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MODELGRUPPEN

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Rational Expectation in the Housing Model

Resumé:

In this paper we examine the possibility of rational expectation in the housing sub-model. To broaden our understanding of the sub-model, we first introduce different restrictions and analyze the model properties. Later we analyze the dynamic properties of the sub-model with rational expectation. We also provide a review of foreign country models with rational expectation.

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Key words: Rational expectation, inflation, housing model

0. Introduction

Expected inflation is central part of the overall structure of ADAM. The forms of expectations will have different implications on the overall model properties. Expected inflation in ADAM affects investments via user cost. Due to the technical requirements[1] of large scale models with rational expectations, we focus on the housing sub-model and consider three options: exogenous expectation, adaptive expectation (AE) and Rational Expectation (RE). In order to better understand the dynamics involved in the housing model, we start out with exogenous expectation and different other restrictions in the sub-model.

1. Expectation formation

The Adaptive inflation expectation in ADAM for the housing model is described as

$$rpibhe_t = \theta \cdot (\frac{pibh_t}{pibh_{t-1}} - 1) + (1-\theta) \cdot rpibhe_{t-1} \ , \ \ 0 < \theta < 1 \qquad (1)$$

Where, *pibh* is price of investment in housing, $rpibhe_t$ is expected inflation. Equation (1) simply says that current expectations of future inflation depend on past expectations and an error adjustment term *(1-θ)* in which current expectations are adjusted according to current actual inflation. The model property will depend on the values of θ and the lag structures in (1), these are explored in TMK12901.

In the isolated housing model *pibh* is exogenous, unless we make some arrangements having (1) in the sub-model would be the same as having exogenous expectation. Even in the entire model *pibh* does not change much, and hence it would be natural to base expectations on market price of houses, *phk*. Thus we use the following AE in the isolated model

$$rpibhe_t = \theta \cdot (\frac{phk_t}{phk_{t-1}} - 1) + (1-\theta) \cdot rpibhe_{t-1} \ , \ \ 0 < \theta < 1 \qquad (2)$$

On the other hand, the theory of rational expectation assumes that expectations are model-consistent and since we have no stochastics it is equivalent to saying perfect foresight. Agents do not make systematic errors in their forecast. A pure forward looking expectation can be written as

$$rpibhe_t = E_t(\frac{phk_{t+1}}{phk_t} - 1) \qquad (3)$$

Equation (3) postulates that on average peoples inflation forecast will be correct and is equal to future values of inflation. An important implication of

[1] PCIM embeds the traditional Fair-Taylor method for solving models with RE, however, the technical requirements are quite involved if one considers the entire model. Such methods are not yet implemented in GEKKO.

(3) is that expected inflation is independent of lagged inflation. This specification can be criticized on the ground that it does not account for the important role played by lagged dependent variable in inflation regression. Alternatively, a hybrid specification as suggested for instance in Sørensen and Whitta-Jacobsen (2005) can be written as

$$rpibhe_t = \delta \cdot (\frac{phk_{t-1}}{phk_{t-2}} - 1) + (1-\delta) \cdot E_t(\frac{phk_{t+1}}{phk_t} - 1) \quad , \quad 0 < \delta < 1 \qquad (4)$$

Expected inflation depends on a weighted sum of its lag and its rationally expected future value.

