175	510	139	144	1138	495	852	-350	6	1	17	100	54
207	1	8100	583	271	425	531	2510	336	648	287	542	129	668	554	367	210	4	2	22	83	54
208	4	8194	924	262	1102	332	537	152	72	187	358	75	507	142	140	11410	9	5	-450	57	22
209	2	8248	240	370	2240	810	310	80	72	693	166	200	541	691	506	1240	1	3	2	96	28
210	1	8509	1147	372	1820	200	448	30	84	511	50	37	200	10	512	3060	6	1	167	67	26
211	1	9657	1113	502	1934	776	972	50	107	624	83	60	115	1308	514	290	1	1	89	99	24
212	2	10212	160	313	2100	430	680	136	65	222	181	273	1303	992	467	1930	9	8	36	95	28
213	1	10889	0	15	1763	502	983	133	0	300	165	133	2929	5143	580	-2920	1	1	84	99	60

3.2 Lower public servants and salaried employees. Rural districts.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
301	1	8105	600	18	1617	482	416	196	0	375	169	114	1257	1000	207	520	1	1	40	99	60
302	1	8230	480	120	1540	0	737	93	38	52	87	0	859	9824	201	-6060	1	1	-29	98	54
303	1	8318	571	777	2650	41	487	167	416	475	29	160	309	240	188	590	4	2	50	99	34
304	1	8364	1046	594	1700	21	837	121	183	421	201	70	939	180	188	270	1	2	4	98	46
305	2	8407	2141	732	1050	620	625	178	41	11	107	43	760	321	143	2410	1	3	-98	94	54
306	1	8565	0	0	2224	706	1778	149	64	2559	309	14	3177	495	75	-4020	4	1	71	100	60
307	1	8587	569	484	1500	893	1188	151	628	70	314	186	400	1188	415	-450	4	2	-62	100	47
308	1	8992	747	898	960	38	290	71	49	104	98	192	503	95	150	4690	1	1	-34	78	60
309	1	9077	420	250	2310	400	824	140	196	0	124	675	760	2703	322	-800	1	1	94	96	47
310	1	9191	300	458	2026	84	1231	196	122	595	283	472	993	483	139	410	4	2	159	98	45
311	2	9216	369	310	1300	375	771	60	150	1858	445	325	703	1430	169	1250	1	3	218	85	54
312	1	9325	576	65	1815	530	878	190	120	28	183	50	2064	780	213	1310	9	1	20	86	60
313	1	9326	588	121	2012	1261	902	111	295	0	168	8	1147	263	534	790	1	1	20	99	60
314	2	9335	425	856	1788	527	721	84	110	69	236	37	1910	0	232	1100	6	3	7	99	32
315	1	9340	440	350	1800	1040	1975	210	350	4205	205	10	865	520	200	-2650	9	1	30	89	55
316	1	9684	723	400	1092	20	688	65	152	285	40	66	355	2443	166	3050	9	2	343	86	35
317	1	10289	1037	847	1440	435	769	28	86	35	167	202	502	136	235	2020	1	2	137	92	38
318	1	10447	360	0	2220	530	1498	228	277	325	193	555	1006	6298	200	-2190	1	1	50	100	54
319	1	10913	480	120	364	0	498	187	450	0	214	58	398	1080	436	2420	6	1	41	98	46
320	1	11079	936	898	1960	746	717	228	396	272	184	62	485	5666	218	160	9	1	-34	100	49
321	1	11782	602	1164	1200	260	375	181	296	1362	166	68	732	240	220	3690	1	2	79	95	60
322	1	13498	360	364	1800	70	1649	243	68	1026	209	196	1511	2425	424	1810	1	2	208	89	42

