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Thomas Thomsen

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Bias-corrected Törnqvist indices

Abstract:

The Törnqvist price index, like other ideal price indices, cannot handle the presence of biased technological trends/biased shifts in preferences, or biased scale effects/income effects.

The paper shows how to correct the Törnqvist price index with regards to such effects. The correction turns out to be quite simple, and can be obtained by means of simple matrix operations.

Even though the bias-corrected Törnqvist index is rather simple to compute, analysis on Danish time series data combined with theoretical considerations indicate that the correction is likely to be small on typical annual data. So in many cases, a traditional Törnqvist price index will be almost as good as, e.g., the true CES price index in the case of a nested CES production or consumption function.

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1. Introduction

It is well known that the Törnqvist index is exact if the data-generating process is a translog cost function (cf. Diewert (1976)). However, the equivalence holds only in the case where the translog cost function does not contain biased time trends or biased scale effects. (Scale effects can be understood as income effects in the case of aggregating consumer goods, and henceforward the term “scale effects” also covers income effects).

The absence of biased trend- and scale effects in the Törnqvist index is theoretically a quite restrictive limitation, since such effects are often observed in the data. For instance, biased time trends are prevalent in most KLEM factor demand systems (for instance augmenting the *K*/*L* ratio), and income effects are prevalent in most consumption systems (for instance increasing the expenditure share of luxury goods as total income/wealth grows).

In order to understand what is precisely meant by *bias* here, we take a look at the well-known share equation in the translog cost function (to simplify with only two factors, and only trend bias):

$$s_1 = a_1 + b_{12} \log(P_1 / P_2) + b_{t1}t \qquad (1)$$

The left-hand side is the i’th share of total costs (or budget), a_1 is an uninteresting constant, and b_{12} is a parameter describing the substitution pattern (curvature of isoquants or preferences). The last term is the *bias* originating from time t. So the parameter b_{t1} indicates by how many percentage points the share of factor/good 1 increases autonomously each year. If, for instance, factor 1 is capital, and $b_{t1} = 0.005$, the share of capital expenses rises by 0.5 percentage points per year on its own. A figure of this magnitude is not uncommon on Danish macro data, and represents capital consuming technical progress.

There can hardly be any doubt that the true (macro) data generating processes often contains biased trend- and/or scale effects, but the question is how big the error is if, for instance, a standard Törnqvist index is used to aggregate factors/goods? The rest of the paper investigates this question, and outlines a relatively simple correction of the Törnqvist index. To anticipate the result, in the case where there are only to factors/goods and only trend effects, it turns out that the Törnqvist price index should be augmented with the following term:

$$\Delta\log(P_Y) = \Delta\log(P_{TQ}) + b_{t1} \log(P_1 / P_2)_{-0.5} \qquad (2)$$

where $\Delta\log(x) = \log(x) - \log(x_{-1})$, and $\log(x)_{-0.5} = [\log(x) + \log(x_{-1})]/2$. P_Y is the bias-corrected price index, P_{TQ} is the standard Törnqvist index, and P_1 and P_2 are the two prices. (The more general formula with more factors/goods and including scale effects is shown later in the paper). So, in the case of aggregating *K* and *L*, and only considering biased time trends, we would compute the ordinary Törnqvist price index, and add the last term in (2). If, for instance, the share of capital rises by 0.5%-points annually due to time, t, we would add 0.005 times the factor price ratio lagged a half year. This sounds easy enough, but where does the b_{t1} parameter come from? Unfortunately this parameter – unless known or conjectured a priori – must be found by means of econometric methods. This may sound complicated, but it turns out the parameters can be estimated by simple OLS, also in the multifactor/-good case (cf. section 4 below).

Even if a parameter like b_{t1} in (1) is not estimated, (2) can be used to check how sensitive the "true" Törnqvist price would be with respect to different values of the autonomous annual time trend in the capital share. As shown later in the paper, this sensitivity is typically quite limited.

