Danmarks Statistik
MODELGRUPPEN

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Comparing ADAM to the textbook

Abstract:

The crowding-out process that closes the unemployment or output gap in ADAM is compared to the similar process in a textbook AS-AD model for a small open economy with fixed exchange rates. ADAM and the textbook describe the same kind of economy, and the interaction between output gap and real exchange rate is crucial in both models. Moreover, the long-run effects look rather similar when we interpret ADAM as an AS-AD model and compare to the parameterized AS-AD model suggested by the textbook. The main difference lies in the dynamics. In order to mimic the hump-shaped output response of ADAM to permanent demand shocks, the stylised autoregressive output gap equation of the textbook model has to be augmented by a lagged output gap.

We also illustrate that if the permanent shock is replaced by a stochastic shock, there is almost no difference between the response of, respectively, ADAM and the textbook model. Besides, the stochastic shock produces an output gap response that looks like a business cycle illustrating the Frisch-Slutsky paradigm.

Nøgleord: udenrigshandel

1. Introduction

Large macro-econometric models like ADAM (Annual Danish Aggregate Model) with 12 industries, 5 institutional sectors, 2,500 endogenous and 1,000 exogenous variables are often met with some scepticism. They seem difficult to understand and far away from the handy macro models you meet in economic textbooks.

In this paper, we compare ADAM to an Aggregate Supply-Aggregate Demand model from the textbook *Introducing Advanced Macroeconomics* by Sørensen and Whitta-Jacobsen. This particular AS-AD model is designed to describe a small open economy with fixed exchange rates. Denmark has a fixed exchange rate policy vis-á-vis the euro, and also the ADAM model assumes fixed exchange rate. Besides, ADAM reflects the traditional synthesis between Keynesian and neo-classical theory. In the short term, ADAM is demand-driven; and in the long term, it is supply-driven, just like the AS-AD model.

Even the AS-AD-model parameters suggested by the textbook are not too different from the parameter vales that ADAM would imply. The noteworthy difference concerns the time lag in ADAM's demand equations, which makes it necessary to introduce lagged output in the AD equation if you want it to mimick ADAM's demand side. There is no lag in the aggregate demand equation of the textbook AS-AD model.

With this extra lag and an appropriate parameterisation, the AS-AD model mimics the hump-shaped output gap response of ADAM to a permanent shock, while the output gap response of the pure textbook AS-AD model is a geometric progression with fixed half-life. Neither the long-winded ADAM response nor the textbook's geometric progression response to a permanent shock resembles a normal business cycle. However, both ADAM and the textbook model produce more normal business cycles when the shock is stochastic.

In the following five sections we formulate an AS-AD model describing ADAM, formulate the output gap equation, evaluate key parameters, estimate the output gap equation mimicking ADAM, and compare the output gap response in ADAM to that of the textbook AS-AD model.

2. A stylized AS-AD model and ADAM

The AS-AD model is presented in Sørensen and Whitta-Jacobsen (2010) chapter 24 p. 730 by the following equations.

AD: $\pi = e^r_{-1} + \pi^f - (1/\beta_1) \cdot (y - \bar{y} - z)$ (1)

SRAS: $\pi = \pi^f + \gamma \cdot (y - \bar{y})$ (2)

Real exchange rate: $e^r = e^r_{-1} + \pi^f - \pi$ (3)

Demand shock:

$$z = -\beta_2 \cdot (r^f - \bar{r}^f) + \beta_3 \cdot (g - \bar{g}) + \beta_4 \cdot (y^f - \bar{y}^f) + \beta_5 \cdot (\varepsilon - \bar{\varepsilon}) \quad (4)$$

π is the inflation in the economy, π^f is foreign inflation, e^r is real exchange rate written as foreign price over domestic price in a common currency, y is output. A bar indicates long-run equilibrium value, so $\bar{y}$ is long-run equilibrium output and $y - \bar{y}$ represents the output gap. Variable z in (1) represents a demand shock. The textbook also introduces a supply shock variable in equation (2), but we shall only work with demand shocks.

The basic AS-AD model consists of the three equations (1), (2) and (3). Equation (1) describes aggregate demand, (2) describes short-run aggregate supply, and (3) defines the real exchange rate. There are two parameters in this AS-AD model: β_1 in demand equation (1) represents the effect of the real exchange rate on output, and γ in supply equation (2) represents the effect of the output gap on inflation. Equation (3) is a definition without parameters.

Equation (4) can be seen as an auxiliary equation describing the demand shock. As exemplified in (4), demand shocks may come from abroad, e.g. foreign interest rate $r^f$and foreign output y^f, or from economic policy, e.g. government demand g, or from a structural break in the residual ε of any of the demand relations that the AD-curve represents. Examples of breaks are a confidence-based shift in the consumption function and a technology-based shift in the investment function.

Taken together, the three AS-AD equations (1), (2) and (3) determine inflation π, output y, and real exchange rate e^r; given the pre-determined variables: lagged real exchange rate, foreign inflation, long-run equilibrium output, demand shock and supply shock.

We now compare to ADAM. The aggregate demand equation in (1) is normalized on inflation, but it is easier to compare to ADAM, if we normalize (1) on output gap:

AD: $y - \bar{y} \; = \beta_1 \cdot e^r + z$ (1ADAM)

This way of writing the AD equation emphasizes that the demand-determined output gap is an increasing function of competitiveness because higher competitiveness means higher market shares. This mechanism is also present in ADAM, although in ADAM, the real exchange rate affects the demand for output, not for output gap, but that is just a formal difference as long as we are not changing long-run output $\bar{y}$.

We shall re-use (2) as the SRAS equation representing ADAM:

SRAS: $\pi = \pi^f + \gamma \cdot (y - \bar{y})$ (2ADAM)

The SRAS equation resembles ADAM's Philips curve, which basically makes wage inflation a function of the gap between actual and long-run unemployment. It should also be mentioned that price inflation in ADAM is represented by domestic inflation which has a coefficient less than one in the

Phillips curve. However, this is no problem to our stylized representation given that the fixed exchange rate makes ADAM's domestic inflation equal to the exogenous foreign inflation.

Also the real exchange rate definition in (3) can be used to represent ADAM:

Real exchange rate: $e^r = e^r_{-1} + \pi^f - \pi$ (3ADAM)

Now, we have three equations representing ADAM as an AS-AD type model of a small economy with fixed exchange rates.