3.3 Skilled workers. Rural districts.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
121	2	4077	336	168	787	200	216	69	66	1490	154	67	302	295	178	-1020	4	3	11	99	54
122	2	4100	252	305	1750	250	77	27	15	30	70	55	90	1527	262	-480	9	3	33	99	22
123	3	4110	612	258	833	103	385	82	35	249	93	28	463	201	182	720	1	4	-11	81	50
124	3	4117	200	170	1445	269	510	66	39	159	83	44	258	59	145	160	9	4	0	81	60
125	3	4149	0	241	960	422	250	82	142	399	157	91	408	111	58	1860	1	4	3	98	51
126	3	4149	312	115	871	261	337	51	86	802	107	27	371	64	326	-50	1	4	0	95	55
127	3	4193	607	416	1473	218	199	60	62	999	39	43	456	242	240	-3040	6	0	-159	92	43
128	3	4343	281	330	1249	123	498	75	151	389	130	51	136	42	203	970	1	4	12	98	53
129	2	4346	210	317	1820	247	280	69	38	37	68	60	122	752	192	-590	9	3	15	92	27
130	3	4464	222	329	1040	347	391	94	119	466	150	56	86	44	139	2760	1	4	-32	95	49
131	2	4475	278	274	1275	190	546	81	54	201	65	50	264	600	320	300	1	3	101	90	22
132	2	4555	160	473	1300	985	361	76	46	25	47	160	315	24	243	10	4	3	115	99	37
133	3	4571	200	430	1457	233	449	75	61	67	20	41	474	53	187	720	1	4	5	95	26
134	2	4582	675	367	1220	595	110	42	100	187	42	82	167	117	225	520	1	3	4	98	28
135	2	4611	306	465	1625	23	425	49	89	207	26	44	78	283	367	1040	4	0	87	63	60
136	2	4836	279	263	1280	56	616	2	18	181	7	30	39	168	221	2610	1	3	524	79	17
137	3	5021	280	184	1360	247	182	80	14	251	116	34	137	896	332	920	1	4	10	99	43
138	3	5168	139	258	1122	108	254	64	44	244	111	45	360	2282	238	-420	1	4	0	98	38
139	1	5251	528	258	2132	600	612	99	104	0	35	72	30	135	482	340	6	1	30	85	50
140	3	5456	353	222	1560	0	465	120	37	38	111	67	171	83	165	2200	1	4	92	96	35
141	2	5646	383	430	1250	352	416	84	109	167	183	202	656	210	180	560	1	3	-3	98	53
142	2	6091	637	458	1485	400	733	132	152	281	91	31	453	150	116	1950	5	3	25	98	26
143	2	6380	496	487	1275	600	153	54	250	362	133	71	235	1229	261	930	1	3	-50	99	42
144	1	6587	552	0	1470	445	537	211	124	24	178	160	1125	595	441	610	1	1	18	99	60
145	1	6993	480	150	1840	0	575	88	308	0	328	104	1568	1261	171	220	9	1	0	100	54
146	1	7062	367	216	2160	700	767	107	222	50	71	42	2064	245	544	-860	3	1	448	67	60
147	1	7771	540	120	1650	0	203	171	16	24	165	35	1267	2381	496	360	9	1	7	99	60
148	1	7884	963	384	2444	557	365	67	105	526	176	95	191	206	391	1240	1	1	63	98	53
149	1	8122	480	60	1975	0	352	62	176	0	334	93	638	87	543	2360	1	1	139	91	60
150	1	8245	924	617	1900	391	260	17	204	200	143	180	854	225	609	650	1	1	86	83	27
151	1	8455	240	100	1750	394	248	70	28	74	155	79	951	3490	570	40	1	1	-26	100	60
152	1	8658	720	197	2356	612	1401	163	533	93	273	50	1068	450	362	-630	6	1	8	92	60
153	1	9781	480	72	2136	0	1183	104	260	325	93	28	824	585	514	2550	6	1	120	93	54
154	2	10922	631	405	1250	0	812	182	28	100	187	109	1510	7863	343	-5630	6	3	-154	99	54
155	1	11320	540	0	1440	0	175	95	0	6522	70	30	1195	399	254	-1330	9	1	44	100	60

