Danmarks Statistik
MODELGRUPPEN

Arbejdspapir*

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Housing model – a cointegration analysis

Resumé:

The paper uses a cointegration analysis for modelling the demand and supply for housing. It is shown that the stock of houses can be better characterized as integrated of order two, and valid statistical inferences can be derived using the statistically well-developed I(1) model by taking a nominal-to-real transformation of the I(2) vector. We document two long term relations – a demand relation between the volume of housing, real house price, income and user cost rate, and a supply relation defining a polynomial cointegration between replacement ratio (Tobin's q) and net investment. The multiplier exercise shows ADAM's housing model behaves no different from a VAR -model.

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1. Introduction

The study of the housing market is always important for a number of reasons. First housing represents households biggest single purchase and constitutes the largest single item of consumer wealth. A rise, for example, in house price creates a positive wealth effect that leads to a significant boost to consumer spending. Second although residential investment constitutes a small share of domestic demand, it is highly volatile and can lead to a recession, cf. Leamer (2007).

Just like any good or service, the interaction of demand and supply determines the equilibrium quantity and price of houses. Nevertheless, the housing market is characterized by inefficiencies and adjusts slowly to market conditions. Disequilibrium in the housing market can originate from shifts in demand and/or supply conditions. Demand side shocks can be triggered by changes in demographics, income or the behaviour of monetary and fiscal authorities that alter the course of interest rates, marginal tax rates or inflation targets, Arestis and González (2013). Changes in construction costs and employment as a result of strikes or labour disputes can generate supply side shocks, Riddel (2004).

In this paper we model the demand and supply for houses in the Danish housing market using the cointegration technique of Johansen (1996). Previously a cointegration analysis of the Danish housing market has been carried out, among others, by Knudsen (1994) and Skaarup and Bødker (2010). The present paper builds on these studies and makes a methodological contribution. By using an information set that is consistent with the housing model of ADAM (Danmarks Statistik, 2013), we have found two cointegrating relations – a demand relation relating the volume of housing to house price, income and user cost rate and a supply relation that defines a polynomial cointegration between the replacement ratio (Tobin's q) and net investment. The demand relation shows that the house price responds more to user cost rates than to income or stock of houses, which points to the lack of efficiency in the housing market. The comparison of the VAR model and the housing model in ADAM uses a multiplier analysis and shows a great deal of similarity between the two. Because the estimated VAR model does not encompass endogenous relations for consumption or wage formation, among others, we cannot compare it to the overall ADAM. A VAR analysis of the housing model that interacts with macro metrics such as unemployment is the objective of a follow up paper to this one. The remaining part of the paper is organized as follow: section 2 provides a brief theory of demand and supply for housing, section 3 presents the dataset, section 4 provides a brief review of Johansen's methodology, section 5 presents the empirical analysis, and section 6 concludes.

2. Theory

The economic framework is based on a simple demand and supply relation for the housing market. The theory of consumer behaviour indicates that the demand for a good or service should be a function of income and of the price of the good or service relative to the price of substitutes. Accordingly, we relate the volume of housing (dwelling stock) positively to household income and negatively to the user cost of dwellings relative to the price of substitutes:

$$h_t = \theta c_t - \beta_1(ph_t + UC_t - pc_t), \beta_1, \theta > 0 \quad (1)$$

Where h_t is dwelling stock, c_t is private consumption which is here used as a measure for income, ph_t is the cash price of houses, pc_t is the consumption deflator which is a proxy for the price of substitutes for housing, and UC_t is the user cost rate. The product $ph_t \cdot UC_t$ translates house prices into an annual rate paid by the owner, cf. Danmarks Statistik (2013). The user cost rate is represented by the nominal bond yield after tax, a residential tax rate, an assumed depreciation rate of 1% and expected inflation. For expected inflation, we use the HP-filtered inflation rate, Δpc_t, see also Knudsen (1994), Danmarks Nationalbank (2003) and Skaarup and Bødker (2010). Equation (1) determines the long term demand for volume of housing.

The supply side describes a stock adjustment equation, where residential net investment is positively related to the ratio between the price of existing houses and construction cost:

$$\Delta h_t = \beta_2(ph_t - pi_t), \beta_2 > 0 \quad (2)$$

Equation (2) corresponds to the classical Tobin's q model, Tobin (1969). When house prices are higher than construction cost (here investment prices pi_t), it pays off to build new houses and residential investment increases. Equation (1) and (2) are candidates for long run cointegrating relations.

