The Optimal Level of Progressivity in the Labor Income Tax in a Model with Competitive Markets and Idiosyncratic Uncertainty [*]

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Abstract

In a world where labor earnings are uncertain and borrowing-constraints are present, progressive taxation is likely to have risk-mitigating benefits for consumption-smoothing agents; higher tax payments are due in periods of life when income is relatively high, and less tax must be paid when income is low. This lowers the probability that the borrowing constraint becomes binding. The question is if this income-smoothing risk-mitigating property of progressive taxation has any value to the consumers?

Simulations using a large-scale computable general equilibrium model with competitive markets show that consumers prefer progressive taxation of labor earnings to proportional taxation - a result that is contrary to the findings in the deterministic framework by Auerbach and Kotlikoff (1987). However, there is a trade-off between the positive risk-migating properties of progression, and the negative distortionary effects; indeed it turns out that there is an optimal level of progressivity.

JEL classification: H21, D91, E21, C68.

Keywords: idiosyncratic income uncertainty, optimal progressivity.

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1 Introduction

Traditional analyses of progressive taxation in competitive Computable General Equilibrium (CGE) models, e.g. Auerbach and Kotlikoff (1987, chapter 8), have concluded that progressive taxation has a negative effect on welfare. The reason is intuitive: in this relatively simple model the single representative agent pays a certain amount of taxes over the life-cycle, and progressivity in the taxation of labor earnings discourages labor supply, and hence lowers welfare.[1] However, it is easy to think of benefits from progressive income taxation that are not captured in this simple framework.[2] For instance progressive taxation is - in an uncertain world - likely to have risk-mitigating benefits for the consumption-smoothing agents: higher tax payments are due in periods of life when income is relatively high - and conversely less taxes must be paid when income is low. In a world with perfect capital markets this is irrelevant: the timing of tax payments (in a cash-flow sense) does not matter as long as the present value of the payment stream is constant. In the rather realistic case that there are imperfections in the capital markets - for instance if the rate of return on savings is lower than the cost of borrowing, or if the consumer is constrained from borrowing against anticipated future income - this conclusion may change.

Inability to borrow against future income is particularly relevant, if the consumer's future income is uncertain. For the consumer progressive taxation lowers variability in after-tax earnings compared to pre-tax earnings, and lowers the probability that the borrowing constraint becomes binding. Does this after-tax income smoothing property of progressive taxation have any value to the consumers? On the firm side, one can make a similar argument, and hedging can be seen as the firm equivalent; for a firm it will generally be optimal to hedge a volatile earnings stream, if taxes are a convex function of earnings (Smith and Stulz, 1985). For the consumer, it may be the case that the positive risk-mitigating effect from progressive taxation can counteract the negative distortionary effects? Or perhaps the risk-mitigating effect can even outweigh the distortionary effects?

This paper attempts to answer this question in a CGE model. Overlapping generations of consumers face idiosyncratic uncertain labor income in each

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[1] A similar conclusion is obtained in models with multiple agents (Altig, Auerbach, Kotlikoff, Smetters and Waliser, 2001); in addition inter-agent redistribution through tax-payments becomes an issue.

[2] With imperfect labor markets the standard result can reverse: progression is good. See Lockwood and Manning (1993).

period of their lives. The uncertainty is only present at the individual level, which means that aggregate variables are not (directly) influenced (i.e. there is no aggregate uncertainty). Since borrowing is permitted, agents have to self-insure against variability in income. The government sector levies taxes on income and uses the revenue for public expenditures. The production side is standard: firms produce the single good using capital and labor according to a constant return to scale technology.