2. The "true" Törnqvist index with trend and scale bias

Since the correction regarding trend and scale bias is mathematically equivalent, only the former is deducted in this section. So we start out with assuming constant returns to scale (CRTS) i.e., that the demand for all factors/goods rises with 1% when the production/utility level rises with 1%.[1]

The translog cost function with trends is given as follows:

$$\log(C) = a_0 + \log(Y) + \sum_i a_i \log(P_i) + 0.5\sum_j\sum_i b_{ij}\log(P_i)\log(P_j) + \sum_i b_{ti} t\log(P_i) + a_t t + 0.5 a_{tt} t^2 \quad (3)$$

$$\Sigma\, a_i = 1,\ b_{ij} = b_{ji},\ \Sigma\, b_{ij} = 0,\ \Sigma\, b_{ti} = 0.$$

The translog-price index is given as $P_Y \equiv C/Y$, which translates into

$$\log(P_Y) = a_0 + \sum_i a_i \log(P_i) + 0.5\sum_j\sum_i b_{ij}\log(P_i)\log(P_j) + \sum_i b_{ti} t\log(P_i) + a_t t + 0.5 a_{tt} t^2 \quad (4)$$

Not surprisingly, under the CRTS assumption the translog price index is independent of the level of Y. Taking the time difference of this index, we get:

$$\Delta\log(P_Y) = \sum_i a_i \Delta\log(P_i) + 0.5\sum_j\sum_i b_{ij}\log(P_i)\log(P_j) - 0.5\sum_j\sum_i b_{ij}\log(P_{i,-1})\log(P_{j,-1}) \quad (5)$$
$$+ \sum_i b_{ti} t\,\Delta\log(P_i) + \sum_i b_{ti}\Delta t\log(P_{i,-1}) + a_t\Delta t + a_{tt}(t^2 - t_{-1}^2)$$

[1] The utility level is of course ordinal, so in the consumption context the CRTS assumption should be understood as absence of income effects (proportionality in the quantities of goods, regardless of income/utility level).

where the first three terms relate to the a_i and b_{ij} parameters, and the rest of the terms to the trend parameters.[2]

The standard Törnqvist price index given as:

$$\Delta\log(P_{TQ}) \quad = \quad \sum_i \frac{s_i + s_{i,-1}}{2}\Delta\log(P_i), \tag{6}$$

If the datagenerating process is a translog cost function of the form (3), the cost shares are of this well-known form:

$$s_i \;=\; a_i \;+\; \sum_j b_{ij}\;\log(P_j) \;\; + \; b_{ti}t \tag{7}$$

So in this case, (6) translates into:

$$\Delta\log(P_{TQ}) \;=\; \sum_i \left(a_i \;+\; \sum_j b_{ij}\;\frac{\log(P_j)+\log(P_j(-1))}{2} \;\; + \; b_{ti}\frac{t+t(-1)}{2}\right)\Delta\log(P_i) \tag{8}$$

Which is equivalent to:

$$\Delta\log(P_{TQ}) \;=\; \sum_i a_i\Delta\log(P_i) \;+\; \sum_i\sum_j b_{ij}\;\frac{\log(P_j)+\log(P_j(-1))}{2}\;\Delta\log(P_i)$$
$$+\;\sum_i b_{ti}\frac{t+t(-1)}{2}\Delta\log(P_i) \tag{9}$$

Now how does this equation compare to the (true) translog price index in (5)? The difficult part is the double sum over (i, j), where it helps to note the following mathematical relationships:

$$\frac{(x+x_{-1})}{2}\Delta x \quad = \quad \frac{x^2 - x_{-1}^2}{2} \tag{10}$$
$$\frac{(x+x_{-1})}{2}\Delta y + \frac{(y+y_{-1})}{2}\Delta x \quad = \quad x\cdot y - x_{-1}\cdot y_{-1} \tag{11}$$

For the diagonal elements ($i = j$) in the double sum the relationship in (10) can be used, and for pairs of off-diagonal elements, the relationship in (11) can be used. Using (10) and (11), and collecting the resulting positive (unlagged) and negative (lagged) terms separately, we can rewrite (9) as