3. The data and time series interpretations

The theoretical model suggests the information set $Y_t = (h_t, c_t, ph_t, pi_t, pc_t, UC_t)'$. Figure 1(A-F) shows the data and important linear combinations. The data are seasonally adjusted and covers the period 1975q1 to 2013q4, lower case letters indicate log transformed values with average 2010 = 0.[1] Panel (A) shows the stock of houses and private consumption, there is an indication of a long-term co-movement between the two. (B) shows the three prices in (1) and (2). Investment prices and consumption deflator tend to move together over time. This is not surprising as consumption prices reflect the overall price movements in the economy. A similar co-movement between house prices and consumption deflator is not obvious from (B). The user cost rate and a measure for inflation are shown in (C) and (D), respectively. The 1987 tax reform that reduced the tax rate applied to interest payments is visible in the user cost rate. It seems also reflected in house prices. House prices showed no growth from the mid-1980s to the early 1990s. In the empirical section we account for this effect by including a level shift dummy. The negotiation for the tax reform in fact started in 1985, in the empirical analysis below we found evidence of a level shift in *1986q1*. (E) shows the relationship between the ratio of housing stock to consumption (with assumed $\theta = 1$) and real house prices. The correlation between the two ratios is very clear, except for a few breaks, for example in the beginning and end of the 1990s. Finally, (F) shows the supply relation (2), as described above, the higher the house prices relative to construction costs the higher will be net investment.

Whether we should model the nominal vector Y_t as integrated of order one, I(1), or order two, I(2), is not straightforward from the outset. For example, the first difference of the housing stock Δh_t displays persistent movements over the whole sample periods, *i.e.* Δh_t can better be characterized as I(1) process. If we assume h_t is I(1), the coefficient β_2 cannot be identified in equation (2), unless the replacement ratio ph_t/pi_t is I(0), which is not the case judging from figure 1(F). Similarly, the nominal prices exhibit persistent movements and hence can better be approximated as I(2) process, cf. Juselius (2006).

[1] The data is obtained from the national bank's quarterly model MONA, cf. Danmarks Nationalbank (2003).

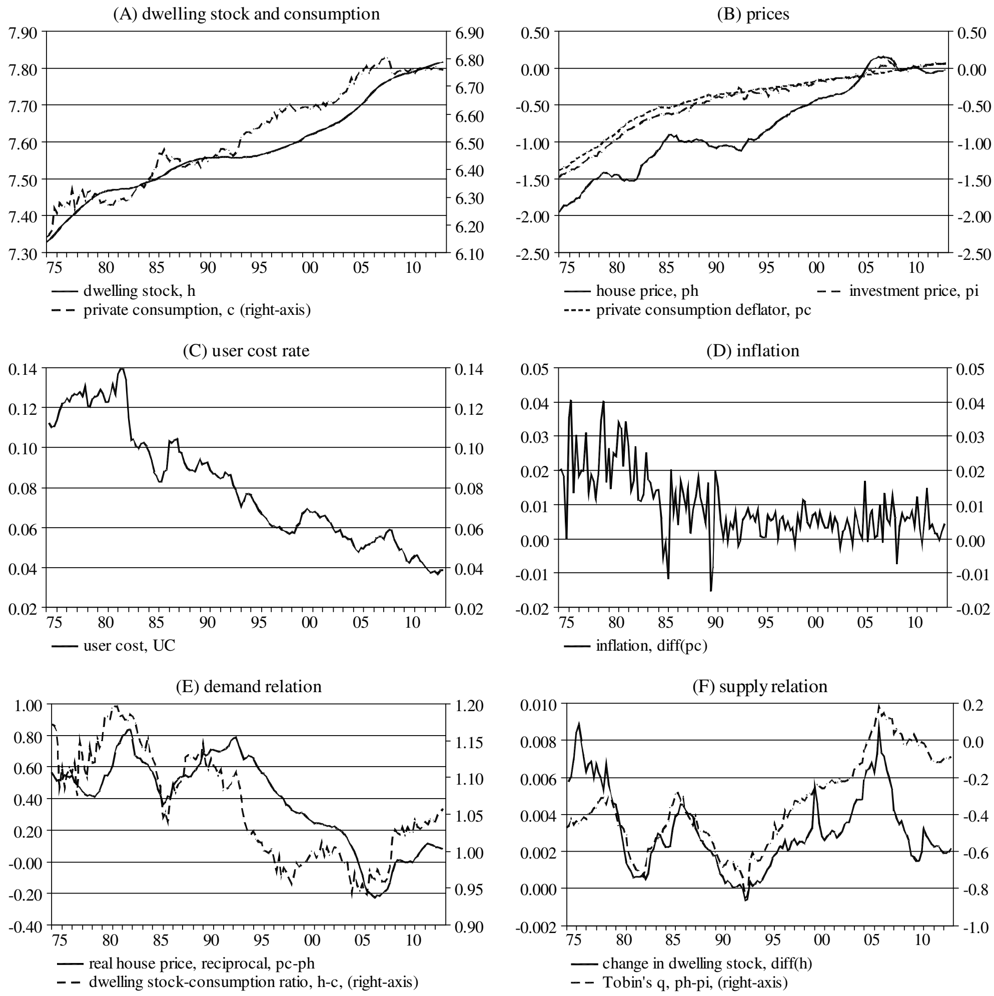

Figure 1. Data and linear combinations, lower case letters indicate logs, average 2010=0