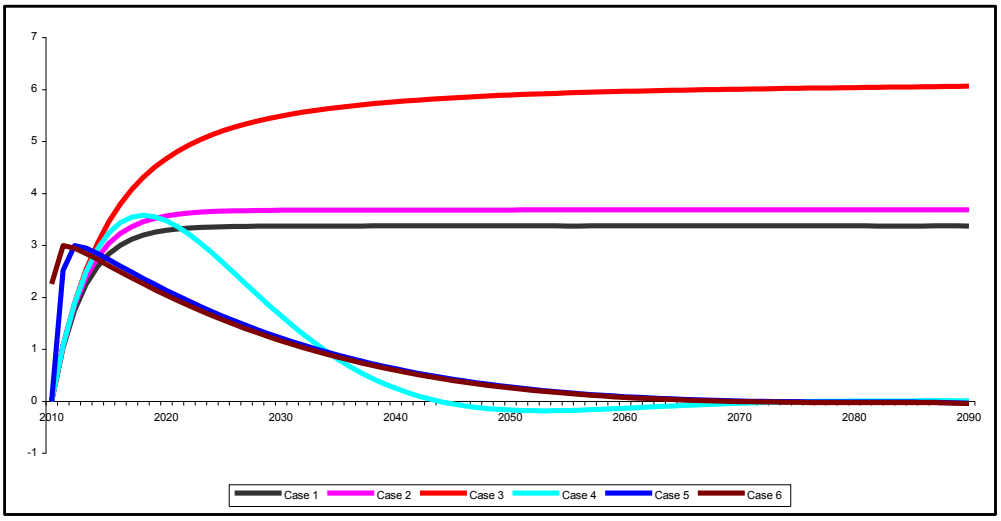
2. Multiplier Analysis

A. The housing model and RE

The reaction of the housing model to standard multiplier experiments (such as government purchase) depends on whether the sub-model is considered alone or together with the rest of ADAM. This is due to the fact that some important variables will appear exogenous in the isolated housing model while they are endogenous in the whole model. To overcome this problem we expand the housing model by including the relations for consumption and wealth. For better understanding, we consider different restrictions in the sub-model such as exogenous inflation expectation and excluding important relations from the sub-model, see below. We then analyze the implications of RE in the sub-model.

The experiment in all cases is a permanent 1 percent increase in wanted capital stock of houses, *fkbhw* (2010-2090). Figure 1 demonstrates the effect on house price (*phk*).

Figure 1. Effect on house price (in pct.), 1 % permanent increase in *fkbhw*



Case 1
This is the most restricted of all the cases. Expectation formations, stock of housing capital, consumption and user costs are exogenous. Here the only effect on house price comes from the increase in housing demand and no indirect effects. Price increases a year after *fkbhw* is increased, this is due to the lag structure in the price equation. In the following years the higher housing prices reduce the demand for houses. The falling demand reduces the growth of prices. Gradually *phk* stabilizes on a higher price level.

Case 2
This is case 1 with endogenous user cost. The initial rise in house price drags down the relative tax in user cost and the fall in user cost raises house price further. As a result house price stabilizes on a higher level than case 1 above.

Case 3
Here we add endogenous consumption to case 2. Besides user cost, there will be a wealth effect on house price. That is the initial increase in house price will raise consumer's wealth, which among other things raises consumption and the increase in consumption exerts upward pressure on house prices. Due to the wealth effect the effect on *phk* is higher than the case in 2 and 1 above. In this model there is no response from higher consumption to lower wealth. This would have decreased the effect on *phk* over time. With no response in the supply of houses, *phk* continues to rise in the long run.

Case 4
Here we relax the restriction on supply of houses (*fkbh*) in case 3. In this case supply of houses responds to the rise in price of houses, and the increase in supply brings down house price. This is the classical Tobin's q. Note that this case can be compared with the AE in ADAM. Since *pibh* does not change in the isolated model it amounts to having exogenous expectation.

Case 5 and 6
So far we had exogenous expectation in the housing model. Case 5 and 6 add a pure forward looking expectation to case 4. In the fifth scenario we assume agents are unprepared in 2010 when shocks start and have no form of expectation, but as of 2011 they have RE. In the sixth scenario we assume agents are prepared and have RE in the whole period (2010-2090).

Under RE agents have full understanding of the working model, when housing demand increases they know it is will not affect house price before next year. Consequently, under case 5 house prices will not increase before 2011, but under case 6 since agents are prepared house price increases already in 2010 as consumers start buying houses in 2010. The difference between these two cases is that consumers react one year before the shocks take effect in case 6 and in the long run they tend to converge, see figure 1.