[2] This formula could be simplified, since $\Delta t = 1$, and $t^2 - t(-1)^2 = t - 0.5$. However, we choose not to simplify here, in order to reuse the formula in the case of scale effects.

$$\Delta \log(P_{TQ}) \;=\; \sum_i a_i \Delta \log(P_i) \;+\; 0.5 \sum_i \sum_j b_{ij} \;\log(P_i) \log(P_j)$$
$$-0.5 \sum_i \sum_j b_{ij} \;\log(P_{i,-1}) \log(P_{j,-1})$$
$$+\; \sum_i b_{ti} \frac{t + t(-1)}{2} \Delta \log(P_i) \qquad (12)$$

If we compare with the translog price index in (5), it is seen that the first three terms are identical. In fact the difference between P_Y and P_{TQ} can be stated as:

$$\Delta \log(P_Y) = \Delta \log(P_{TQ}) + \sum_i b_{ti} \Delta t \frac{\log(P_i) + \log(P_{i,-1})}{2} + a_t \Delta t + a_{tt}(t^2 - t_{-1}{}^2) \qquad (13)$$

or written more simply using the notation of a half-period (logarithmic) lag:

$$\Delta \log(P_Y) = \Delta \log(P_{TQ}) + \sum_i b_{ti} \Delta t \log(P_i)_{-0.5} + a_t \Delta t + a_{tt}(t^2 - t_{-1}{}^2) \qquad (14)$$

where, as noted before, $\log(x)_{-0.5} = [\log(x) + \log(x_{-1})]/2$. From (14) it is seen that the growth rate of a "naive" Törnqvist price index (P_{TQ}) without trend correction is wrong due to the unbiased contributions from a_t og a_{tt} (depending on t alone), and from the biased contributions from b_{ti} times Δt times the log price lagged half a period. Note that t is typically constructed so that $\Delta t = 1$.

In a nested production or consumption function context, the last two terms in (14) can be discarded, since an unbiased trend at some level in the production/consumption function can always be picked up as a biased trend a level higher up in the nesting structure. For instance, consider a ((*KL*)*E*) production function, where *K* and *L* substitute at the lowest nesting level, and the *KL*-aggregate and *E* (energy) substitute at the highest nesting level. If, for instance there is an autonomous time trend $a_t = -0.02$ at the lowest level, this means that the price of the *KL*-aggregate should decresase autonomously by 2% percent per year, corresponding to a productivity gain. But even if the price of the *KL*-aggregate is not corrected with respect to this unbiased effect, the effect from a_t (or a_{tt}) will be picked up at the next level in the nesting structure (as a *biased* trend in the *KL*-aggregate when estimated together with *E*).[3]

Thus, unbiased trends are not interesting at any level except the highest in a nested production or consumption function, since an unbiased trend at a lower level can always be represented (and estimated) as a biased trend one step higher up in the nesting structure. Keeping the above in mind, the "sufficiently true" price index can be restated as:

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[3] Another way of stating this is that if a nested ((*KL*)*E*) translog cost function were estimated with a_t as a free parameter at the lowest (*K* contra *L*) level, this parameter would be perfectly correlated with the trend parameters at the highest (*KL*-aggregate contra *E*) level, and hence unidentifiable.

$$\Delta\log(P_Y) = \Delta\log(P_{TQ}) + \sum_i b_{ti}\Delta t \log(P_i)_{-0.5} \qquad (15)$$

If the data-generating process is translog cost function, this index contains all the information necessary to yield true unbiased estimates of substitution patterns higher up in the nesting structure. As it is also shown in section 1, in the case of only two factors/goods, and bearing in mind that the b_{ti}'s sum to 0, the formula reduces to:

$$\Delta\log(P_Y) = \Delta\log(P_{TQ}) + b_{t1}\log(P_1 / P_2)_{-0.5} \qquad (16)$$

where it is assumed that $\Delta t = 1$, since t is typically constructed that way. So with two factors/goods, the growth rate of the true index is given as the growth rate of the standard Törnqvist price index plus the share trend parameter (b_{t1}) times the factor price ratio lagged a half period.[4] If $b_{t1} = 0$, the two indices are equivalent, thus establishing that the Törnqvist price index is indeed a so-called superlative index in the absence of biased effects.[5]