Analyses using models with idiosyncratic earnings uncertainty and borrowing constraints started with the seminal papers by Aiyagari (1994, 1995)[3]. Subsequently the models have been used to analyze precautionary savings behavior and social security issues [Hubbard and Judd (1987); Hubbard, Skinner and Zeldes (1994, 1995); İmrohoroğlu, İmrohoroğlu and Joines (1993, 1995) ; Huggett and Ventura (1997); Huang, İmrohoroğlu and Sargent (1997)][4], effects of a flat tax reform [Ventura (1999)], and capital income taxation [İmrohoroğlu (1998)]. The only other paper focusing on proportional versus progressive taxation is Castaneda, Diaz-Gimenez and Rios-Rull (1999). However, most of these models have either been partial equilibrium, used a Ramsey formulation, or had exogenous labor supply. In contrast this paper presents general equilibrium calculations with overlapping generations of consumers with endogenous labor supply. Apart from the new features (idiosyncratic uncertainty and borrowing constraints), the model is kept as close as possible to Auerbach and Kotlikoff (1987, chapter 8); yet the added features change the model fundamentally. Firstly, the presence of uncertainty in earnings gives the consumers a precautionary savings motive. As pointed out by Engen and Gale (1996) savings of the precautionary type are less sensitive to the rate of return than pure life-cycle savings; therefore the savings elasticity is likely to be lower in a situation with precautionary savings[5]. Secondly, the framework makes it possible to include the positive risk-mitigating effects from progressive taxation - these cannot be captured by the single-agent representative framework used by Auerbach and Kot-

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[3] Aiyagari (1994) investigates the precautionary savings effects in a Ramsey model (but is not like the present paper concerned with policy applications).

[4] This class of models is well suited for analysis of social security, since the models can capture the risk-sharing properties of social security in a way that is not possible within the deterministic single-agent representative framework of Auerbach and Kotlikoff (1987). An overview of the analysis of social security can be found in İmrohoroğlu, İmrohoroğlu and Joines (1999).

[5] This effect could potentially be important: Engen and Gale (1996) report that the response in savings to changes in consumption taxation is 80 per cent smaller in a stochastic life-cycle model compared to a certainty life-cycle model. This smaller savings elasticity seem to be better in accordance with empirical evidence (Deaton, 1992).

likoff (1987).

In the simulations two taxes are present: a capital income tax and a labor income tax, but only the latter tax will be changed in the simulations. The results of the analysis turns out to be quite surprising, and challenge the conventional wisdom derived from a simple deterministic model: in fact the simulations show that consumers *ex-ante* actually *prefer* progressive to proportional taxation of labor income. Hence the smoothing (or risk-mitigating) property of progressivity more than outweighs the negative effects from the increased distortion, and it turns out that there is in fact an optimal level of progression. The sensitivity analysis shows that the conclusions depend on the specification of uncertainty - the more volatile the earnings the larger the benefits from progressivity.

This paper is organized as follows. Section 2 contains a description of the model used - this includes a description of the consumers, the producers and the government sector. Section 3 describes how the model is calibrated, as well as the numerical solution methods employed. Section 4 presents the results of the central case, which includes aggregate effects, life-cycle effects, as well as distributional effects. Section 5 presents a sensitivity analysis of the results, and finally section 6 summarizes the findings.

2   A General Equilibrium model

The model described here is a relatively standard stochastic general equilibrium model[6]. To simplify matters, we will only look at the stationary state of the model, which means that all time indices are removed in the formulae below.

2.1   The Consumers

The consumers in the model are almost similar to Auerbach and Kotlikoff (1987) (the "A-K model"): they live for J periods, and seek to maximize their life-time utility function subject to their budget constraint. Their utility function is additive separable, and they incur utility from consumption and leisure. They face a borrowing constraint in every period, and as well as (the usual) life-time budget constraint. Labor income arise from sale

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[6]Closest related work is İmrohoroğlu, İmrohoroğlu and Joines (1995) and Huggett and Ventura (1997).

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of labor services - the income from this is uncertain, due to a stochastic productivity term (described below). Since income in each period is uncertain, solving the consumers' problem is easier formulated as a dynamic programming problem, which is the way it is presented below.