Although the I(2) model can be a better choice for modelling Y_t, it is statistically involved and complicated, instead we consider the I(1) model that can be obtained by transforming the nominal vector into a real vector. Following Danmarks Nationalbank (2003) and Danmarks Statistik (2013), we transform the nominal vector Y_t to a real vector $X_t = [(h-c)_t, \Delta h_t, (ph-pc)_t, (pi-pc)_t, \Delta pc_t, UC_t]'$. Both the nominal and real vectors are of dimension $6x1$. The working hypothesis is that there are two I(2) trends in the nominal data – one driving the stock of houses and consumption and the other driving the price variables. By taking ratios we cancel both I(2) trends, and to avoid loss of information we also include the first differences of the stock of houses and the consumption deflator. The transformation can be tested by estimating an I(2) model for Y_t and testing for long run homogeneity.[2] The following section provides a brief review of the I(1) model to be used for modelling X_t in the subsequent empirical section.

[2] The I(2) model with two or more I(2) trends is statistically involved and it's difficult to uncover stationary long run relations. Instead we have tried estimating two I(2) models by splitting the nominal information set into two parts - one for the demand relation $Y_{1t} = (h_t, c_t, ph_t, UC_t)'$ and another consisting of nominal prices $Y_{2t} = (ph_t, pi_t, pc_t)'$. There is evidence of long run homogeneity between the stock of houses and consumption in Y_{1t}, and price homogeneity in Y_{2t}. Note the cointegration relation is invariant to changes in the information set (Juselius, 2006). If the homogeneity holds in the smaller information set, we also expect it to hold in the larger information set.

4. The econometric approach

The empirical analysis is based on a VAR(k) model for the p-dimensional vector X_t, which can be re-parametrized in changes and levels as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \mu_1 t + \varphi D_t + \varepsilon_t \tag{3}$$

Where $\varepsilon_t \sim iid(0, \Omega), t = 1, \dots, T$ and Ω is the covariance matrix of ε_t, $X_t = [(h-c)_t, \Delta h_t, (ph-pc)_t, (pi-pc)_t, UC_t]$', and the initial values $X_{-k+1}, \dots, X_0$ are considered fixed. Note that we have dropped the inflation variable from the real information set, partly to simplify the empirical analysis and partly because it is included in the user cost rate in the form of expected inflation. The k matrices of autoregressive coefficients $(\Pi, \Gamma_1, \Gamma_1, \dots, \Gamma_{k-1})$ are each of dimension pxp, μ_0 and μ_1 are a vector of constants and linear drift terms. Finally, D_t is a vector of dummy variables (step dummy, permanent impulse dummy, and transitory impulse dummy) and φ is the corresponding vector of coefficients, and all parameters are unrestricted, see Johansen (1996) and Juselius (2006).

The cointegrated I(1) model, H_r, is formulated as a reduced rank restriction on Π as $\Pi = \alpha\beta'$ where α and β are pxr of rank $r < p$, and Γ is unrestricted. A correct specification of the deterministic components is important in the VAR model, and it has to be restricted in certain way to avoid undesirable consequences. In the I(1) model, unrestricted constant cumulates to a linear trend and unrestricted trend cumulates to a quadratic trend. We need to restrict the deterministic components appropriately to avoid higher order trends as it is not important for our analysis. In the empirical analysis, we restrict the trend in order to allow linear trends in all components of the model, including the cointegration relation, and exclude higher order trends. The specification of the dummy variables is also equally important. Unrestricted permanent blip dummy in ΔX_t cumulates to a level shift in X_t. The effect of including a step dummy is similar to including unrestricted constant, it has to be restricted in a way that allows level shifts in all directions of the model including the cointegration relations. It is not always clear from the outset whether the level shift and linear trends cancel in the cointegration relation, alternatively the long run exclusion can be tested. Finally, unrestricted transitory blip dummies cumulate to a blip in X_t with no serious consequences and can be ignored, cf. Juselius (2006) for further discussion.