3. Biased scale effects

Given the above analysis regarding biased trend effects, it is relatively easy to compute the impact of biased scale effects on the "true" Törnqvist price index. The starting point is to note the mathematical similarity between the use of t and log(Y) in the "full" translog cost function, repeated below:

$$\begin{aligned}\log(C) = a_0 + a_y\log(Y) + \sum_i a_i\log(P_i) + a_t t + 0.5\sum_j\sum_i b_{ij}\log(P_i)\log(P_j) \\ + \sum_i b_{yi}\log(Y)\log(P_i) + 0.5a_{yy}(\log(Y))^2 + \sum_i b_{ti} t\log(P_i) \qquad (17) \\ + b_{ty} t\log(Y) + 0.5a_{tt}t^2\end{aligned}$$

$$s_i = a_i + \sum_j b_{ij}\log(P_j) + b_{yi}\log(Y) + b_{ti}t \qquad (18)$$

$$\Sigma a_i = 1,\ b_{ij} = b_{ji},\ \Sigma b_{ij} = 0,\ \Sigma b_{yi} = 0,\ \Sigma b_{ti} = 0.$$

The b_{yi} parameters express biased scale effects, and it is noted that for each trend parameter b_{ti}, a_t and a_{tt}, there is an equivalent scale parameter b_{yi}, a_y and

[4] It is seen that the *level* of the relative factor prices influences the *growth rate* of the true price index. This may seem strange, but in fact any difference in the levels of P_1 and P_2 will be picked up by the trend parameters one level higher up in the nesting structure. It is, however, advisable to scale the levels of P_1 and P_2 so that they are equal in some base year. Otherwise there will be a fixed growth rate in the true index originating from the (arbitrary) scale difference in prices.

[5] The translog cost function in terms is provably flexible enough to be termed a flexible functional form (FFF), capable of approximating the true cost function (and thus its dual production or utility function) to a second-order degree. See also Diewert (1976).

a_{yy}. This symmetry stems from the fact that the translog cost function can be understood as a second-order Taylor expansion in log(P_i), log(Y) and t.

From this fact, and from the fact that both t and Y can be treated as exogenous variables, it should hardly be any surprise that the bias-corrected Törnqvist index in the case of both trend and scale bias turns out to be as follows:[6]

$$\Delta\log(P_Y) = \Delta\log(P_{TQ}) + \sum_i b_{ti}\Delta t\log(P_i)_{-0.5} + \sum_i b_{yi}\Delta\log(Y)\log(P_i)_{-0.5} \qquad (19)$$

The b_{ti} term is the by now familiar biased trend term also seen in (15), whereas the b_{yi} term stems from possible scale bias. As before, any unbiased trend- or scale parameters will be picked up as biased trend/scale effects one level higher up in the nesting structure (i.e., the parameters a_t, a_{tt}, a_y, a_{yy} and b_{ty}). Hence, it is not necessary to include any of these unbiased parameters in the "sufficiently true" Törnqvist index (19). The last term in (19) represents bias due to the level of production/utility, and if factor 1 is capital, b_{y1} = 0.5 would mean that a 1% increase in the production level would increase the capital share by 0.5 percentage points.

4. OLS-estimation of the bias-corrected Törnqvist index

In order to use the bias-corrected Törnqvist price index, one has to know the parameters b_{ti} and b_{yi}. Unless these are somehow known a priori, they have to be estimated econometrically. From successive pairs of two observations of time series data generated with a translog cost function with trend/scale bias, it is not possible to deduct the true bias-corrected price index (as is the case if there is no bias in the data-generating process). The intuition is quite clear: how should it be possible to distinguish whether a movement in a cost share from t-1 to t is due to price substitution, or trend effects, or scale effects, or a combination? With more observations, however, it is possible to reveal the effects statistically.[7]

The above implies that there is no avoiding estimation of a factor demand or consumer demand system, if the true price index is to be known. And estimating a n-factor or n-good translog cost function may sound cumbersome and even error-prone, if the user has limited experience with estimation of multi-equation systems with cross-restrictions between the parameters.