Finally, to bring this AS-AD model close to ADAM we shall use ADAM with exogenous wages to calculate the demand shock z. If we call ADAM without wage equation ADAM_K with K as in Keynes, the auxiliary equation determining z can be written:

Demand shock: $z = ADAM\_K\ (\nabla\, r^f, \nabla\, g, \nabla\, y^f, \nabla\, \varepsilon, etc.)$ (4ADAM)

Operator ∇ indicates difference to baseline, so that $\nabla\, r^f$ is the assumed change in interest rate, $\nabla\, g$ is assumed change in government demand etc. Thus, the resulting demand shock z reflects the changes to ADAM's exogenous variables, and z is the effect on the output gap $y - \bar{y}$ calculated by ADAM_K. ADAM's definition of output and output gap is discussed in section 4.

ADAM_K is a short-term model in the sense that wages are exogenous, which keeps the labour market from clearing, so that permanent demand changes have permanent impact on output and unemployment. However, ADAM_K does contain production functions and capital formation, so it is more than a traditional short-term model because you need a higher capital stock to produce more. The ADAM_K model resembles old ADAM versions from the era of incomes policies when the wage rate was treated as exogenous and assumed to be determined by the policy makers.

There are many exogenous variables in the ADAM model including a large number of fiscal instruments and the residuals of its behavioural equations. With (4ADAM) to calculate the demand shock, the AS-AD model representing ADAM can be used for any demand-shock calculation that ADAM can be used for.

3. The stylised output gap equation of the AS-AD model

The three basic AS-AD model equations imply that the output gap can be explained by itself and by the exogenous demand shock. More specifically, equation (2) and (3) can be used to replace the real exchange rate in (1) by an accumulation over time in output gap, i.e. $e^r = -\gamma \cdot \sum \hat{y}$, where $\hat{y}$ represents the output gap $y - \bar{y}$. This gives us the following output gap equation:

$$\hat{y} = -\beta_1 \cdot \gamma \cdot \sum \hat{y} \; + z \qquad (5)$$

The textbook presents the same gap equation without cumulating term:

$\hat{y}_{+1} = \beta \cdot \hat{y} + \beta \cdot (z_{+1} - z), \qquad \beta = 1/(1 + \gamma \cdot \beta_1) \qquad$ (5’)

cf. p. 732 in Sørensen and Whitta-Jacobsen (2010). The parameter β describes the speed of adjustment in the output gap and you come from (5) to (5’) by leading the variables 1 period, taking first order differences, and collecting terms.

Our formulation in (5) emphasizes that the real exchange rate reflects the cumulated output gap, and this mechanism controls the gap in the long run. An equivalent output gap can be made from the three equations in the AS-AD model representing ADAM:

$\hat{y} = -\beta_1 \cdot \gamma \cdot \sum \hat{y} \ + z \qquad$ (5ADAM)

Consequently, the long-run multiplier expressions are identical, and it follows from the common output gap equation that a permanent change in z has a long-run effect of zero on the output gap.

There are two parameters in (5ADAM), β_1 and γ. The higher the price elasticity β_1 of aggregate demand, the lower is the permanent domestic price increase needed to neutralize a permanently higher z. And the larger the output-gap coefficient γ from the short-run supply equation, the smaller is the accompanying output gap reaction. The size of the accompanying output gap reaction can be measured by $\sum \hat{y}$.

4. Evaluating the two AS-AD parameters

To quantify the long-run impact on the real exchange rate, we need to evaluate β_1, the elasticity of output demand with respect to the real exchange rate.

In the textbook, the real exchange rate refers to GDP deflators that are assumed proportional to the hourly wage rate in the absence of productivity shocks. Wages and prices are more different in ADAM, where the use of imported capital makes the wage elasticity of value added deflators less than one. Moreover, the ADAM price of exports has its wage elasticity further reduced by the content of imported inputs in exports, while the textbook price of exports is represented by a GDP deflator.

Thus, ADAM has a more complicated price formation but its wage formation resembles the price formation in the textbook AS-AD model. Consequently, we base the real exchange rate in the ADAM-related AS-AD on the hourly wage rate, which we determine by the short-run AS equation (2ADAM).

With the foreign wage rate and exchange rate exogenous, we can calculate the real exchange rate elasticity β_1 of output demand as the long-run output effect of reducing the hourly wage rate by 1 per cent in ADAM with exogenous wage, i.e. in ADAM_K. This model calculation is easy but it is not obvious which ADAM variable we should use for measuring the output effect.

The textbook focuses on GDP, which makes it natural to use the ADAM-calculated impact on GDP, or perhaps on GDP at factor cost to avoid the effect from indirect taxes in fixed prices. However, there is no explicit measure of long-run GDP in ADAM and it is difficult to quantify the output gap if we let the AD equation (1ADAM) describe ADAM's GDP response.

It conforms better with the structure of ADAM to focus on unemployment. ADAM has an explicit measure of long-run unemployment and the gap between actual and long-run unemployment determines the wage change in ADAM. This makes it straightforward to let the unemployment gap illustrate the output gap. More specifically, a positive unemployment gap means that unemployment is above its long-run value, so the output gap is illustrated by the unemployment gap with a minus.

Given the long-run unemployment and labour force, ADAM also describes long-run employment so we can also use ADAM's employment or ADAM's "desired employment" as activity measure.

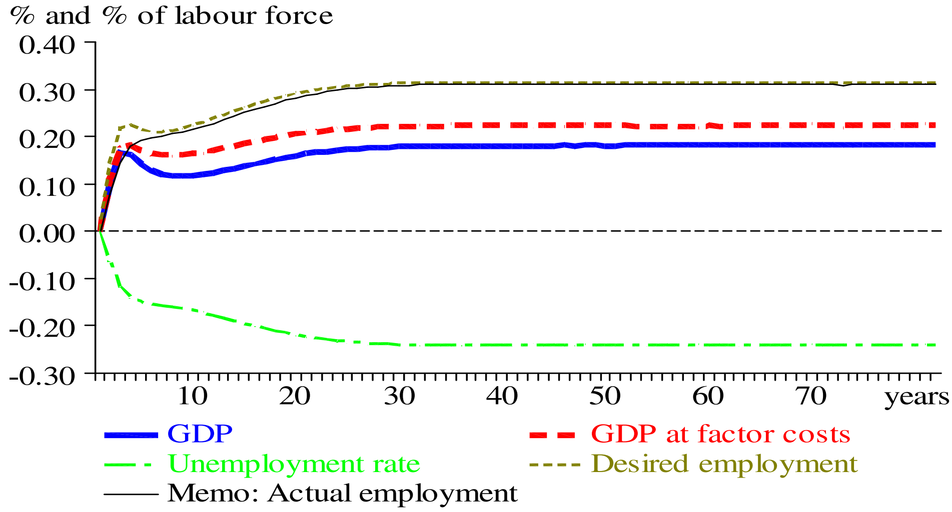
Desired employment N^d is derived from the production functions of ADAM by cost minimization at given output Y, given working hours per employee, and given relative factor prices, e.g. user cost/wage rate: u/w. In the first year, there is little response in factor prices and working hours are exogenous, which makes the immediate change in desired employment proportional to the immediate output change. In a longer run, the change in relative factor prices will affect labour productivity and introduce a wedge between desired employment and output.