2.1.1 A recursive approach

The recursive maximization problem for a representative consumer (with the start-of-period assets a_{j-1} and the productivity category d_{j-1}) is given by:

$$V_j\left(a_{j-1}, d_{j-1}\right) = \max_{\{c_j, l_j, a_j\}} \left[\frac{1}{(1-1/\gamma)} u\left(c_j, l_j\right)^{(1-1/\gamma)} + \beta \sum_b \pi_b\left(d_{j-1}\right) V_{j+1}\left(a_j, b\right) \right] \tag{1}$$

with the budget constraint:

$$a_j = (1+r)\,a_{j-1} + w\left(1 - l_j\right) e_j\left(d_{j-1}\right) - pc_j - TAX \tag{2}$$

where the agent is subject to the liquidity-constraint:

$$a_j \geqq 0 \quad (\forall j) \tag{3}$$

the consumption-constraint:

$$c_j \geqq 0 \quad (\forall j) \tag{4}$$

and the leisure constraint:

$$1 \geqq l_j \geqq 0 \quad (\forall j) \tag{5}$$

where

a_j is the end-of-period assets, $e_j\left(d_{j-1}\right)$ is the productivity in period j for labor with the start of period[7] productivity d_{j-1}, c_j is the consumption in period j, l_j is the leisure enjoyed by generation j, γ is the household's intertemporal elasticity of substitution, β is the one-period discount factor, $\pi_b\left(d\right)$ is the probability that next period's labor productivity will be b given

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[7]Rather this *was* their productivity at the end-of-last period, since assets and productivity use end-of-period notation. Thus the productivity level d_{j-1} is active (earns labor income) in period j. Therefore $e_j\left(d_{j-1}\right)$ belong together.

it is d in this period[8], r is the one-period interest rate, w is the wage and p is the consumer price level. TAX is taxes paid, and is the sum of a consumption tax, an interest income tax and a labor income tax:

$$TAX = TAX_C\left(pc_j\right) + TAX_A\left(ra_{j-1}\right) + TAX_L\left(\left(1 - l_j\right)e_j\left(d_{j-1}\right)w\right) \quad (6)$$

where TAX_C is the consumption tax, TAX_A is the taxation of interest income, TAX_L is the taxation of labor income.

Notice the dating rules used: the end-of-period convention.[9] The first period for the consumer is $j = 1$, which means that the consumer's problem when entering the economy is to solve $V_1(a_0, d_0)$. Since the consumers live for J periods, we exogenously set $V_{56}\left(\cdot\right) = 0$.

Earnings

The labor productivity falls in one of D categories, and is age-dependent through the function $e_j\left(d\right)$. To simplify matters we assume that productivity can be separated in an age- and a category effect, such that $e_j\left(d\right)$ can be written as the product, $e_j\left(d\right) = \widehat{e_j}\overline{e}_d$. The transition between categories of labor productivity is a first order Markov process, with age-*in*dependent transition probabilities π. One can think of the labor productivity, $e_j\left(d\right)$, as following a random walk with a drift (a special non-linear hump-shaped drift), $\widehat{e_j}$, as well as the random walk component, $\overline{e}_d$.

Uncertainty in earnings is caused by variability in the labor productivity. As formulated above the earnings uncertainty is a first order Markov process, where the earnings level tomorrow only depends on earnings level today (the Markov formulation makes it possible to introduce "persistence" in productivity - see the random walk formulation below).

Taxes

The tax-functions (TAX_C, TAX_A and TAX_L) allow for progressive taxation schemes. We use the same specification[10] of the progression system as

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[8]This implies that $\sum_d \pi_b\left(d\right) = 1$.

[9]Also notice the timing of information: at the time the consumer solves the problem $V_j\left(a_{j-1}, d_{j-1}\right)$, he knows the relevant productivity in the j'th period period (which is d_{j-1}) and his assets at the beginning of the period (a_{j-1}). In other words the uncertainty about productivity concerns future productivity, and not the productivity in this period.