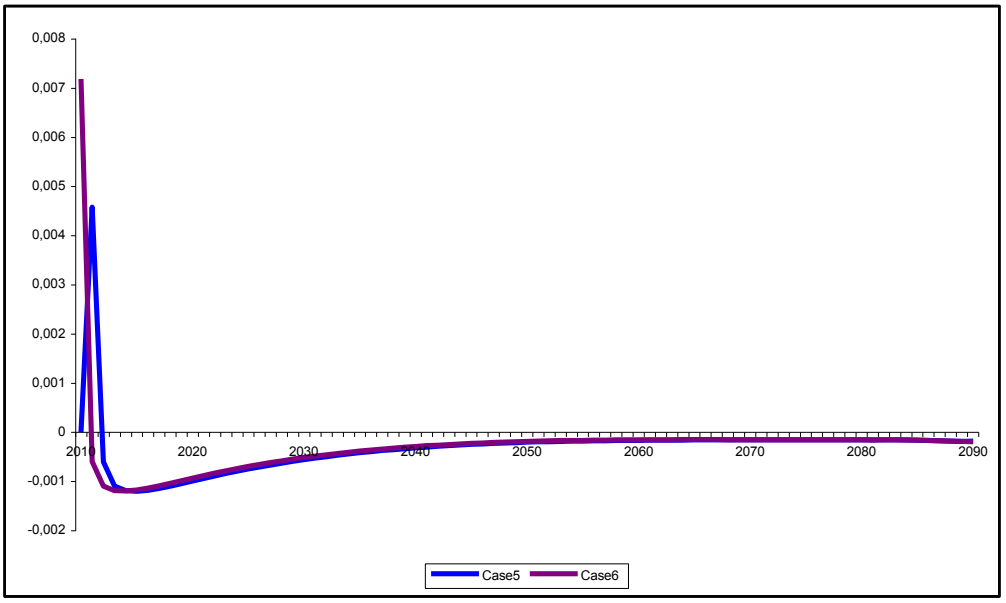
Recall case 4, the percentage increase in *phk* reaches maximum after some times and starts coming down when the effect from Tobin's q dominates the wealth effect. Under RE, since agents know this in advance they start selling houses before *phk* actually starts declining, these will reduce *phk* earlier than it

would with no form of expectation. The sharp turns in price can be explained as follow. The initial jump in *phk* increases wealth which in turn increases price in the following year. After the second year the downward pressure on prices from supply of houses dominates the upward pressure on prices from wealth, the net effect will be a fall in price. In general, with RE there is smooth convergence back to equilibrium and less fluctuation in price.

B. Expected inflation under RE

Figure 2 shows the effect on expected inflation when agents have RE. The sharp tern in *phk* in case 5 and 6 is reflected in expected inflation. Expected inflation first rises then falls and becomes negative after which it continually increases back to equilibrium.

Figure 2. Effect on expected inflation (in diff.), 1 % permanent increase in *fkbhw*

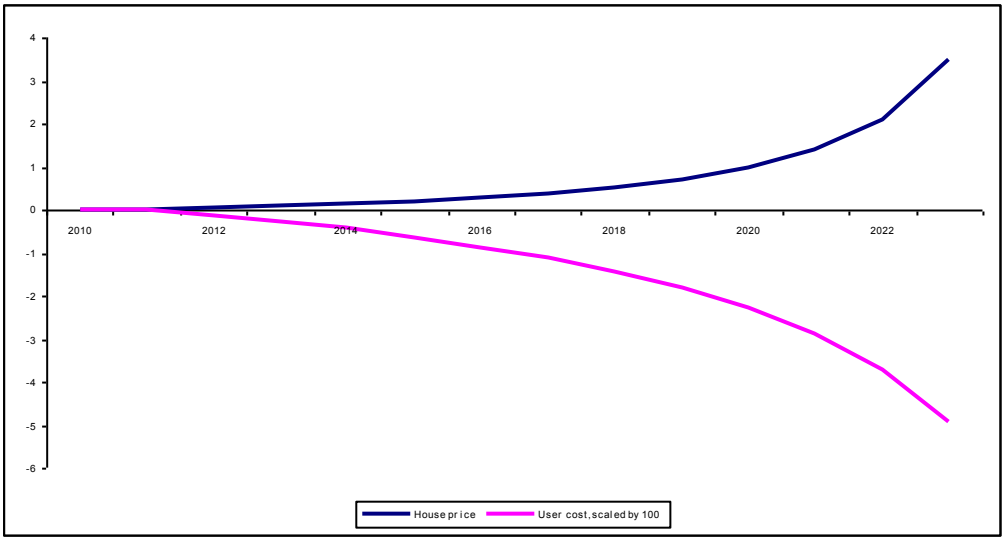


C. Adaptive and hybrid expectation

We have so far focused on RE and different restrictions in the housing model. It would be natural to see the same experiment under adaptive and hybrid expectation, we start with the former.