3.4 Unskilled workers. Rural districts.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
241	2	5030	263	334	2040	20	922	70	99	141	76	23	98	115	247	920	9	3	-42	93	53
242	1	5091	360	489	1560	357	298	112	256	365	159	26	89	358	280	40	1	2	0	99	54
243	2	5112	176	316	2165	475	110	16	58	81	61	108	291	746	238	460	1	3	84	94	34
244	2	5151	429	411	1330	347	293	46	84	78	41	47	105	35	267	1940	1	3	139	72	33
245	1	5176	670	655	1046	0	343	146	88	473	135	14	234	167	78	180	4	2	1	99	54
246	2	5215	117	248	1020	51	317	35	51	137	58	46	407	17	233	1420	1	0	353	63	15
247	2	5361	540	411	1185	228	319	96	58	181	116	112	377	648	300	740	1	3	23	98	49
248	2	5399	707	287	1300	319	365	77	43	264	77	60	413	120	280	3570	9	3	419	54	52
249	2	5447	396	553	1820	441	338	70	212	782	104	119	128	115	113	380	1	3	149	88	21
250	1	5472	540	0	1828	700	720	109	0	45	200	60	150	557	379	100	1	2	0	100	50
251	2	5474	264	414	1380	250	544	157	164	322	131	95	637	160	244	530	4	3	25	89	50
252	2	5505	561	447	1200	260	758	107	118	293	125	54	416	451	278	160	1	3	41	89	41
253	2	5562	415	536	1045	398	283	26	43	134	41	143	137	146	274	160	1	3	1	99	50
254	3	5576	296	424	1030	100	320	53	41	46	139	72	192	5300	197	-8330	4	4	157	97	36
255	2	5580	497	454	1268	469	254	66	107	289	208	130	510	480	350	680	6	3	-166	83	51
256	2	5842	270	322	1300	310	566	53	59	235	99	51	385	25	247	3110	1	3	37	90	31
257	1	5947	488	907	1820	135	390	132	167	241	103	242	127	229	443	170	1	1	-4	71	44
258	1	6005	360	168	2040	474	757	115	200	0	224	0	686	390	460	190	1	1	0	99	54
259	2	6158	476	481	2080	234	415	86	73	81	113	54	350	6	217	2090	1	3	382	94	16
260	2	6290	376	410	1208	350	419	62	50	403	189	70	430	780	338	-70	1	3	133	80	28
261	2	6399	420	637	1560	234	125	101	80	343	117	96	70	1125	365	2400	1	3	0	99	60
262	1	6521	384	584	2205	275	411	103	90	39	118	28	250	30	253	130	1	2	0	70	50
263	2	6669	547	296	1232	660	379	114	106	141	170	148	499	377	477	1840	1	3	7	72	48
264	2	6924	408	326	1820	1000	612	41	211	63	196	104	608	187	88	250	9	3	12	98	40
265	2	6987	607	481	1680	560	569	56	59	73	149	45	567	157	432	2950	2	3	-275	97	42
266	1	7035	618	489	1300	435	652	130	161	157	71	72	77	338	265	2040	4	1	449	89	47
267	1	7066	88	145	1300	542	608	98	55	139	79	197	1015	1094	415	-60	4	1	155	78	28
268	1	7448	480	10	2240	705	971	111	138	0	293	206	951	1976	349	-1030	4	1	13	100	53
269	1	7686	554	436	1040	0	730	33	140	121	72	80	285	124	444	3680	9	1	279	48	46
270	1	7723	240	36	1560	464	467	99	0	111	88	0	1051	57	341	3210	4	1	11	100	53
271	1	7931	1567	620	1040	652	412	114	122	45	59	133	265	7265	526	-8960	1	1	155	95	34
272	1	7970	335	401	2184	1020	1057	57	217	520	185	90	471	75	492	210	1	1	55	91	29
273	1	7982	840	22	927	620	1635	207	34	1089	389	134	1171	202	137	160	1	2	0	95	60
274	1	8058	1675	403	2000	78	375	104	13	52	118	59	318	174	398	1170	1	1	124	93	6
275	1	8101	504	4	2576	200	1149	149	312	8	110	14	2415	1311	430	-1200	4	1	105	96	60
276	1	8236	180	0	2250	488	1124	104	250	0	239	0	1240	1614	417	-110	4	1	-4	100	53
277	1	8259	360	36	2010	696	381	83	84	55	141	84	1060	2000	442	440	1	1	3	99	54
278	2	8494	728	893	2600	287	2009	153	83	775	240	74	443	410	144	-2240	9	3	1864	23	17
279	1	9396	1084	6	2000	549	306	104	180	34	192	0	1119	492	468	2770	1	1	-229	91	51
280	1	9439	240	153	1560	1225	604	73	195	11	38	64	548	1120	398	40	1	1	148	76	38
281	1	10478	339	557	3172	314	710	52	86	31	56	115	427	1380	465	1780	1	1	88	68	17