The model (3) with a reduced rank restriction on Π and a level shift and trend restricted to the cointegration space can be written as:

$$\Delta X_t = \alpha\tilde{\beta}'\tilde{X}_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varphi_s \Delta D_{s,t} + \sum_{i=1}^{k-1} \varphi_{s,i} \Delta D_{s,t-i}$$

$$+\varphi_p \Delta D_{p,t} + \varphi_{tr} \Delta D_{tr,t} + \mu_0 + \varepsilon_t \tag{4}$$

Where $D_{s,t}$ is 0 for 1975q1 to 1985q4 and 1 for 1986q1 to 2013q4, $\tilde{X}_{t-1} = (X'_{t-1}, D_{s,t-1}, t)$, $\tilde{\beta}' = (\beta', \gamma'_0, \beta'_0)$, $D_{p,t}$ is a permanent blip dummy of the form (0,0,1,0,0) and $D_{tr,t}$ is a transitory blip dummy of the form (0,0,1,-1,0,0) to be specified in the empirical analysis.

5. Empirical result

The first step in the empirical analysis is to get a well specified model with appropriate lag length. This requires appropriate specification of deterministic terms and inclusion of dummies for outliers and breaks in the data. We have identified and included a level shift dummy *D86q1* corresponding to the tax reform and transitory and permanent blip dummies for outliers. The Schwarz and Hannan-Quinn information criteria points to a lag length of $k = 1$. There is a significant residual correlation at a lag length of one, but not at a lag length of two. Based on a likelihood ratio test, we can restrict k=*4* to k=*3 and* k=*2*, and $k=3$ to $k=2$, see table A1 in the appendix. We have also estimated an unrestricted VAR(4) model in level and tested for the significance of the additional regressors at X_{t-3} and X_{t-4}, they are significantly rejected. In practice a well specified model seldom needs a lag length above two, cf. Juselius (2006). Multivariate normality is rejected for a lag length of two. Increasing the number of dummies for outlying residuals does not seem to improve the multivariate normality test, see table A2 in the appendix. However, the estimates of the VAR models are generally robust to deviations from normality (Juselius, 2006). Continuously plugging in dummies is also expensive in terms of the degrees of freedom. In the following we continue with a VAR(2) model and include dummies only for very significant outlying residuals.

Rank determination

The cointegration rank divides the p dimensional vector into r relations toward which the process is adjusting and p-r stochastic trends which are pushing the system. The Likelihood-Ratio test for cointegration rank, often called the trace test, relies on the trace statistic of Johansen (1996). Table A3 reports the trace test and the estimated eigenvalues.

The trace test is based on the null hypothesis of p-r unit roots. The null of p-r = 5 and p-r = 4 is rejected based on both the standard and Bartlett corrected trace test statistics. The null of p-r = 3 cannot be rejected based on the Bartlett corrected test statistics, but it is rejected by the standard trace test. Hence, we have two choices – a rank of 2 based on the Bartlett corrected statistics and a rank of 3 based on the standard test statistics. The rank choice will influence all subsequent analysis and thus has to be determined with all the available information.

Based on the eigenvalue roots of the model a rank of 1 and to a lesser degree a rank of 2 can be tolerated. Table A4 reports the unrestricted eigenvalues of the companion matrix for each choice of rank. For a rank of zero, we can see three of the eigenvalues are close to the complex unit circle, which suggests a rank of 2. Restricting the largest eigenvalue to 1 introduces one more large eigenvalue, this is a sign of I(2)ness that remains in one or more of the variables after the nominal-to-real transformation, cf. Juselius (2006). Based on the theoretical candidates for long run relations and the statistical analysis, a rank of two is maintained in the following.

Identification of the long run structure

Guided by the theoretical discussion in section 2, we now continue to identify the two cointegrating relations. The long run structure is identified by imposing restrictions on each of the cointegrating relations. As a starting point we estimated the unrestricted cointegrated relation and normalize on $(ph - pc)_t$ for the first relation and on Δh_t for the second relation. We then imposed restrictions on the first and second long run relations consistent with the demand relation (1) and the supply relation (2), respectively. In addition we also imposed restrictions on the adjustment matrices α_1 and α_2. We impose restriction on the adjustment coefficients guided by the t-values on one hand and the theoretical framework on the other hand. For example, the user cost rate consists of interest rates, taxes and expected inflation, although it is certain that the user cost will affect households demand for housing, the vice versa is not straight forward. Interest rates in Denmark are exogenously determined from abroad, because of the fixed exchange rate policy Denmark follows vis-à-vis the Euro. The overall structure is over identified, and it cannot be rejected with a p-value of 0.13. The test of zero restrictions on the adjustment coefficients alone cannot be rejected with a p-value of 0.08. Table 1 reports the identified long run structure.

Table 1. The long run structure

	$(h\text{-}c)_t$	$(ph\text{-}pc)_t$	Δh_t	$(pi\text{-}pc)_t$	UC_t	t	$D86q1s$
β_1	2.389	1	0	0	12.285	0	0.547
	(3.689)				(6.463)		(5.545)
β_2	0	-0.007	1	0.007	0	.0003	0
				(8.526)		(6.461)	
α_1	0	-0.047	-0.001	0	0		
		(-6.149)	(-2.951)				
α_2	0	6.553	-0.118	0	0		
		(5.914)	(-4.073)				

LR test of restriction: $\chi^2(11) = 16.287$ [0.1308], t-values are given in parenthesis. Transitory and permanent blip dummies are included.