[10]This specification represents a simplification. It disregards the fact that most actual tax systems are piece-wise linear tax systems, and not of the continuously increasing marginal tax type considered here.

Auerbach and Kotlikoff (1987, ch. 8), and assume that the marginal tax rates take the form:

$$\tau = \underline{\tau} + \kappa B \tag{7}$$

where $\underline{\tau}$ is the marginal tax applicable at zero income (the "intercept"), B is the taxable amount (the "base"), and κ is the progressivity parameter (the "slope"). If $\kappa = 0$ we have a proportional tax system. The average rate of a system given by equation (7) is

$$\overline{\tau} = \underline{\tau} + \kappa \frac{B}{2} \tag{8}$$

This formula is applied similarly to taxation of interest income, labor income and consumption, and we have the tax functions:

$$TAX_A(B_A) = B_A \overline{\tau}_A = B_A \left(\underline{\tau}_A + \kappa_A \frac{B_A}{2} \right)$$

$$TAX_L(B_L) = B_L \overline{\tau}_L = B_L \left(\underline{\tau}_L + \kappa_L \frac{B_L}{2} \right)$$

$$TAX_C(B_C) = B_C \overline{\tau}_C = B_C \left(\underline{\tau}_C + \kappa_C \frac{B_C}{2} \right)$$

Utility

The annual utility function is the CES-function

$$u(c, l) = \left[c^{(1-1/\rho)} + \alpha l^{(1-1/\rho)} \right]^{\frac{1}{(1-1/\rho)}} \tag{9}$$

where α is a taste parameter reflecting the joy of leisure, and ρ is an elasticity of substitution between leisure and consumption.

The consumer has two state-variables: the assets (a_j) and the labor-productivity (d_j), where the first is a value and the second is a category, i.e. the first is continuous and the second is discrete. Notice that assets are not allowed to be negative at any point in time.

2.1.2   Solving the consumer's problem

The optimization problem facing an individual is one of finite-state, finite horizon dynamic programming[11]. The decision rules can be found by back-

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[11] Using the terminology in Rust (1996) we are dealing with a *Discrete Time Discounted Markov Decision Process.*

wards recursion from the last period of life.

We start by reducing the number of control-variables by substituting the budget constraint into the Value-function[12]. First we isolate c_j in the budget constraint:

$$c_j = \frac{(1+r)\,a_{j-1} - a_j + (1-l_j)\,e_j\,(d_{j-1})\,w + \lambda - TAX}{p}$$

and substitute this and the CES-utility-function (9) into the Value-function:

$$
V_j\left(a_{j-1}, d_{j-1}\right) \hspace{5cm} (10)
$$
$$
= \max_{\{l_j, a_j\}} \left[\frac{1}{(1-1/\gamma)} \left(\left[c_j^{(1-1/\rho)} + \alpha l_j^{(1-1/\rho)} \right]^{\frac{1}{(1-1/\rho)}} \right)^{(1-1/\gamma)} + \beta \sum_b \pi_b\,(d_{j-1})\,V_{j+1}\,(a_j, d_j) \right]
$$

Last period

Since death is certain beyond period J (hence $V_{56} = 0$) and there is no bequest motive in the model, the choice in period J is to consume everything that is left, plus whatever income is generated in that period. With $a_J^* = 0$, we can simplify the last-period problem. Consumption is:

$$c_J = \frac{(1+r)\,a_{J-1} + (1-l_j)\,e_J\,(d_{J-1})\,w - TAX}{p}$$

and the associated utility is

$$
V_J\left(a_{J-1}, d_{J-1}\right) = \max_{\{c_J, l_J\}} \left[\frac{1}{(1-1/\gamma)} \left(\left[c_J^{(1-1/\rho)} + \alpha l_J^{(1-1/\rho)} \right]^{\frac{1}{(1-1/\rho)}} \right)^{(1-1/\gamma)} \right]
$$
$$
\tag{11}
$$

We obtain the argmax for l_J (which we later will call l_J^*) and for c_J (which we later will call c_J^*) by solving equation (11) numerically.