3.5 Agricultural workers. Rural districts.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
121	2	3103	165	320	1300	304	65	28	51	25	53	50	59	66	193	-100	6	3	2	69	25
122	2	3118	310	244	910	386	55	17	58	213	55	64	19	100	226	80	4	3	0	99	33
123	2	3122	237	320	1000	150	152	42	45	215	67	87	152	25	90	530	6	3	0	97	18
124	3	3126	181	139	1213	194	220	48	29	90	59	76	274	68	164	-80	1	4	143	92	52
125	2	3168	172	436	1071	154	390	41	58	77	54	98	111	72	202	150	4	3	0	98	54
126	2	3195	60	409	1430	214	65	15	110	219	48	63	95	183	69	660	6	3	1	98	27
127	2	3225	230	227	1300	125	125	67	51	5	59	27	109	15	72	1420	4	3	97	98	28
128	3	3240	258	366	867	130	313	120	60	266	74	24	109	123	180	290	1	4	-11	87	35
129	2	3248	359	455	1560	5	115	18	46	28	64	74	44	47	186	280	4	3	19	86	27
130	4	3265	146	188	1080	47	342	30	31	98	39	29	27	1078	120	-1400	6	5	-4	52	46
131	3	3282	665	189	907	41	256	53	21	163	71	18	23	50	233	720	4	4	13	87	23
132	3	3295	60	157	1213	63	245	73	27	450	63	43	42	52	71	1540	1	4	110	81	38
133	2	3383	239	204	1040	72	621	56	57	552	71	45	31	30	241	90	1	3	3	84	27
134	3	3440	199	303	1137	460	376	87	33	359	51	44	56	65	220	380	1	4	-6	87	40
135	2	3522	378	367	1000	355	161	50	40	479	51	52	117	50	244	300	1	3	30	59	17
136	2	3562	264	381	1400	180	164	24	104	137	96	52	118	78	245	30	4	3	9	67	36
137	2	3615	567	248	1135	225	231	54	49	20	66	32	212	100	66	490	4	3	26	99	17
138	2	3669	240	297	1820	57	260	38	205	124	18	62	33	26	58	40	1	3	1	100	46
139	2	3772	301	373	1020	250	265	85	40	97	124	50	211	204	257	940	1	3	4	84	27
140	2	4001	207	222	1560	14	296	57	47	121	49	52	694	21	86	970	6	3	105	98	19
141	1	4013	519	341	540	33	298	88	188	92	65	74	588	102	262	290	3	1	104	84	60
142	1	4046	472	259	1364	125	261	106	100	84	62	40	135	75	79	640	6	1	353	88	60
143	2	4054	201	253	1334	208	402	37	40	233	61	32	318	41	109	740	1	3	31	97	18
144	2	4076	196	313	1170	360	396	20	151	44	55	107	5	712	230	50	1	3	38	63	25
145	1	4304	784	226	780	699	193	52	350	60	124	65	92	195	106	0	2	1	230	32	60
146	1	4350	142	457	960	126	258	38	224	17	56	209	201	97	438	1010	6	1	49	33	26
147	1	4353	44	425	800	100	55	29	110	303	52	95	245	102	454	820	2	1	28	75	46
148	2	4451	270	370	1300	207	694	65	47	548	105	39	121	24	282	560	1	3	39	98	23
149	2	4585	614	344	1150	258	205	38	49	66	37	36	61	100	235	1870	9	9	130	51	28
150	1	4634	184	186	1278	292	127	39	70	72	169	66	495	72	277	1190	1	1	174	65	42
151	2	4844	920	880	1200	30	430	38	63	22	66	15	100	90	232	1070	9	3	0	78	24
152	1	4883	810	725	1000	500	738	28	143	16	51	71	135	42	60	-870	2	1	199	47	60
153	1	5029	150	15	1312	617	901	101	205	20	127	30	976	120	108	100	1	1	10	100	60
154	1	5108	508	386	520	365	726	85	152	467	265	33	350	250	152	740	4	1	-17	99	48
155	1	5225	91	143	1920	420	436	43	79	17	59	5	699	125	468	200	5	1	4	69	60
156	1	5319	348	439	525	364	228	71	80	0	75	0	752	97	410	104	6	1	2	99	48
157	2	5359	211	301	1920	308	470	72	97	162	44	60	919	283	221	-130	4	3	3	83	23
158	1	6573	383	355	2800	0	562	83	246	14	25	95	1250	13	358	-340	4	1	121	95	50
159	2	7199	200	61	624	166	127	27	63	689	58	9	214	1049	135	13350	1	3	-39	47	54
160	1	8104	360	566	2062	661	476	75	559	22	68	143	1113	1263	540	40	1	1	3	99	39

Appendix D

Detailed account of the expenditure items in the questionnaire of the Danish consumption survey 1955.