The long term demand and supply relations can be written, respectively, as:

$$\frac{ph_t}{pc_t} = -2.39(h_t - c_t) - 12.29UC_t - 0.55D86q1_{s,t} \quad (5)$$

$$\Delta h_t = 0.007(ph_t - pi_t) - 0.0003t \quad (6)$$

Equation (5) is the inverse demand function to (1) with an elastic of 2.39 with respect to the stock of houses consumption ratio. It follows that the demand price elasticity is estimated at 0.42 (=1/2.39), which is the same as the estimate in Knudsen (1994), and slightly above the estimate in ADAM (Danmarks Statistik, 2013). The estimates in Skaarup and Bødker (2010) and MONA (Danmarks Nationalbank) are relatively high. The semi-elasticity of user cost with respect to house prices is estimated to be 12.29, which implies that a 1 percentage point increase/decrease in user cost following an increase/decrease in the after tax interest rate, or property tax or expected inflation, results in a 12 percent decrease/increase in real house prices. The estimated coefficient to D86q1s suggests that the tax reform of the mid-eighties implied a 55% per cent reduction in the long term real house prices. That might be an exaggeration but the depressed developments in house prices are obvious in figure 1B right after the tax reform.

The supply relation (6) is a polynomial cointegration relation and represents the estimated Tobin's q relation, where the replacement ratio is made stationary by the adjustment in the stock of houses with a coefficient of 0.007. The estimated coefficient represents the short-term supply elasticity, the long-term supply elasticity is infinite, corresponding to a horizontal supply curve at the construction cost level, cf. Danmarks Nationalbank (2003).

The short run structure

The identification of the short run structure is facilitated by keeping the identified long-term structure fixed and treating $\tilde{\beta}'\tilde{X}_{t-1}$ as predetermined stationary regressors as ΔX_{t-1}. We first estimated a multivariate dynamic equilibrium error correction model, which is achieved by pre-multiplying the reduced form with a pxp matrix $A_0 = I$. The system is estimated with full information maximum likelihood, which is exactly identified by the $p-1$ zero restrictions on each row of A_0. Further zero restrictions are over identifying. The parsimonious system reported in table 2 is achieved by imposing over identifying restrictions. Economic theory is not precise about the short run structure, we relied on simplification and empirical evidence when imposing over identifying restrictions. The parsimonious system cannot be rejected with a likelihood ratio test for 61 over identifying restrictions with a p-value of 0.43. The reader is invited for equation-by-equation scrutiny of the estimated coefficients.

Table 2. Parsimonious multivariate equilibrium correction model

	Δ(h-c)t	*Δ(ph-pc)t*	$\Delta^2 h_t$	*Δ(pi-pc)t*	*ΔUCt*
Δ(h-c)t-1	-0.246		0.005	0.231	
	(-3.56)		(1.91)	(2.26)	
Δ(ph-pc)t-1	-0.170	0.372	0.005	0.189	
	(-4.07)	(6.04)	(3.34)	(3.01)	
$\Delta^2 h_{t-1}$	-4.121				
	(-2.09)				
Δ(pi-pc)t-1				-0.149	
				(-2.56)	
ΔUCt-1		-1.001			0.319
		(-2.19)			(4.43)
ECM1t-1		-0.046	-0.001		
		(-6.52)	(-3.94)		
ECM2t-1		6.256	-0.100		
		(5.57)	(-3.85)		
	-0.001	0.110	0.003	0.001	0.0003
	(-0.667)	(4.62)	(5.38)	(0.607)	(-1.24)

The column heading is the dependent variable in each equation and the row headings are the predetermined regressors, t-values are given in parenthesis. ECM1 and ECM2 are the identified long run demand and supply relations:

$ECM1_t = \frac{ph_t}{pc_t} + 2.39\left(\frac{h_t}{c_t}\right) + 12.29UC_t + 0.55D86q1_t$

$ECM2_t = \Delta h_t = 0.007\left(\frac{ph_t}{pi_t}\right) - 0.0003t$

Dummies are not reported.

LR test of restriction: χ^2 (61) = 62.33 [0.4287].

Multiplier Analysis

In the following we set up a small model consisting of the estimated relations in table 2 and carry out a multiplier experiment, which is one way of illustrating the properties of the parsimonious multivariate error correction model. We compare the multiplier results with a similar experiment using ADAM's housing model.