ADAM's employment adjusts to desired employment in the long run, and we can use the gap between desired employment and long-run employment as an output gap. If we do that, the ADAM output gap can be calculated as:
$\hat{y} = N^d(Y, u/w) - N^*$
Long-run employment N^* is equal to labour force minus long-run unemployment.

The advantage of this gap measure is that total desired employment N^d is a simple sum of desired employment in ADAM's 12 industries and N^d can be compared directly to structural employment. The approach seems easier than calculating potential GDP on the basis of 12 industries to get a GDP gap.

Now we calculate the impact of 1 per cent lower hourly wage rate using ADAM _K where the wage rate is exogenous. The resulting impact on normal GDP at fixed market prices, on GDP at fixed factor costs, on unemployment and on desired employment are illustrated in figure 1. For comparison, the impact on actual employment is also shown.

Figure 1: Impact of hourly wage rate minus 1% (β_1 estimate), ADAM_K



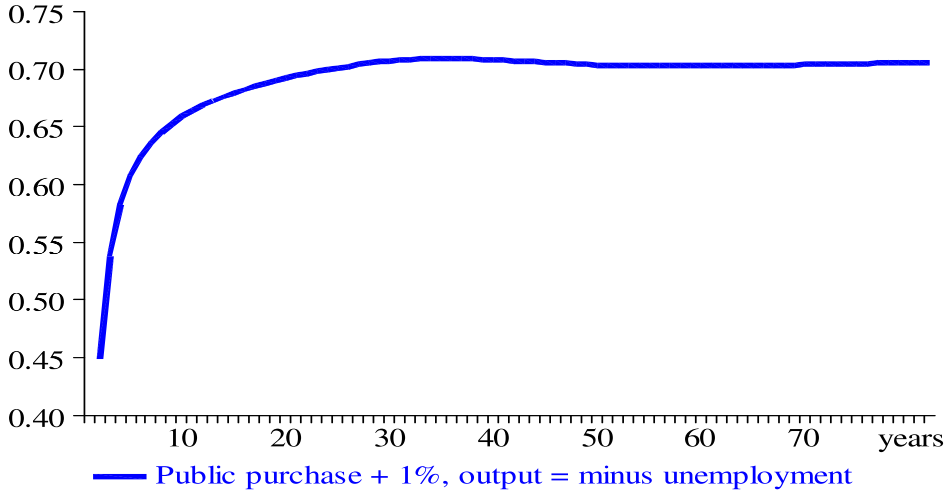
The long-run impact on ADAM's endogenous variables is reached after some 30 years with housing investments as the last major component to reach its new equilibrium. Long-run impact is 0.18 per cent on standard GDP, 0.22 per cent on GDP at factor cost, minus 0.24 percentage points on unemployment rate, and 0.31 per cent on desired employment.

The 1 per cent lower price of labour decreases labour productivity, which helps to explain that the effect on employment is larger than the effect on GDP. Besides, the lower domestic wage decreases real income and private consumption, and a lower share of private consumption will reduce GDP in market prices by reducing the content of indirect taxes. The lower content of indirect taxes does not affect GDP at factor costs, which expands by more than standard GDP in market prices. Finally, the unemployment rate drops by only 0.24 percentage point because the 0.31 per cent increase in employment is partly neutralized by a higher labour force that partly adapts to higher employment in ADAM.

The output impact of 1 per cent lower wage rate is also the output impact of 1 per cent higher real exchange rate defined as in the AS-AD model, so now we have four β_1 estimates for ADAM's AS-AD model, ranging from 0.18 to 0.31. This is clearly lower than the β_1 estimate of 0.72 suggested in the textbook. However, the difference is mostly formal reflecting that the textbook calibrates β_1 to reflect the price elasticity of output, while our ADAM calculation describes the wage elasticity of output. The difference between the ADAM calculation of β_1 and the textbook calibration is scrutinized in appendix.

To quantify the speed of adjustment in the stylised AS-AD models, we also need to determine the parameter γ for the output gap in the supply equation. For this assessment, we choose to represent ADAM's output gap by minus the unemployment gap. And in order to estimate γ, ADAM is used to calculate the effect of a permanent 1 per cent increase in public purchase. More specifically, γ is derived as the long-run effect on real exchange rate, ∇e^r, divided by the long-run effect on ADAM's cumulated output gap: $\nabla \sum \hat{y}$, cf. the notation used in section 3. This gives us a γ-estimate close to 0.70, cf. figure 2. This reflects the coefficient of 0.55 for the unemployment gap in ADAM's wage relation plus the impact from the inflation term and other variables in the wage relation.

Figure 2: Wage impact/cumulated output gap (γ estimate), ADAM



Thus, we have a γ of 0.70 for the stylised AS-AD model representing ADAM, clearly higher than the γ of 0.3 in the 2010 edition of the textbook. It seems natural that the textbook suggests a lower γ, when the ADAM parameter concerns a wage change and the textbook parameter concerns a price change.

In an ADAM calculation, it takes almost 3 per cent wage change to produce a 1 per cent price change of manufactured exports suggesting that the wage elasticity of prices is one third. Thus, the wage-related γ of 0.70 corresponds to a price-related γ of around 0.23 or somewhat lower than the textbook γ of 0.3. Actually, the 0.23 is closer to the γ of 0.2 suggested in the 2005 edition of the textbook. Note also, that a price-related demand elasticity β_1 of 0.72 for the textbook model corresponds to a wage-related β_1 of 0.24 for ADAM, if the wage elasticity of prices is one third in ADAM.

The speed of output gap adjustment in the stylised AS-AD model can be described by a β parameter combining β_1 and γ, $\beta = 1/(1 + \gamma \cdot \beta_1)$, cf. (5') in the section on output gap equations. Equation (5') also implies that, without shocks, the output gap will adjust towards zero according to the simple autoregressive equation:
$\hat{y}_{+1} = \beta \cdot \hat{y}$.
The textbook parameters, $\beta_1 = 0.72$ and $\gamma = 0.3$, imply a β of 0.82 and a half-life of 3.5 years ($0.82^{3.5}$=0.5), while the ADAM-calculated parameters , $\beta_1 = 0.24$ and $\gamma = 0.70$ (output represented by minus unemployment), imply a β of 0.86 and a half-life of 4.5 years.