However, given that we are willing to accept a relatively unsofisticated estimation of the share system in (18), it turns out that the estimation can be performed by simple OLS. With two factors/goods, (18) can be estimated easily, but with more than two factors, the problem is more complicated. However, with three factors/goods the system of factor shares can be written as follows (abstracting here from scale effects):

[6] Later on we return to the question of what to do in the consumer case where Y is the (unmeasurable, and thus endogenous) utility level.
[7] See e.g. Diamond/McFadden/Rodriguez (1978).

$$\begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & (\mathbf{p}_1 - \mathbf{p}_3) & \mathbf{0} & (\mathbf{p}_2 - \mathbf{p}_3) & \mathbf{t} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & (\mathbf{p}_2 - \mathbf{p}_3) & (\mathbf{p}_1 - \mathbf{p}_3) & \mathbf{0} & \mathbf{t} \end{pmatrix} \boldsymbol{\beta} \quad , \qquad (20)$$

where $\mathbf{s}_i$ are vectors of cost shares, $\mathbf{p}_i$ are vectors of prices (in logarithms), $\mathbf{t}$ is a vector with the time trend, $\mathbf{1}$ og $\mathbf{0}$ er vectors with 1's and 0's, and $\boldsymbol{\beta} = (a_1, a_2, b_{11}, b_{22}, b_{12}, b_{t1}, b_{t2})$ is a vector of parameters. By rewriting the system in this way, the cross-restrictions on the b_{ij}'s are automatically enforced, and hence the system can be estimated by means of simple OLS. This simple estimator presupposes that the variance of the error terms is equal (and independent) for s_1 og s_2, whichs seems to be a justifiable hyphotesis if the benefit is the ability to calculate the b_{ti}'s in such a simple way.[8] The above method generalizes to $n > 3$ factors, and the b_{yi} parameters can be estimated in the same manner as the b_{ti} parameters (by means of vectors of log(Y) instead of vectors of the time trend).

Hence, permitting the assumption that the error terms have equal variance and no covariance between different cost shares, the a_i, b_{ij}, b_{ti}, and b_{yi} parameters of the translog cost function can be estimated by means of OLS, which in turn means simple matrix operations (inversion and multiplication). Compared to multi-equation maximum likelihood with cross restrictions on the parameters, this OLS procedure is a lot simpler.[9]

It should be emphasized that in the suggested procedure, the number of parameters grows proportionally to the square of the number of factors/goods, so the method is not intended to find the biased trend- and/or scale parameters aggregating dozens of factors/goods. This might exhaust or challenge the degrees of freedom, and might yield implausible parameter estimates.

5. Scale-effects in the consumer demand case

The reader might ask herself what to do when dealing with consumer goods, since in that case Y should be interpreted as the utilitly level (which is unobservable). In order to simplify the discussion, we consider a two-good aggregation with biased income (scale) effects only

$$\Delta\log(P_Y) = \Delta\log(P_{TQ}) + b_{y1}\Delta\log(Y)\log(P_1/P_2)_{-0.5} \qquad (21)$$

In order not to change/confuse the notation, Y is used to denote utility. Given knowledge of Y, the parameter b_{y1} can be estimated from

$$s_1 = a_1 + b_{11}\log(P_1/P_2) + b_{y1}\log(Y) \qquad (22)$$

[8] It does not matter which of the s-equations is suppressed (here s_3), cf. Barten (1969).

[9] A simple AREMOS procedure that computes the bias-corrected Törnqvist index for any number of factors/goods by means of the trick in (20) has been constructed, and is available upon request.