Because of the nominal-to-real transformation, a variable can appear more than once as part of a given dependent variable. For example, both equation 1 and 3 can be normalized on h_t, and hence cannot be identified. Alternatively, equation 1 can be normalized on c_t defining an equation for consumption. A classical equation for consumption includes, among others, income and wealth as the main explanatory variables, hence attributing equation 1 to consumption can be misleading. Alternatively, one can estimate a smaller system of VAR equations conditioning on consumption, i.e. by a priori fixing consumption to be weakly exogenous to the full information set. Provided the exogeneity assumption is not rejected, valid statistical inference about the VAR model based on the full information set can be made using the smaller VAR conditioning on consumption. Here, we settle with the system of VAR equations from table 2 and to facilitate comparison with ADAM's housing model we set up our VAR model excluding equation 1 from table 2. A multiplier experiment using all five equations in table 2 is presented in the appendix.

The multiplier experiment in ADAM is calculated using October-2015 model version, shocks are calculated from 2016 onwards using a standard projection as a baseline. Whereas in our simple VAR model, shocks are calculated using the residuals of the estimated equations in the historical periods and to facilitate comparison the quarterly frequency is collapsed to annual figures. Below we consider two examples, a demand side and a supply side shock.

Figure 2 shows the effect of a permanent 1 percentage point increase in the user cost rate that could be due to changes in the tax rate, interest rate or expected inflation. The quantity of housing demanded decreases for each price level, i.e. the demand curve for housing shifts leftward, and house prices decline consequently. In the following years, price changes guide the market into a new equilibrium through the contraction in supply of houses, which in turn exerts upward pressure on house prices. The adjustment in net investment continues as long as house prices are different from construction costs. The market is characterized by sustained periods of disequilibrium. In the long term house prices are unchanged and stock of houses decline permanently. The long-term impact on houses prices is the same in the two models, but the short and medium term reactions are somehow different. The immediate impact on house price is larger in ADAM, and the effect peaks relatively quick and starts to change course before that of the VAR model, which reflects the difference in the estimated parameters, see also Danmarks Statistik (2013).

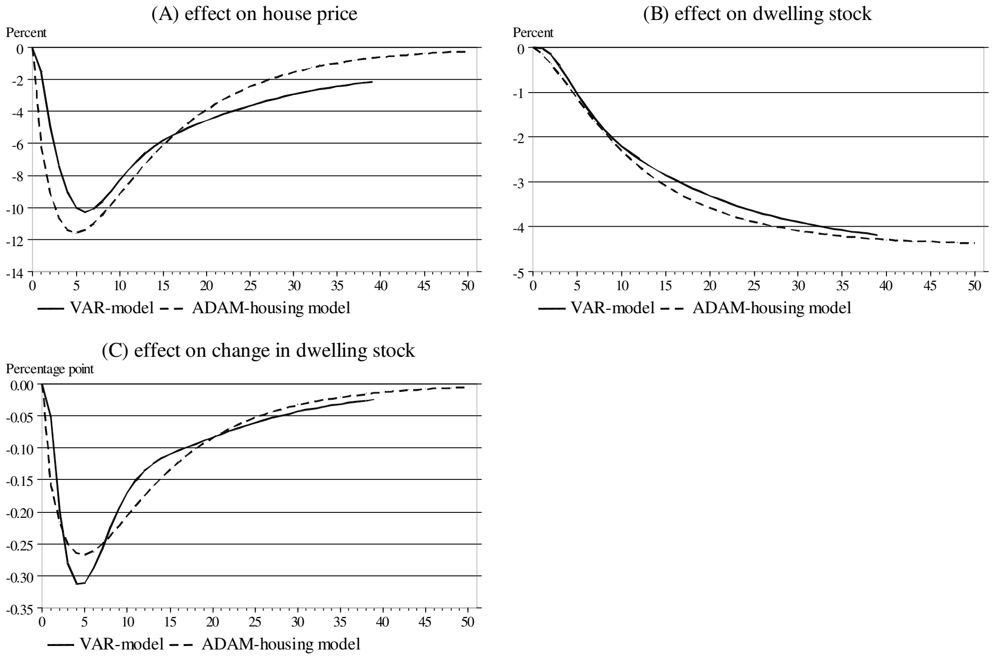

Figure 2. The effect of a permanent 1 percentage point increase in user cost rate

Figure 3 presents the effect of a supply side disturbance on the housing market. In the experiment, construction cost is permanently increased by 1 percent, the result is a permanent fall in housing investment and a permanent rise in house prices. The short term dynamics in our VAR model shows some peculiar movements, for instance there is a small positive effect on net investment initially, as it is very small we do not attach much significance to it. The long-term impacts are comparable in the two models.