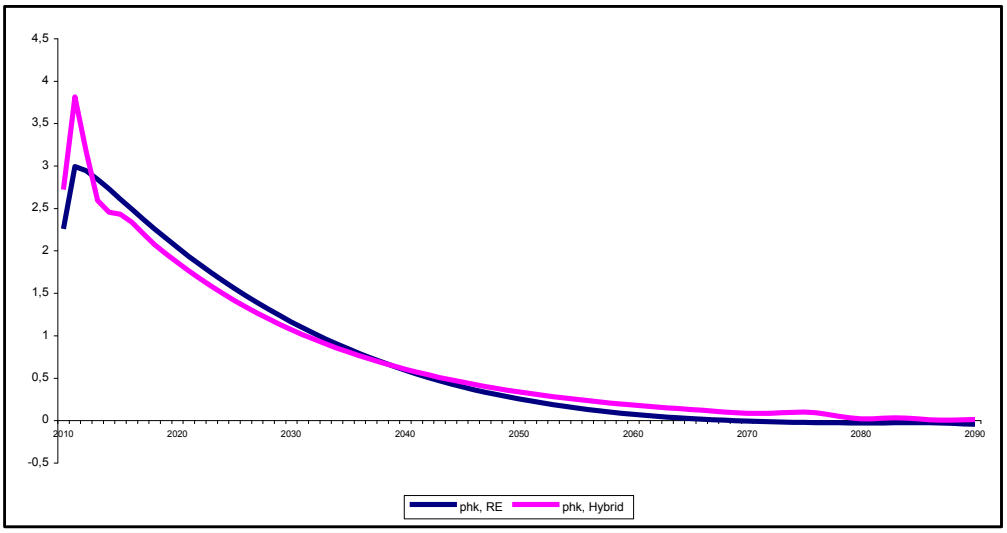
When expectation is adaptive (as in eq. 2) house price is unstable in the long run. This is because the initial increase in *phk* raises expected inflation, the combined effect will be a reduction in user cost. A reduction in user cost in turn raises *phk.* House price and user cost reinforce each other and in the end user cost becomes negative leading to a housing bubble, due to this the model fails to solve after some time, figure 3 demonstrates.

Figure 3. Effect on house price and user cost (in pct.), 1 % permanent increase in *fkbhw*



Finally, figure 4 compares the hybrid model with $\delta = 0.2$ to the pure forward looking expectation. We have seen AE leads to housing bubble, when this is combined with RE, the net effect is a higher initial increase in house price compared to the pure RE. The higher the value of δ the higher is the initial over shoot. There is also a slight tendency for prices to take longer time to reach equilibrium with hybrid expectation.

Figure 4. Effect on house price (in pct.), 1 % permanent increase in *fkbhw*



3. Rational Expectation in Foreign Country Models

Because of the technical requirements in simulating large scale models with rational expectations, we have only considered the housing model in ADAM. By contrast the following models implement the REH in a large model. The purpose here is not to make a comparison as it would be inappropriate since the theoretical setups and the underlying assumptions used in the different models are different. It is rather to illustrate how the assumptions of RE changes the dynamic properties of each models with a view to our own analysis of the housing sub-model.

A. The US macro model[2]

The Federal Reserve Board (FRB) has had different variants of macro models at different times. FRB/WORLD is the most popular of these models. FRB/WORLD is a version which links the foreign Multi-Country Model, FRB/MCM, with the domestic economy model, FRB/US.

FRB/WORLD ascribes an important role to expectations. Recognizing that no single assumption regarding the formation of expectations is likely to be appropriate in all circumstances and that it may be useful to see how different specifications in this regard affect system properties, this model has been designed to have the flexibility to be simulated under alternative assumptions about how expectations are formed. Two options are considered: adaptive and rational expectations. The different expectation formations are in particular, implemented in FRB/US.