This discussion of parameters β_1 and γ summarizes the perhaps moderate difference between the two stylised AS-AD models relating to textbook and ADAM, respectively. However, we are far from explaining the full difference between the textbook and ADAM, because the stylised dynamics of the AS-AD model that we relate to ADAM is quite different from the dynamics of the actual ADAM model. Consequently, we have to enhance the dynamics of the AS-AD model before it can mimic ADAM.

5. An estimated ADAM output gap equation

In the AS-AD model, the dynamic adjustment of the output gap is created solely by the short-run supply equation in (2), which determines the adjustment path for the real exchange rate. On the demand side, output adjusts immediately to the exchange rate according to equation (1).

In ADAM, the Phillips curve produces similar dynamics in the nominal wage rate and thereby also in the real exchange rate based on wages. However, in ADAM it also takes time for prices to react to wages and time for foreign trade to react to prices. Moreover, the gradual increase in the Danish wage level will be accompanied by a gradual increase in the real income of consumers. This Harberger-Laursen-Metzler effect implies that a positive demand shock will be accompanied by higher private consumption increasing the need for crowding out via higher wage rates. Moreover, the higher demand triggers a hump-shaped reaction in investments in ADAM reflecting the accelerator mechanism. All in all, adjustment in ADAM is more complicated than adjustment in the textbook AS-AD model.

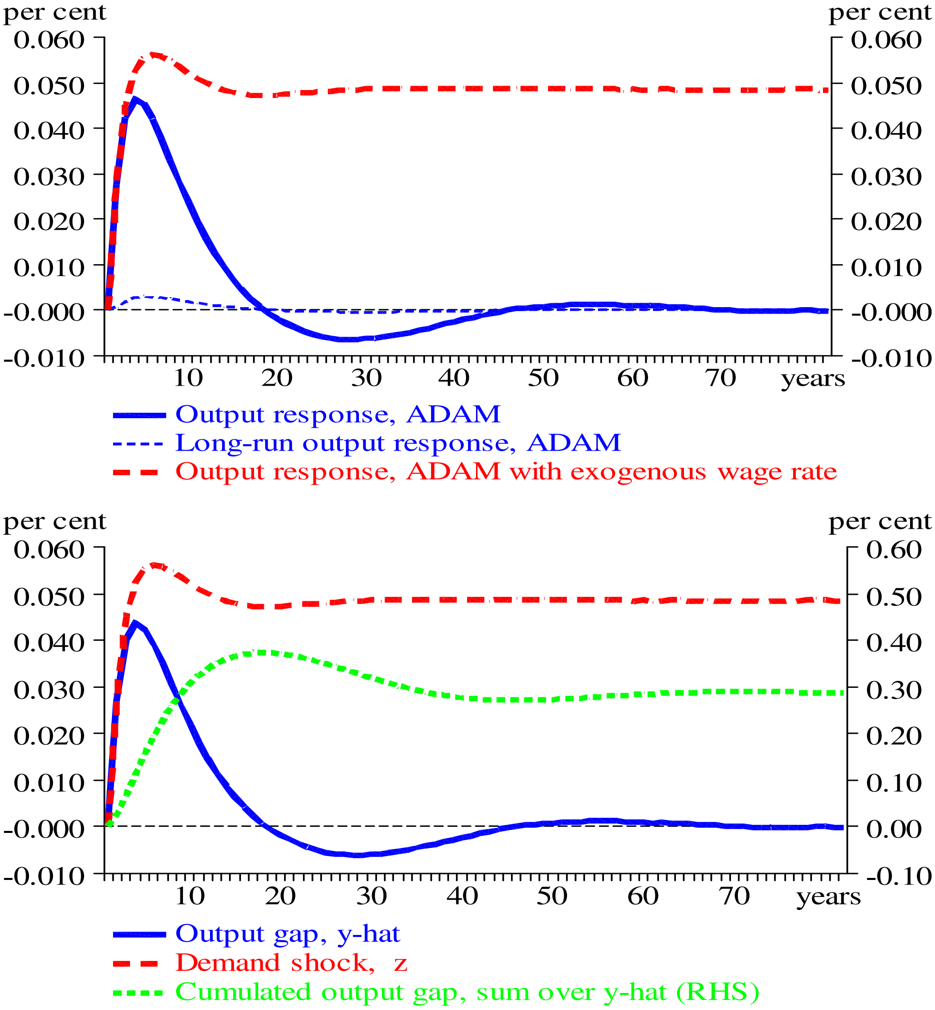
In order to mimic the richer dynamics of ADAM, we shall put the output gap equation in (5ADAM) on a less restricted error-correcting form:

$$\Delta\hat{y} = \alpha_1 \cdot \Delta z - \theta \cdot \hat{y}_{-1} + \alpha_2 \cdot \sum \hat{y}_{-1} + \alpha_3 \cdot z_{-1} \qquad \text{(5ADAM*)}$$

In this equation, we have introduced the lagged output gap on the right-hand side with parameter θ, and we are not restricting the coefficient of the lagged exogenous shock z_{-1}. Thus, there are four parameters to estimate in the error-correcting equation (5ADAM*), θ, α_1, α_2 and α_3. In (5ADAM), there is just one parameter to estimate, $\beta_1 \cdot \gamma$. The error-correcting form in (5ADAM*) is encompassing the stylised AS-AD equation (5ADAM) that would emerge if we estimated θ to 1, $\alpha_1$and α_3 to $1/(1+\gamma \cdot \beta_1)$, and α_2 to $-\gamma \cdot \beta_1/(1+\gamma \cdot \beta_1)$.

The parameters in (5ADAM*) can be estimated on the basis of ADAM calculations. More specifically, we re-use the demand shock from the previous section. The effect of the shock is calculated with both ADAM and ADAM without wage relation, i.e. by ADAM_K. The results of 1 per cent higher public purchase are shown in figure 3 where the upper panel shows the impact on output in ADAM and ADAM_K plus the impact on long-run output in ADAM while the lower panel shows the impact on the three variables $\hat{y}$, $\sum \hat{y}$, and z, which we use for estimating (5ADAM*). The impact on output gap $\hat{y}$ is the impact on output minus long-run output in ADAM, $\sum \hat{y}$ is accumulated $\hat{y}$ starting in year 1, and z is the impact on output in ADAM_K.