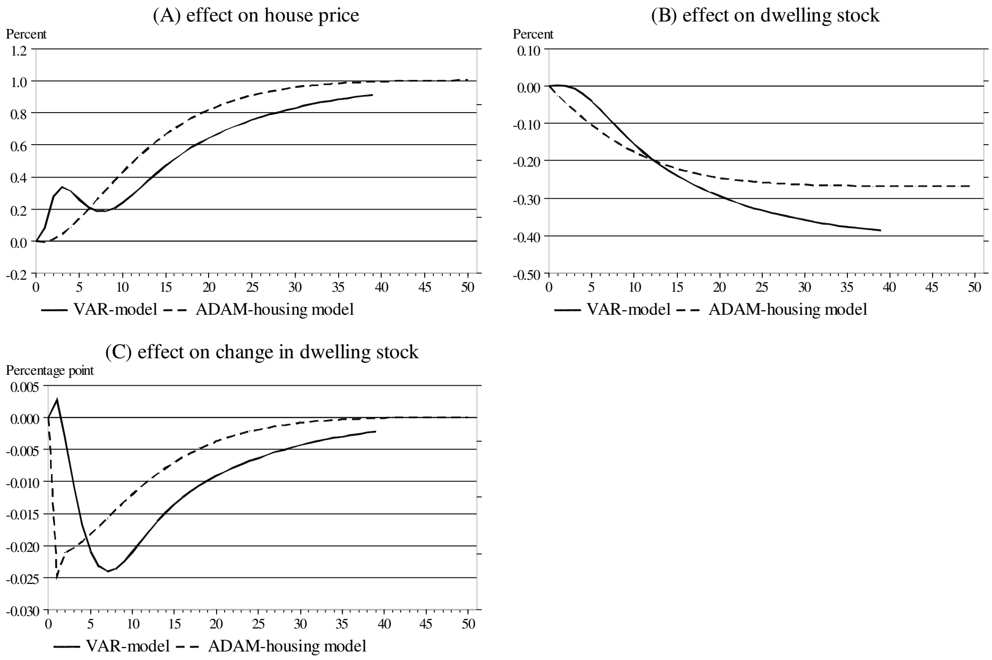

Figure 3. The effect of a permanent 1 percent increase in construction cost

6. Conclusion

The paper applied the cointegration technique of Johansen to the Danish housing market. We have found two cointegrating relations – a demand relation between dwelling stock, house price, income and user cost rate and a supply relation between replacement ratio and net investment. The long and short term parameters are estimated more precisely, supporting the corresponding estimates in ADAM. The multiplier exercise shows similarity between our VAR model and ADAM's housing model and demonstrates the slow adjustment of the housing market to disequilibrium.

References

Arestis and González (2013). Modelling the Housing Market in OECD Countries, Levy Economics Institute, Working Paper No. 764.

Riddel M. (2004). Housing-Market Disequilibrium: an examination of housing-market price and stock dynamics 1967–1998, Journal of Housing Economics 13 (2004) 120–135.

Danmarks Nationalbank (2003). MONA: A Quarterly Model of the Danish Economy, Danmarks Nationalbank.

Danmarks Statistik (2013). ADAM: A Model of the Danish Economy, Danmarks Statistik.

Johansen, S. (1995). ‘ A Statistical Analysis of Cointegration for I(2) Variables’, Econometric Theory, Vol. 11,pp. 25-59.

Johansen, S. (1996). Likelihood-based Inference in Cointegrated Vector Autoregressive Models, 2nd edition, Oxford University Press.

Juselius, K. (2006). The Cointegrated VAR Model, Methodology and Applications, Oxford University Press.

Knudsen, D. (1994). Residential Investments and House Prices in Denmark, Economic Modelling 11 (2).

Leamer E., Edward (2007). Housing is the Business Cycle, National Bureau of Economic Research, working paper 13428.

Tobin, J. (1969). A general Equilibrium Approach to Monetary Theory, Journal of Money, Credit and Banking, Vol 1, No. 1, pp. 15-29.

Skaarup, M. and S. Bødker (2010). House Prices in Denmark: are they far from equilibrium? Finansministeriet, working paper no 21/2010.