Adaptive expectations are implemented by assuming that all agents share a common small vector autoregression (VAR) model of the economy that includes past observations of inflation, interest rates, output, and long-run expectations of inflation and interest rates. Under rational assumptions expectations are generated using the predicted values from the model itself. Solutions under RE are implemented using a variant of the extended Fair-Taylor method. The future path of exogenous variables is assumed to be known in advance. For ease of computation, a loglinearized version of the U.S. model is made that can be solved efficiently using Anderson and Moore's (AIM) implementation of the Blanchard-Kahn solution method. Figure 5 demonstrates the effect of a 1% of GDP increase in government purchase under adaptive expectation/VAR expectation and rational expectation.

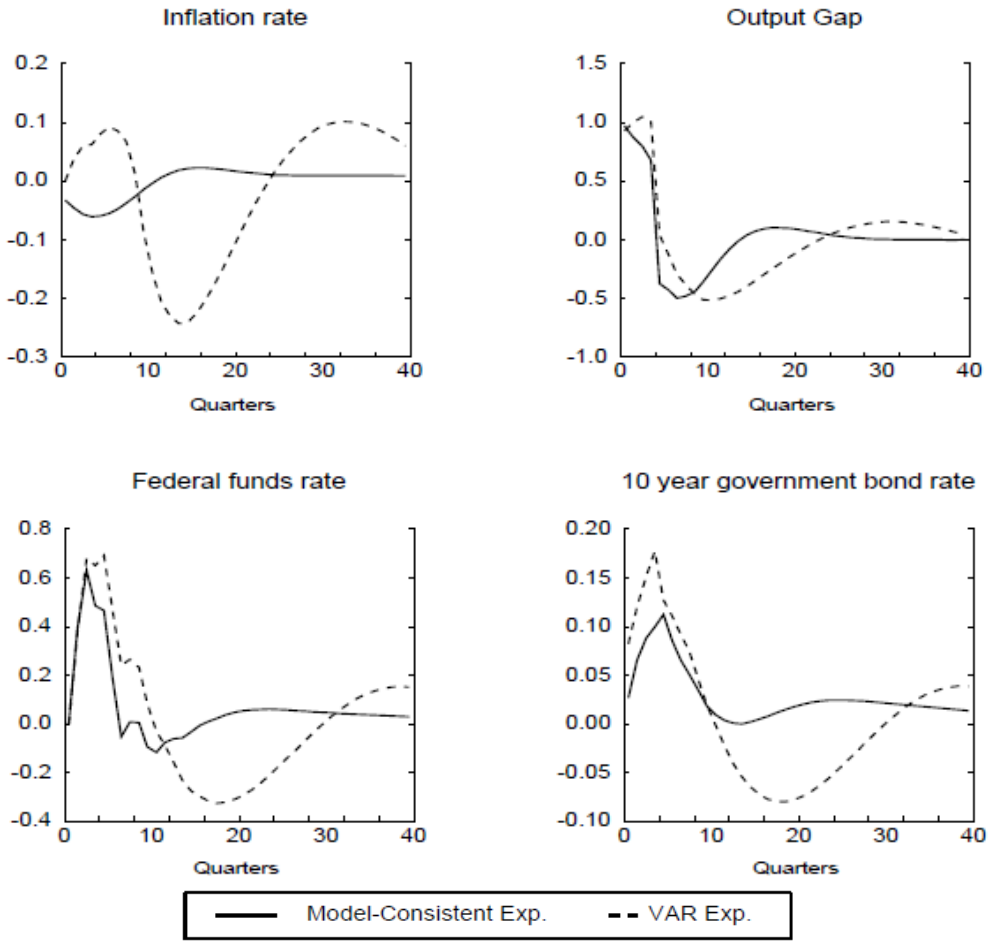
Looking at the figure, the simulated outcomes for the output gap and short and long interest rates are quite similar under the two expectations assumptions. However, the peak reduction in inflation under VAR expectations is about three times that under model-consistent expectations.

The exogenous rise in aggregate demand is initially reflected one-for-one in output. Much of the initial surge in activity is generated by productivity

[2] For a detail review see Brayton et al. (1997). The Evolution of Macro Models at the Federal Reserve Board. Federal Reserve Board, Washington, D.C. 20551.

improvement due to the sluggish adjustment of hours. The high level of activity leads to an increase in interest rates. The cessation of the spending shock then drives output significantly below baseline as the effects of the rise in interest and exchange rates start to take hold. This overshooting of output is then corrected over time.