Figure 3: Public purchase + 1%, ADAM-calculated responses, effect on output is minus effect on unemployment



In the first couple of years, it is difficult to distinguish the calculation on ADAM from the calculation on ADAM_K, but thereafter the difference widens. In the long run, the output impact is zero in ADAM but permanently positive in ADAM_K. The impact on output is represented by the impact on unemployment with a minus.

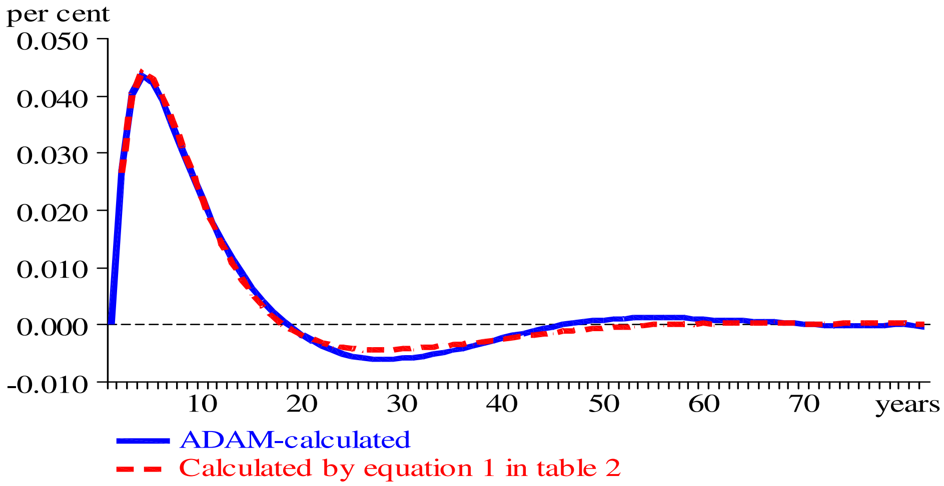
The time lag in the wage-indexation of unemployment benefits creates a slight temporary drop in the replacement ratio, and this drives the slight temporary increase in the ADAM-calculated long-run output that is visible in the upper panel in figure 3. In the long run, the wage rate has adapted and is growing in parallel to the base line, which makes the lag in the wage indexation irrelevant implying that higher public purchase has no long-run effect on output.

The lower panel of figure 3 illustrates that the cumulated output gap overshoots its long-run target for a number of years. This overshooting is reflected in the wage rate of in ADAM, and it is the overshooting of the wage rate that forces the output gap to cross the zero line and undershoot its target of zero for a number of years.

The result of estimating (5ADAM*) based on this demand shock is shown in table 2. There is no stochastics in the ADAM calculation, so we cannot indicate standard errors or formal significance of the four parameters, but the high fit is illustrated by the high R square and by figure 4, where we compare the ADAM-calculated output gap to the output gap calculated by inserting the exogenous shock z in the output gap equation (5ADAM*).

Table 2: Estimate of output gap equation (5ADAM*)					
	θ	α_1	α_2	α_3.	R2
:					
	0.136	0.938	-0.0120	0.070	0.998
Public purchase + 1 %. Effect on output represented by minus effect on unemployment					

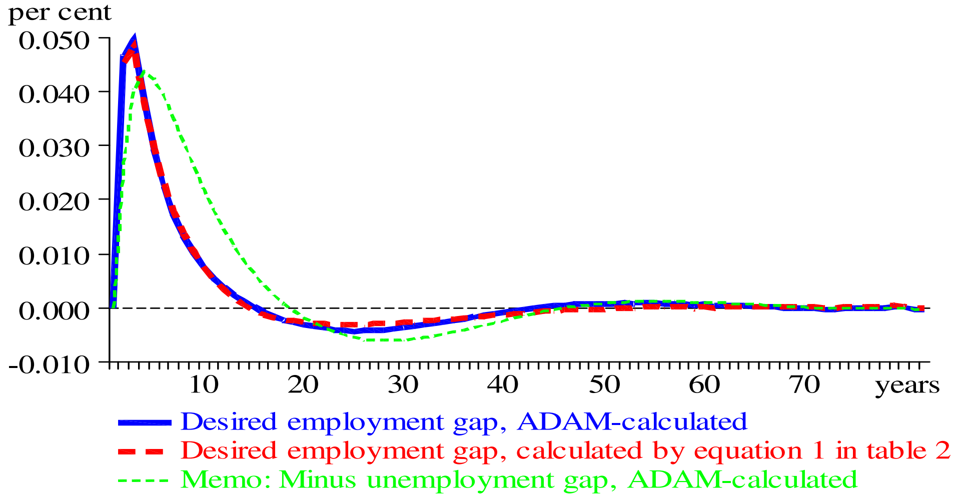
Figure 4: Approximating ADAM's output gap, government purchase + 1%



ADAM is a large model and it is not evident that we would achieve a high fit when estimating the rather simple equation (5ADAM*). However, the fit in table 2 is hardly a coincidence, because the fit of the parameterized (5ADAM*) is also high for other demand shocks.

We can also try out the gap in ADAM's desired employment, cf. figure 5. The figure confirms that the gap between actual and long-run desired employment responds more quickly to the government purchase shock than the unemployment gap. This reflects that desired employment is proportional to output in the short run. Figure 5 also illustrates that the equation in table 2 can mimic the ADAM-calculated desired employment gap rather closely. For this calculation, the shock variable z in (5ADAM*) is the impact on desired employment gap calculated by ADAM with exogenous wage rate,

Figure 5: Desired employment and unemployment, government purchase + 1%

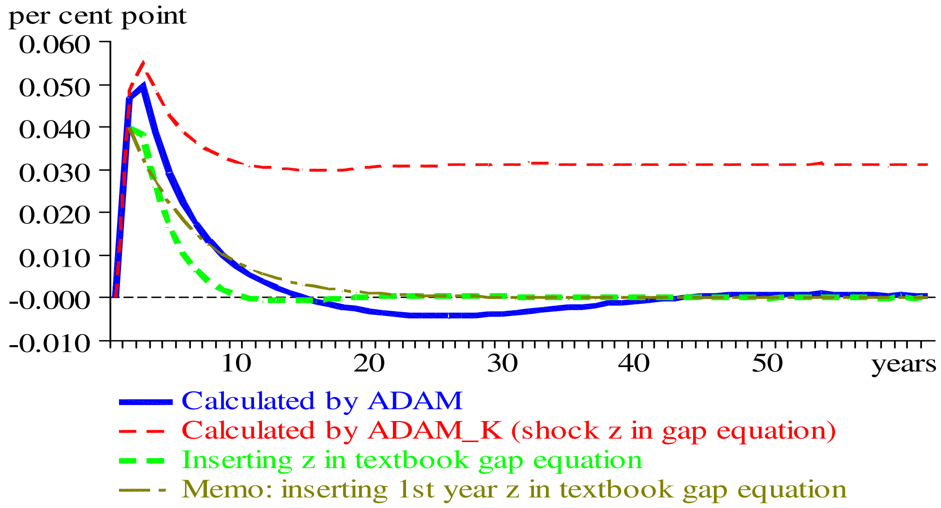


6. Gap response in ADAM and textbook

To illustrate the difference between the gap response in ADAM and in the textbook equation, we calculate the impact of 1 per cent permanent increase in public purchase using ADAM and using the textbook output gap equation. To get the shock variable *z* for the output gap equation, we calculate the impact on desired employment gap using ADAM with exogenous wage rate.