Appendix

Table A1. Lag length determination

MODEL SUMMARY

Model	k	T	Regr	Log-Lik	SC	H-Q	LM(1)	LM(k)
VAR(5)	5	153	39	4126.498	-47.530	-49.823	0.014	0.492
VAR(4)	4	153	34	4101.980	-48.031	-50.031	0.010	0.035
VAR(3)	3	153	29	4083.122	-48.607	-50.312	0.001	0.492
VAR(2)	2	153	24	4065.938	-49.204	-50.615	0.008	0.353
VAR(1)	1	153	19	4025.652	-49.499	-50.617	0.000	0.000

Lag Reduction Tests:

VAR(4) << VAR(5)	ChiSqr(25)	= 49.035[0.003]
VAR(3) << VAR(5)	ChiSqr(50)	= 86.753[0.001]
VAR(3) << VAR(4)	ChiSqr(25)	= 37.717[0.049]
VAR(2) << VAR(5)	ChiSqr(75)	= 121.12 [0.001]
VAR(2) << VAR(4)	ChiSqr(50)	= 72.085[0.022]
VAR(2) << VAR(3)	ChiSqr(25)	= 34.367[0.100]
VAR(1) << VAR(5)	ChiSqr(100)	= 201.69 [0.000]
VAR(1) << VAR(4)	ChiSqr(75)	= 152.66 [0.000]
VAR(1) << VAR(3)	ChiSqr(50)	= 114.94 [0.000]
VAR(1) << VAR(2)	ChiSqr(25)	= 80.571[0.000]

SC : Schwarz Criterion
H-Q : Hannan-Quinn Criterion
LM(k): LM-Test for autocorrelation of order k

The models include a level shift dummy D86q1 and permanent and transitory dummies: D76Q1P, D77Q3T, D79Q1P, D83Q2P, D86Q1P, D90Q1P, D93Q1T, D96Q1T, D97Q1P, D00Q1P, D06Q3P.

Table A2. Test for misspecification of the unrestricted VAR(2)

	Univariate tests			
Equation	AR(1-5)	ARCH(1-4)	Hetero test	Normality
(h-c)$_t$	0.9786[0.4338]	0.19998[0.9379]	0.80487[0.7180]	3.4460[0.1785]
(ph-pc)$_t$	0.18389[0.9682]	1.1689[0.3282]	1.4440[0.1096]	8.5197[0.0141]
Δh_t	2.8504[0.0182]	5.3098[0.0006]	1.8039[0.0244]	19.936[0.0000]
(pi-pc)$_t$	0.79705[0.5539]	0.87222[0.4829]	0.76307[0.7676]	25.934[0.0000]
UC_t	0.42993[0.8270]	0.19234[0.9420]	0.79434[0.7307]	2.8426[0.2414]
	Multivariate tests			
AR(1-5)	0.99584[0.5007]			
Hetero test	0.91571[0.8386]			
Normality test	49.316[0.0000]			

Note: AR is the test of autocorrelation of order 1-5, figures in square bracket are p-values. The model includes a level shift D86q1s and the following transitory and permanent blip dummies: D75Q1P, D75Q4P, D76Q1P, D77Q2P, D77Q3T, D79Q1P, D82Q1P, D83Q1P, D83Q2T, D87Q1P, D90Q1T, D93Q1T, D94Q1P, D96Q1T, D97Q1T, D00Q1T, D05Q1P, D06Q3T, D08Q1P.

Table A3. The trace test of the cointegration rank and the eigenvalue roots of the model

p-r	r	eigenvalues	trace	trace*
5	0	0.332	153.119[0.000]	142.670[0.000]
4	1	0.191	90.125[0.001]	80.563[0.009]
3	2	0.156	56.994[0.008]	47.303[0.083]
2	3	0.114	30.489[0.057]	25.411[0.188]
1	4	0.072	11.621[0.179]	11.296[0.199]

Note: figures in square bracket are p-values, trace* is the small sample Bartlett corrected trace test statistics. The model includes a trend and level shift restricted to the cointegration relation and unrestricted constant, and no other deterministic components as the asymptotic distribution of the test is sensitive to deterministic components.

Table A4. Moduli of the companion matrix

0.971	0.971	0.890	0.671	0.488	0.488	-0.361	0.324	0.090	0.090
1	0.974	0.974	0.669	0.488	0.488	-0.355	0.334	0.061	0.061
1	1	0.904	0.823	0.477	0.477	0.357	-0.342	0.106	0.003
1	1	1	0.918	0.599	0.361	0.361	-0.347	0.112	-0.003
1	1	1	1	0.842	0.351	0.351	-0.339	0.111	-0.093
1	1	1	1	1	0.583	0.388	-0.314	-0.114	0.017

Multiplier analysis using the full VAR in table 2.

Here we include equation 1 from table 2 that was dropped from the multiplier exercise in the main text. It can be re-arranged and written as an equation for consumption. In the present case house price permanently falls in the long run following a permanent increase in the user cost rate. The permanent reduction in the house price reflects that consumption follows the fall of the dwelling stock due to equation 1. Consequently, the full VAR in table 2 implies a long-run response, where housing demand is falling pari passu with housing supply. We should expect some feedback to the housing market from a consumption relation, for instance if house values enter the wealth term of the consumption relation. However, the one-to-one correspondence between consumption and dwelling stock implied by equation 1 is too much.

Figure A1. The effect of a permanent 1 percentage point increase in user cost rate