Figure 5. Four-quarter, 1% of GDP increase in government purchases



The inflation response under rational expectations is *negative* in response to the boom in spending. This is due to the forward-looking nature of price and wage setting. Although unemployment falls below baseline during the year of the shock, and rises above baseline thereafter, the weighted sum is positive. Thus a small transitory reduction in inflation results. If the shock were to last longer, the initial response of inflation would in fact be positive because the weighted sum of future unemployment would be negative. The pro-cyclical pattern of inflation observed in the case of VAR expectations can be traced to expectations errors. In general, RE yield less over shooting and fluctuation.

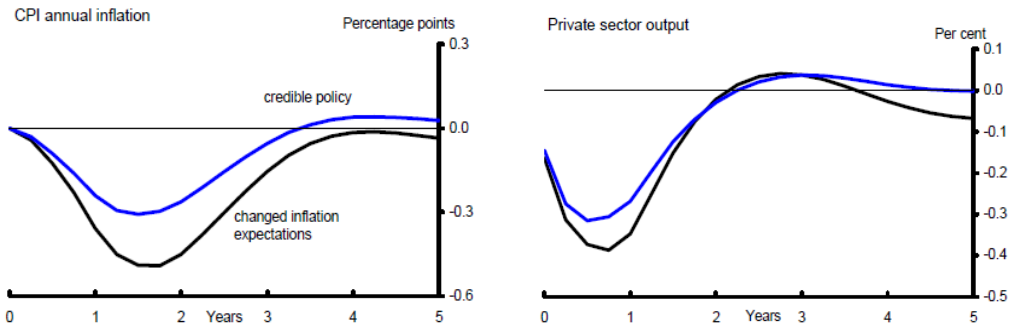
B. The Bank of England Quarterly Model (BEQM)[3]

BEQM contains explicit forward-looking representations of agents' expectations about the future. These include expectations about future labour income, aggregate demand, the exchange rate, and so on. Models with fully forward-looking agents can exhibit unrealistic dynamic properties if agents are assumed to have perfect foresight, they might adjust their behaviour immediately in response to future anticipated events. But in reality the economy does not 'jump' about in this fashion. That partly reflects the fact that it is often costly for households and firms to change their behaviour very rapidly. In addition, firms and households do not have perfect foresight. Instead, they have to form expectations on the basis of limited information. BEQM incorporates both of these features. In particular, it is structured in such a way that assumptions about the speed of adjustment and the amount of information available to agents can be changed in order to help policy makers to assess how these assumptions could affect the future path of the economy.

Figure 6 illustrates the effect of a 1 percentage point rise in nominal interest rates for four quarters. In BEQM interest rates react endogenously to movements in inflation and output that come about because of some other shock. Here shocks instead start from interest rate and propagate to the rest of the model.

The responses of CPI inflation and private sector output accord with the theoretical setups of the model. Following the unanticipated monetary policy tightening, output contracts, profits and real wages fall, and inflation (eventually) falls.

Figure 5. Effect of a nominal interest rate shock, a 1 percentage point rise in four quarters



The blue line is based on the assumption that the unexpected change in interest rates does not affect agents' long-run inflation expectations. But the response of the economy to a change in interest rates depends on the credibility of the inflation target. In particular, as inflation expectations become more firmly anchored around the inflation target – the target becomes more credible – a change in the short-term interest rate is likely to have less impact.

[3] For a detail reviews see Harrison R. et al. (2005). The Bank of England Quarterly Model, publications group, Bank of England.

The black line shows the effect on inflation and private sector output when agents wrongly perceive that the unexpected increase in interest rate may have been triggered by a reduction in the targeted rate of inflation. It is based on the assumption that agents revise down their expectation of the targeted rate of inflation and expect a prolonged period of tighter policy in order to achieve that perceived lower target. It illustrates the sensitivity of the simulation exercise to assumptions about expectations.