1. *Dwelling*

- Ordinary rent
- Expenditure on maintenance, etc.
- Mortgage payments
- Taxes on land and buildings
- Water rates, etc.
- Glass insurance
- Other insurance
- Expenditure in conn. with purchases of real property

2. *Fuel and lighting*

- Contribution to central heating
- Coal
- Coke
- Fuelwood
- Kindling
- Peat and patent fuel
- Lignite
- Oil for heating
- Town gas
- Bottled gas
- Electricity
- Kerosene
- Electric bulbs
- Matches
- Other expenditure

3. *Food¹⁾*

- Expenditure of foodstuffs bought (incl. beer, wine, and spirits for household use)
- Expenditure on regular eating out

4. *Tobacco*

- Cigars, cigarillos, cheroots
- Cigarettes
- Cigarette tobacco and paper
- Pipe tobacco

¹⁾ This item was taken up for special investigation in a separate food-survey, cf. *Statistiske Efterretninger* 1958, No. 46.

Chewing tobacco and snuff
Pipes
Pipe cleaners
Tobacco purses, lighters, etc.

5. *Clothing*

27 different items of men's clothing
25 different items of women's clothing
Repair of clothing

6. *Footwear*

5 different items of men's footwear
5 different items of women's footwear
Repair of footwear
Shoe-laces, polish

7. *Washing and cleaning*

Outside washing, mangling, ironing
Outside cleaning, pressing, proofing
Soda and other softening agents
Brown soap, soft soap
Soap flakes, soap powder, bar-soap
Detergents, dish-washing preparations
Self-acting washing preparations
Starch, bleaching solution, dye tablets
Scouring powder
Methylated spirits, hydrochloric acid, cleaning liquids
Lacquer and varnish
Floor polish and mop-oil
Other expenditure

8. *Durables*

13 different items of furniture, lamps, and ornamental objects
11 different items of bedclothes and table-linen
10 different items of kitchen utensils and table ware
Washing machines
Wringing machines
Kitchen ranges, cookers, and ovens
Refrigerators and ice-boxes
Mixers
Vacuum-cleaners
Sewing machines

Perambulators
 Brushes
 Buckets, tubs
 Irons
 Tools and implements (excl. those for professional use, hobby, and garden)
 Other acquisitions apart from transport equipment
 Repair of durables

9. *Personal hygiene*

Bath, pedicure, ultra-violet ray treatment
 Hairdresser and beauty culture
 Hand soap, bathing soap, shaving soap
 Hair-washing preparations
 Toothpaste
 Nail brushes, sponges, face cloths
 Combs, hairbrushes, hairpins
 Razor, shavers
 Toothbrushes and tooth glasses
 Hair-lotion, brilliantine, cream, perfume, lipstick, powder, nail-polish, and other
 cosmetic articles
 Other expenditure on personal hygiene

10. *Books, newspapers, etc.*

Books
 Newspapers
 Weekly and monthly magazines
 Periodicals

11. *Sports, hobbies*

Consumption at restaurants
 Beer, wine, and spirits outside the usual household consumption
 Radio and television
 Gramophone
 Musical instruments
 Theatre, cinema, and concerts
 Other entertainment (sports games, etc.)
 Holiday dwelling
 Other holiday expenditure, incl. holiday transport
 Garden and domestic animals
 Sports, subscriptions, and accessories
 Other recreation

12. *Transport*

Public transport facilities

Taxi

Acquisition and maintenance of bicycle

Acquisition and maintenance of motor-assisted bicycle

Acquisition and maintenance of motor-car and motor-cycle

Petrol and oil

Taxes and insurance

Other transport expenditure

13. *Union fees and subscriptions*

Unemployment insurance

Fire and burglary insurance

Health insurance

Other insurance (excl. life and superannuation insurance)

Fees and subscriptions to trade and professional associations

Other fees and subscriptions (excl. sports and motor associations)