In order to avoid the unobservable utility Y, it can be replaced by $Y \equiv C/P_Y$, where C is total costs (the budget). Hence we get:

$$\Delta\log(P_Y) = \Delta\log(P_{TQ}) + b_{y1}\Delta\log(C/P_Y)\log(P_1/P_2)_{-0.5} \tag{23}$$

If b_{y1} is known from other sources, (23) can be solved for $\Delta\log(P_Y)$, and in that case, the growth of the "true" Törnqvist index is given as:

$$\Delta\log(P_Y) = \frac{\Delta\log(P_{TQ}) + b_{y1}\Delta\log(C)\log(P_1/P_2)_{-0.5}}{1 + b_{y1}\log(P_1/P_2)_{-0.5}} \tag{24}$$

Similarly, in the translog share equation we can use C/P_Y instead of Y:

$$s_1 = a_1 + b_{11}\log(P_1/P_2) + b_{y1}\log(C/P_Y) \tag{25}$$

Since P_Y occurs in (25), the b_{y1} parameter cannot be estimated from this equation alone, but instead one can use the following iterative procedure:

(1) Start out computing P_Y as the standard Törnqvist price index.
(2) Estimate b_{y1} from (25) given P_Y
(3) Compute a new P_Y from (24), given b_{y1} from (2).
(4) Go to (2), using the new P_Y from (3)

Under normal circumstances, the procedure will converge in few iterations. If the b_{yi}'s are known from other sources, we get the following formula in the general case with more than 2 goods and including biased time trends (the generalization of (24)):

$$\Delta\log(P_Y) = \frac{\Delta\log(P_{TQ}) + \sum_i b_{ti}\Delta t\log(P_i)_{-0.5} + \sum_i b_{yi}\Delta\log(C)\log(P_i)_{-0.5}}{1 + \sum_i b_{yi}\log(P_i)_{-0.5}} \tag{26}$$

6. Does the correction matter?

If we again consider the simple two-factor/good system without biased scale effects, the relationship is the following:

$$\Delta\log(P_Y) = \Delta\log(P_T) + b_{t1}\log(P_1 / P_2)_{-0.5} \quad (27)$$

If the cost shares are assumed approximatively constant, this system can be formulated as:

$$\Delta\log(P_Y) = \alpha\,\Delta p_1 + (1-\alpha)\,\Delta p_2 + \beta\,(p_1 - p_2)_{-0.5} \quad (28)$$

where α is the cost share of factor 1, and β is the number of percentage points the cost share of factor 1 rises autonomously per year.

If, for instance, we assume that p_1 and p_2 are IID ~ N(0,1), the first two terms in (28) will be $N(0, 2\alpha^2+2(1-\alpha)^2)$, while the last term will be $N(0, 2\beta^2)$. Hence, the first two terms together have a standard deviation of $\sqrt{(2\alpha^2+2(1-\alpha)^2)}$, which will be a figure between 1 og 1.41 depending on the cost share, whereas the last term has a standard deviation of $\sqrt{2} \cdot \beta$. If, for instance β is estimated to be 0.005, i.e. a relatively large increase of 0.5%-points per year, the standard deviation of the last term is the modest figure 0.007. So the standard deviation of the last term is more than 100 times smaller than the standard deviation of the first two terms together. For reasonable biased trends, we might generously assume max 2%-points per year, implying 80%-points over 40 years. And even in that case, the first two terms will overshadow the last term.

However, this conclusion is invalidated if the p's are non-stationary. Or more precisely if the ratio $(p_1–p_2)$, corresponding to $\log(P_1/P_2)$ includes a (stochastic) trend. If this is so, the last term in (28) might be of significance, provided that the trend in the relative price, and the parameter b_{t1} are both significant enough. Still, it should be noted that given e.g. 40 observations, it is quite difficult to obtain a significant bias effect. If, as above, we assume that $bt_1 = 0.005$ (being a rather large figure), and that the relative price changes by 2% per year (again a rather large trend in the relative price), the ratio P_1/P_2 will change by a factor 2.2, and $\log(P_1/P_2)$ by 0.79 over 40 years. The contribution from $bt_1 \cdot \log(P_1/P_2)$ will thus change by 0.004 (i.e. a total rise in the Törnqvist price index of 0.4%) over the 40 years; an effect that can easily vanish in the noise from the first two terms in (28).