Increasing public purchase by 1 per cent in ADAM produces a change in desired employment that at first is close to the *z* variable calculated by ADAM with exogenous wage rate. However, the difference to z widens over time, and after 15 years the ADAM-calculated gap in desired employment crosses zero. In the long run, the ADAM-calculated gap becomes zero, but the calculated full cycle of the desired employment gap looks much longer than a normal business cycle when the gap is closing after 40 years, cf. figure 6.

Figure 6: Desired employment, government purchase + 1%



To calculate the impact of the same shock in the textbook's AS-AD model, we parameterize the textbook output gap equation (5) by setting β_1 to 0.72 and γ to 0.3, i.e.:
$\hat{y} = -0.216 \cdot \sum \hat{y} \ + z \Leftrightarrow \hat{y} = 0.822 \cdot \sum \hat{y}_{-1} \ + 0.822 \cdot z.$

In this equation, the immediate impact of the exogenous shock z on output gap $\hat{y}$ is close to 20 per cent less than z. We insert the z variable shown in figure 6 and get an output gap response that crosses zero and closes after 10 years. This means that the output gap of the textbook equation crosses the x-axis five years earlier than ADAM, and the subsequent undershooting of the x-axis seems too small to matter. Basically, this output gap does a half cycle of ten years, so it is adapting faster than the ADAM-calculated output gap.

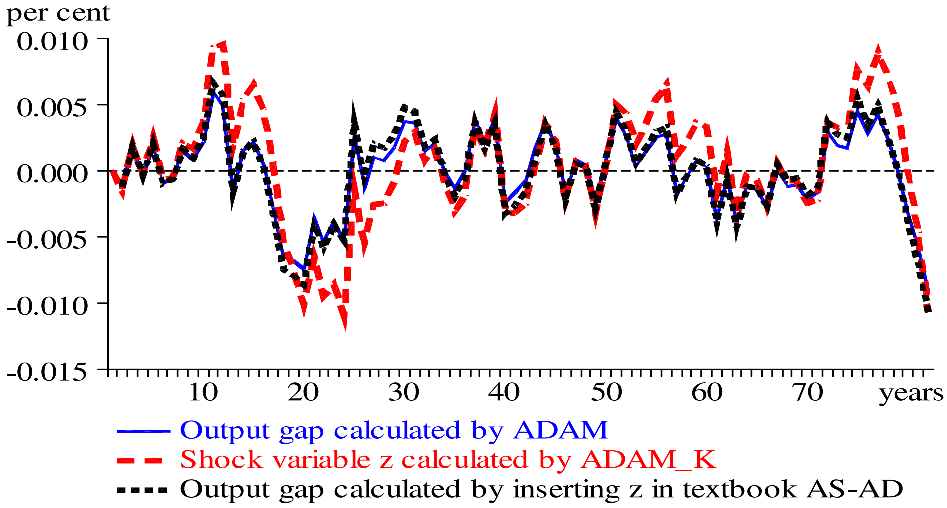
It must be added that the textbook does not suggest using a shock variable with a peak like z to represent a permanent 1 per cent increase of public purchases. With no time lags in its aggregate demand equation, the textbook would suggest a constant shock, for instance equal to the first year value of the z variable shown in figure 6. This would produce an output gap following a schematic geometric progression, also illustrated in figure 6.

Neither ADAM nor the geometric progression produce a gap that looks like part of a normal business cycle. But this is no surprise. We are calculating the response of a permanent deterministic shock to public expenditure, and the textbook says explicitly that business cycles are created by stochastic shocks.

This point is illustrated thoroughly in the textbook, and we can make a simple illustration for ADAM by introducing a stochastic white noise shock in the consumption function. The response in ADAM's output gap, i.e. the desired employment gap is shown in figure 7. It looks more like business cycles than the ADAM response to a permanent shock.

For comparison, we introduce the exact same stochastic shock in ADAM_K with exogenous wage rate. The resulting output gap is somewhat similar to the output gap calculated by the normal ADAM model indicating that the crowding out mechanism of ADAM is not reshaping completely the stochastic shock to domestic demand. Moreover, the output gap calculated by ADAM with exogenous wage rate is used as shock variable z in the parameterized textbook equation. The output gap calculated by ADAM_K and the output gap calculated by the textbook equation are shown in figure 7.

Figure 7: Stochastic consumption shock, output gap response



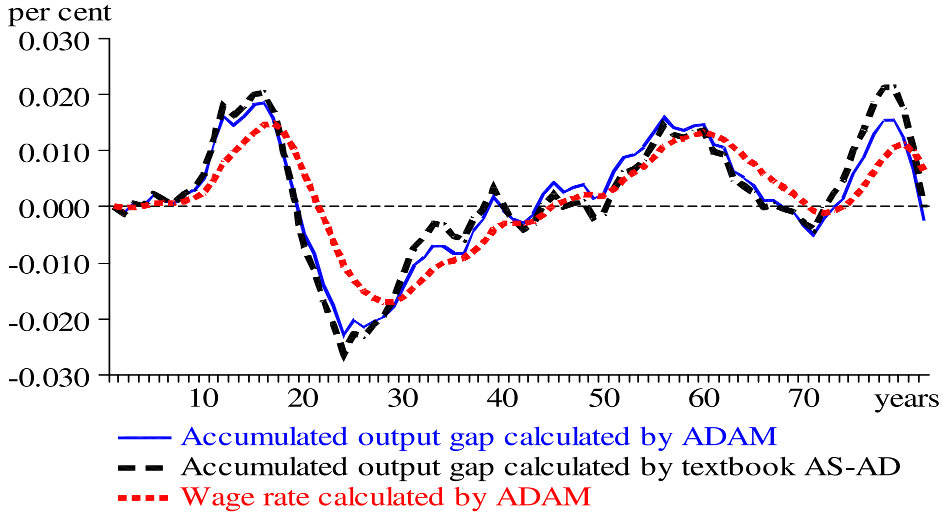
The main impression is that the ADAM- and textbook-generated output gaps resemble each other quite a lot. This is because the shock is stochastic. In case of a permanent and deterministic shock, the same gaps look rather different in the long run, cf. the previously shown figure 6. The close resemblance in figure 7 is reflected if we re-estimate the output gap equation (5ADAM*) on the basis of the stochastic shock to ADAM and ADAM_K. The re-estimated (5ADAM*) gets a first-year coefficient of 0.75 for the shock variable z, close to the 0.822 in the textbook equation.