The response of inflation is much sharper when agents believe the target has been reduced, reflecting the effects of lower expected inflation on price setting. After the first four quarters, the absence of further policy surprises (relative to an unchanged target inflation rate) leads agents to correct their expectations gradually towards the true target. The effect on real variables is somewhat smaller but consistent with that on inflation: the initial fall in private sector output is bigger, though there is little difference between the two lines by the end of the third year.

4. Conclusion

This paper has taken a step by step approach to understanding the housing model and examined the theory of rational expectation in the sub-model. Under RE convergence back to equilibrium is smoother and there is less over shooting than the case with AE. This is also observable in the foreign country models. Over all, the assumption of RE does not seem to significantly lower the crowding out time in the model.

Appendix

Rational expectation and stability: the housing sub-model

The paper *jnr07mar11* shows the lower the price elasticity, b2, is in the house price equation the more unstable is the housing model.

```
log(fKbhw)   = Log(Cpuxh/pcpuxh)
             + b1/(1+(exp(t1*tid+t2)/exp(4.3))**(-25))
             + b2*log(pcpuxh/(buibhx*phk))
             + b3 ;
dlog(phk)      = a1*dlog(fcpuxh)
             + a2*diff(buibhx)
             + dlog(pcpuxh)
             + a3*(Log(fKbh(-1))-log(fKbhw(-1)))
             + a4*d06
             + a5*(rho-led)
```

A rational expectation ensures a reasonable degree of stability and faster crowding out in the housing model with lower values for *b2,* see the figures below.

Multiplier Experiment

The experiment is: **UPD jrfkbhw 2010 2108 + 0.01**

Figure 1. Effect in house price, *phk*, in pct.
Expected inflation, rpibhe = phk(+1)/phk-1

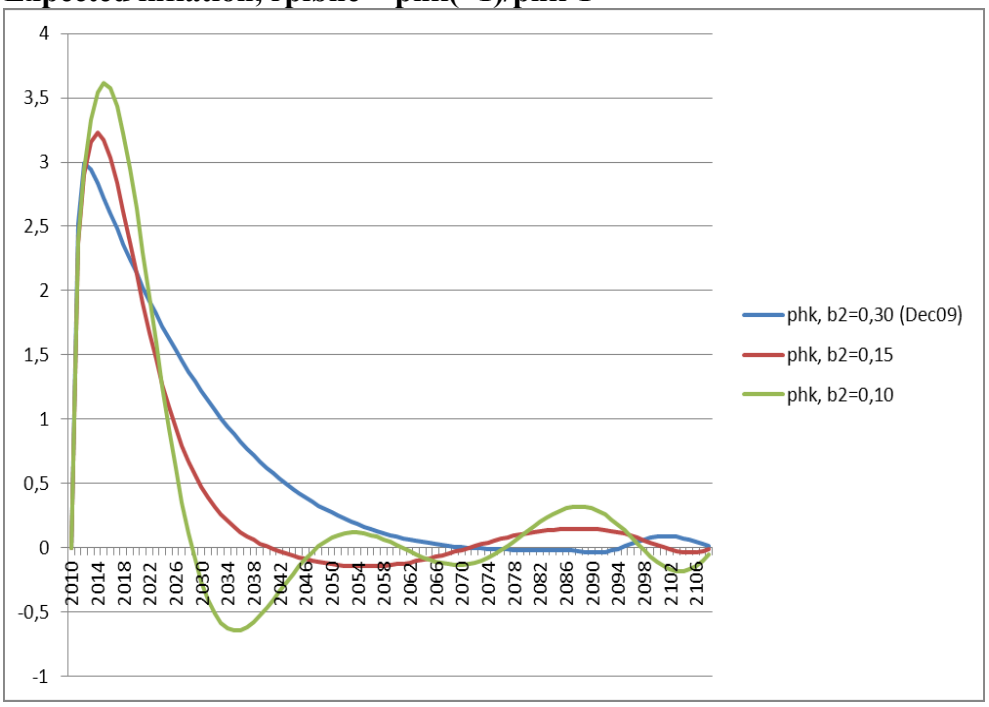


Figure 1. Effect in house price, *phk*, in pct.
Expected inflation, rpibhe = ((phk(+10)/phk)0.1)-1**