The above considerations carry over to the question of biased scale effects. In the same manner as was the case regarding the biased trend effect, in many cases a biased scale effect may tend to disappear in the noise from the factor prices, unless there is a strong trend in the factor price ratio.

7. Empirical illustration

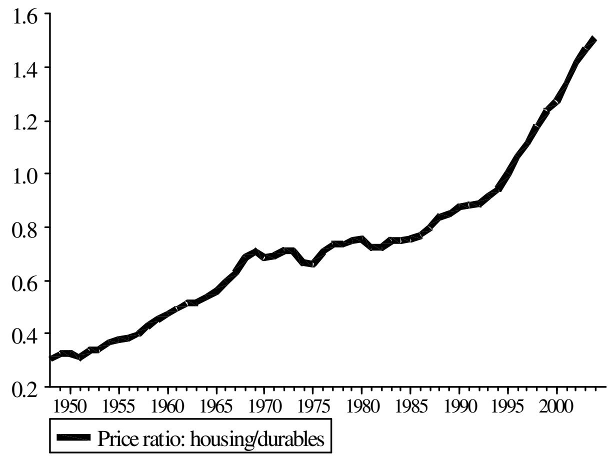
In order to find an example of a significant bias-effect on Danish macro data, I have investigated pairwise aggregation of production factors (combinations of the five production factors in the macroeconomic model ADAM), and similarly for consumer goods (there are 11 of these in ADAM, excluding leisure).[10]

So the experiment has been to investigate $(5 \cdot 4)/2 = 10$ combinations of production factors, and $(11 \cdot 10)/2 = 55$ combinations of consumer goods, in order to find examples where the bias-correction is of significance.

To keep the experiment simple, I have focused on trend bias only. In the consumer case, an income effect would be the standard way to incorporate a bias, but real income ($Y \equiv C/P_Y$) is typically quite trended anyway, and the experiments are for illustrative purposes only.

To judge the impact of the bias-correction, two indices with and without correction have been compared, for each of the above-mentioned combinations. The largest effect, by far, was measured when aggregating the consumption of housing relative to the consumption of durable goods. The price ratio between these types of goods is quite non-stationary, as seen below, reflecting the fact that price of housing has grown on average 7% over the period, whereas the price of durables has grown by approximately 4% over the period (the price level even decreases after 1994).

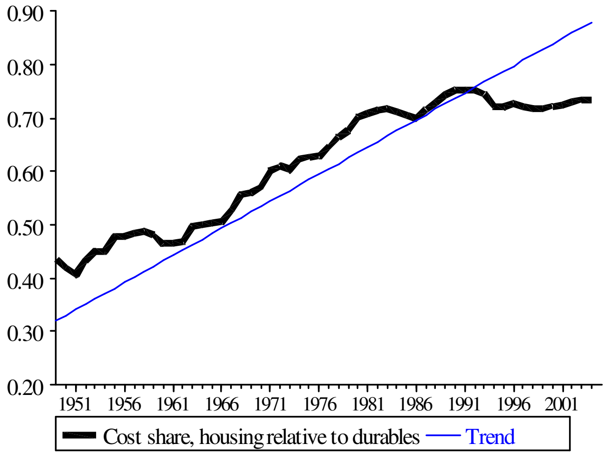
Figure 1. Historical price ratio between housing and durables



The cost share of housing versus durables is seen below:

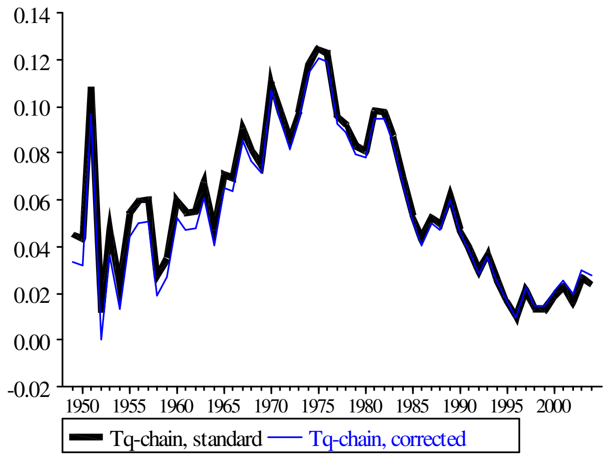
[10] The production factors are buildings, machinery, labout, energy and materials. The consumer goods are food, stimulants, other non-durables, heating, gasoline, cars, durables, housing, collective trasport, other services, and tourist travel.