We also note that, on average, both output gaps keep closer to zero than the demand shock z calculated by ADAM_K with exogenous wage rate. This reflects that both ADAM and the textbook model tend to crowd out the demand shock. However, the crowding out mechanism via wage and price is not creating a lasting difference between the exogenous shock and the resulting output gap, neither for ADAM nor for the textbook model.

The impact on wage and price can be described by the impact on the accumulated output gap, and the two accumulated output gaps respond rather similarly reflecting the similar response of the output gaps.

We are here using the desired employment gap to represent ADAM's output gap, and the lag between desired and actual employment in ADAM helps to explain the lag between the response of the accumulated output gap and the response of ADAM's wage rate, also illustrated in figure 8.

Figure 8: Stochastic consumption shock, accumulated output gap response



If the output gap seems stationary, the accumulated output gap seems non-stationary. This difference is illustrated by the accumulated output gaps in figure 8 that have fewer and longer cycles than the output gaps in figure 7. Basically, the accumulated gaps have just three peaks and one trough away from the baseline, and figure 7 confirms that the output gaps deviate from the exogenous demand shock in four separate periods.

7. Conclusion

We have compared the large macro econometric model ADAM describing the Danish economy to a small stylized textbook model describing a small open economy with fixed exchange rates. The two models are quite different in size, but their basic mechanism is similar. The important difference concerns the dynamics. For instance, the hump-shaped demand shock multiplier of ADAM looks rather different from the schematic autoregressive response of the textbook model, and the two models respond differently to a permanent demand shock.

Notwithstanding their different response to a permanent shock, ADAM and the textbook model respond rather similarly to a stochastic demand shock, and their response resembles business cycles. The similarity of the output gap responses to the stochastic shock seems to reflect that the sluggish price formation of the models is only correcting the more persistent fluctuations in the output gap towards zero.

Appendix: Price elasticity in the text book and in ADAM

In the textbook, the price elasticity β_1 of output with respect to the real exchange rate is evaluated by means of the following expression explained in the textbook:

$$\beta_1 = \frac{[(M\ /\ Y) \cdot (\ \epsilon_E + \epsilon_M - 1)] + [\epsilon_D \cdot (D\ /\ Y)]}{1 - D_Y}$$

Import ratio $M\ /\ Y$= 0.3, export + import price elasticity $\epsilon_E + \epsilon_M$= 3, demand share $D\ /\ Y\ =$ 0.8, domestic demand elasticity with respect to real exchange rate, ϵ_D= -0.3, derivative of output with respect to total demand D_Y=0.5.
With the quoted textbook values, the expression sets β_1 to 0.72.

We now want to relate the expression to ADAM's AS-AD model: (1ADAM), (2ADAM) and (3ADAM), and we start by rewriting the expression as (A1):

$$\beta_1 = [(M\ /\ Y) \cdot (\ \epsilon_E + \epsilon_M - 1)] + [\epsilon_D \cdot (D\ /\ Y)] + [\beta_1 \cdot D_Y] \qquad \text{(A1)}$$

In ADAM's AS-AD model the real exchange rate is wage based, and (A1) formulates the output impact of 1 per cent lower Danish wage rate as a sum of three square parentheses. The first parenthesis contains the output contribution from net export responding positively to the lower wage. The second contains output contribution from domestic demand responding negatively to the lower wage. The third contains output contribution from domestic demand responding positively to the higher output. All three output contributions can be identified in an ADAM calculation.

Consider now the central national accounts relation where GDP in market prices reflects domestic demand and net foreign trade:

$$GDP = CO + CP + IH + IXH + E - M$$

CO is public consumption, CP private consumption, IH housing investments, IXH other investments, E exports, and M imports. The simple relation holds in current prices.

We are not interested in current prices, but the percentage volume change in GDP from its baseline can be calculated as a weighted sum of percentage volume changes from baseline in the right-hand-side variables. Weights indicate the shares of the variables in baseline GDP evaluated at lagged prices. Using weights and volume changes from the end of the baseline period where all weights and volume changes have stabilized, the following relation holds.

$$GDP\% =$$
$$CO\% \cdot 0.270 + CP\% \cdot 0.495 + IBH\% \cdot 0.067 + IXBH\% \cdot 0.148 + E\% \cdot 0.566 - M\% \cdot 0.546$$

Where $X\%$ is the percentage volume change in variable X from X's baseline, and weights sum to 1 (=0.270+0.495+0.067+0.148+0.566-0.546). From the calculation on 1 per cent lower wage rate in ADAM with exogenous wage, we can insert volume changes from baseline:

$$GDP\% = 0.18 =$$
$$0 \cdot 0.270 - 0.31 \cdot 0.495 - 0.28 \cdot 0.067 + 0.29 \cdot 0.148 + 0.69 \cdot 0.566 - 0.14 \cdot 0.546 =$$
$$-0.31 \cdot 0.495 - 0.28 \cdot 0.067 + 0.29 \cdot 0.148 + 0.69 \cdot 0.566 - 0.14 \cdot 0.546$$

GDP and weights are measured in market prices, i.e. including indirect taxes. Private consumption includes the bulk of indirect taxes, and if we reduce private consumption by 20 percent and change the weights for private consumption and other items correspondingly, the national account relation calculates the 0.22 per cent change in GDP at factor costs. GDP at factor cost must be the ADAM output concept that comes closest to the textbook, and with 0.22 for ADAM's β_1 it is less than one third of the textbook's β_1 of 0.72.

To simplify the comparison, we ignore a couple of finer points in ADAM: We use the 0.31 per cent fall in private consumption for housing investments as well instead of the 28 per cent fall calculated by ADAM. Thus, we have a 0.31 per cent fall in households demand consisting of consumption plus housing investments. By doing that, we are ignoring the effect of a small fall in the relative price of housing consumption when the wage rate drops 1 per cent at given import prices. We also use the GDP change of 0.22 per cent for other investments ignoring the 0.29 per cent calculated by ADAM. The ignored difference reflects a compositional change towards foreign-competing industries like manufacturing and agriculture that are capital intensive.