Figur 2. Cost share housing/durables, and estimated trend



In the figure, the estimated trend is also depicted, being of a magnitude of close to 1%-points per year (this is the largest biased trend measured between any pairs production factors or consumer goods). Such a strong trend combined with a non-stationary relative price ratio does indeed produce a difference between the standard and bias-corrected Törnqvist index:

Figure 3. Standard and bias-corrected Törnqvist indices



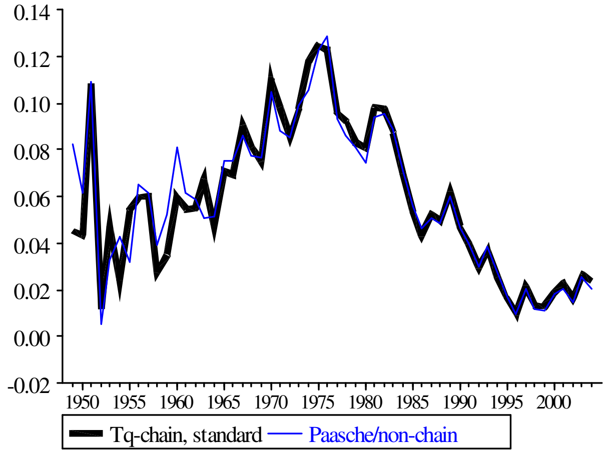
The thick curve is log-differences of the standard Törnqvist price index aggregating housing and durable goods, wheras the thin curve is the bias-corrected index. As it is seen, there *is* an effect, but hardly enough to revolutionize any parameter estimates based on this index higher up in a nesting structure.

In all other of the investigated combinations of aggregations, the effect is smaller. The next-largest correction-effect among the consumer goods is only about half the magnitude of Figure 3.

In the factor demand case, the differences are even smaller, and the largest biased trend measured between the production factors was between machinery and labour, amounting to some 0.4%-points per year.

To put the bias-correction in perspective, the standard Törnqvist chain index can be compared to a Paasche fixed-base (non-chain) price index as used in ADAM until recently.

Figure 4. Standard Törnqvist and Paasche (non-chain) indices



Moving from a Paasche fixed-base to a Törnqvist chain index seems to matter more or at least as much on the aggregate price index, as correcting the Törnqvist index with respect to trend bias.

8. Conclusion

The paper has shown that it is relatively simple to correct the Törnqvist price index with regards to biased trends and/or scale effects. The correction formula is simple and gives insight, but depends upon some ”deep” parameters describing the annual trend in the cost shares and/or effects of production/utility level on the cost shares.

The “deep” parameters are typically not known a priori, but even if they are not known, one can use the correction formula to judge what kind of bias to expect given different (sensible) values for the deep parameters. In many cases, as shown in section 7, the correction does not amount to much, and so a traditional Törnqvist price index may be a very good approximation to the true index.

As shown in section 4, it is possible to obtain relatively good estimates of the above-mentioned trend- and scale parameters by means of simple OLS; that is, by simple matrix operations. But the conclusion is that even though this procedure is relatively straightforward, in most cases – and as it is shown in section 7 even in the presence of relatively strong bias effects – the bias correction is not that large. Or at least this is what was found on Danish annual macro data.

So it seems that when estimating nested production or consumption functions, in many cases one can replace the true price index from a lower nesting level (e.g., a CES price index) with a standard Törnqvist index, even in the presence of trend- or bias effects at the lower nesting level. This implies a tremendous simplification regarding the estimation of such systems, since it makes possible independent estimation of the different nesting levels.

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