The simplifications just mentioned disappear in the rounding-off errors, so that the expression for ADAM's β_1 still holds:

$\beta_1 = 0.22 =$
$-0.31 \cdot 0.448 - 0.31 \cdot 0.073 + 0.22 \cdot 0.162 + 0.69 \cdot 0.619 - 0.14 \cdot 0.597 =$
$-0.31 \cdot 0.521 + 0.22 \cdot 0.162 + 0.69 \cdot 0.619 - 0.14 \cdot 0.597$

In order to identify the negative impact on households' purchasing power, we formulate the -0.31 per cent change in household expenditure as the GDP volume increase of 0.22 per cent minus the purchasing-power impact of the lower wage rate. And to identify the price-driven impact on imports, we formulate the 0.14 per cent increase in imports as the GDP increase minus the fall in imports relative to GDP. This gives us:

$\beta_1 = 0.22 =$
$(0.22 - 0.53) \cdot 0.521 + 0.22 \cdot 0.162 + 0.69 \cdot 0.619 - (0.22\text{-}0.08) \cdot 0.597$

We can now recollect terms on the right hand side and get three output contributions similar to the ones suggested in (1A) above:

$\beta_1 = 0.22 =$
$[0.619 \cdot 0.69 + 0.597 \cdot 0.08] + [-0.53 \cdot 0.521] + [0.22 \cdot 0.086] =$
$[E / Y \cdot \epsilon_E + M / Y \cdot \epsilon_M] + [-\epsilon_D \cdot (D / Y)] + [\beta\_1 \cdot D\_Y]$ (A2)

Export ratio E / Y = 0.619, import ratio M / Y = 0.597, export wage elasticity ϵ_E = 0.619 and import wage elasticity ϵ_M = 0.08, demand share D / Y = 0.521, domestic demand elasticity with respect to real exchange rate, ϵ_D= -0.53, derivative of output with respect to total demand D_Y=0.086. The squared parentheses in (A1) and (A2) can be compared one by one.

The first parenthesis in (A1) and in (A2) indicates the output effect from net foreign demand driven by higher competitiveness. The value of the parenthesis is 0.6 in (A1) and 0.475 in (A2). The difference reflects that a substantial difference between the responses of foreign trade is partly offset by higher trade shares in the ADAM calculation and by a difference in method.

The high import and export shares in the ADAM calculation reflect the present level when GDP is measured at factor costs. The competitiveness-driven 0.69 per cent increase in exports is a simple ADAM-calculated result, and it should be straightforward to discuss the difference to the textbook.

In the textbook, the sum of export and import price elasticity is set to 3. It seems a fair guess that the export price elasticity accounts for 2 of the 3, and 2 is clearly higher than 0.69. However, the 0.69 does not represent ADAM's price elasticity. It represents ADAM's wage elasticity. One per cent lower wage rate produces a moderate 0.26 per cent long-term fall in the total export price, partly because prices of energy, agricultural products and sea freight are exogenous, but also because there is a content of imports in exports.

Thus, the difference between 0.69 and 2 per cent for the impact on exports reflects the difference between wage and price elasticity. The wage rate in ADAM would have to be reduced by 3.85 per cent to produce 1 percent lower export price and that would produce 2.7 per cent additional export volume. Also the difference between 0.08 and 1 per cent for the impact on imports is related to the difference between wage and price elasticity because 1 per cent lower wage rate reduces Danish output prices by less than 1 per cent. Besides, the residually derived negative price-driven impact on imports of 0.08 per cent is probably too modest, because the volume-driven impact is probably higher than the GDP increase due to high import content in investments and exports.

Finally, the approach to the GDP impact of net exports differs between the textbook and our ADAM calculation. The textbook is calculating the nominal impact of net exports on GDP as it subtracts the assumed price fall of 1 per cent from the sum of export and import price elasticity. However, if we are interested in the impact on real GDP we should not correct for the price change. If the textbook calculation were revised accordingly, the β_1 estimate would ceteris paribus increase from 0.72 to 1.32.

The second parenthesis indicates the GDP contribution from the Harberger-Laursen-Metzler effect, i.e. from domestic real income and demand driven by 1 per cent change in the real exchange rate. The textbook sets the contribution to -0.24 per cent ($= -0.3 \cdot 0.8$), which is close to the -0.28 per cent ($= -0.53 \cdot 0.521$) in the ADAM calculation. The higher negative impact on the purchasing power, -0.53 against -0.3, reflects the higher import content in the ADAM calculation. While the lower ADAM weight for private demand in GDP, 0.521 against 0.8, reflects that ADAM's business investments follow GDP and not household demand. Besides, the GDP share of exogenous public demand is higher and there is a small positive net export in ADAM's baseline.

The third parenthesis can be seen as a multiplier contribution where the higher GDP increases the demand for GDP. In the textbook, the multiplier contribution is 0.36 ($= 0.72 \cdot 0.5$) reflecting a D_Y coefficient of 0.5. In the ADAM calculation, the multiplier contribution is only

$$0.019 = 0.22 \cdot (0.521 + 0.162 - 0.597) = 0.22 \cdot 0.086$$

ADAM's D_Y coefficient is only 0.086 because the import content is deducted, which reduces the multiplier contribution in a small open economy. A lower D_Y coefficient would reduce the textbook estimate of β_1.

Summing up, we may revise the formal textbook calculation, e.g. replace the nominal change in foreign trade: $(M\,/\,Y) \cdot (\,\epsilon_E + \epsilon_M - 1)$ by the real change: $({}^{M}/_{Y}) \cdot (\,\epsilon_E + \epsilon_M)$ and reduce the D_Y coefficient because of the import content.

However, the basic issue is the price concept as mentioned in the main text. The textbook is referring to price changes of 1 per cent and it would take wage rate changes of 2 to 4 per cent to produce these price changes. Consequently, it is as expected that the price elasticity of GDP is higher than the wage elasticity.

Litterature:

Statistics Denmark, 2013. ADAM – a model of the Danish economy.

Sørensen, P.B. and H.J. Whitta-Jacobsen, 2005. Introducing Advanced Macroeconomics. Mc Graw Hill.

Sørensen, P.B. and H.J. Whitta-Jacobsen, 2010. Introducing Advanced Macroeconomics. Mc Graw Hill.