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[12]By substituting consumption (c) away, we reduce the number of control variables to two. We could equally well have substituted the end-of-period assets (a) away - in this case the consumer would explicitly choose labor and consumption (and implicitly the end-of-period assets). But as noted in chapter 12 in Judd (1998), it is convenient to have control variables that are also state variables.

Second-last period (and forwards...)

In all periods before the last period, we calculate the optimal plan for the consumer by numerically solving equation (10). This is done in a manner described in section 3.4.

Optimal individual policy rules

Denote the optimal consumption for a consumer with the decision problem $V_j(a_{j-1}, d_{j-1})$ by $c_j^*(a_{j-1}, d_{j-1})$ the optimal leisure by $l_j^*(a_{j-1}, d_{j-1})$, and the optimal end-of-period asset-holdings by $a_j^*(a_{j-1}, d_{j-1})$.[13] We will refer to these as the individual policy rules, and for short refer to them as c_j^*, l_j^* and a_j^*.[14]

2.2 Aggregation

Finally some aggregation and equilibrium conditions for the consumer:

2.2.1 Population transition

At any point in time the agents in the economy have characteristics in the $(a, d) \in (A, D)$ space (the two state variables). Let $\gamma_j(a, d)$ denote the number of individuals with assets a, productivity level d and age j. This is a stock variable, and measured at the end of the period; in other words $\gamma_{j-1}(a, d)$ are alive and active in period j of their lives (remember that there is no mortality).

Calculating how many individuals are in which group - and the transition between groups - is done in the following manner:

> **Initial:** Individuals entering the economy have no assets, but their distribution on productivity categories is exogenously determined. The probability that an agent starts in category d is η_d. Thus we have that the individuals active in the first period

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[13] Consider the optimal choice of assets for an individual that has been active for 1 period. His optimal end-of-period-one assets are denoted $a_1^*(a_0, d_0)$, - since we know that he entered the economy with zero assets it will in fact be $a_1^*(0, d_0)$.

[14] If we think of these as functions we have $a^* : A \times D \to A$, $c^* : A \times D \to \mathsf{R}_+$ and $l^* : A \times D \to \mathsf{R}_+$, where A is asset holdings, and D is productivity levels.

are:

$$\gamma_0 (0, d) = \eta_d \tag{12}$$

Transition: In the second period individuals can have positive assets in addition to a productivity category. A complicating matter is that the transition in the system is endogenously determined by the consumer's choice of control variable. Thus, for $j = 1, 2, ..N - 1$ we have:

$$\gamma_j (\widehat{a}, b) = \underbrace{\sum_{d \in D} \pi_b (d)}_{} \underbrace{\sum_{a \in A} \gamma_{j-1} (a, d) \; \chi \left[a_j^* (a, d) = \widehat{a} \right]}_{} \tag{13}$$

where the first term is the (exogenous) transition probability from any productivity category to category b, and the second term is the number of individuals in the previous period who chose to their next-period assets to equal a_j^* where $\chi (\bullet)$ is an indicator function (that assumes 1 if true, and 0 if false, the condition being whether individuals choose the end-of-period assets we are considering: $\widehat{a}$). Thus we sum over all individuals that (choose) to transit to the state $\gamma_j(a, b)$.

Terminal: At the final period, the equation above reduces to:

$$\gamma_J (a, b) = 0 \tag{14}$$

2.2.2   Aggregate factor supply

The next aggregation issue is the aggregate labor-supply. The total labor-supply in efficiency units is given by:

$$L = \sum_{j=1}^{J} \sum_{a \in A} \sum_{d \in D} \gamma_{j-1} (a, d) \left(1 - l_j^* (a, d) \right) e_j(d) \tag{15}$$

The aggregate savings can be calculated in a similar fashion. The capital stock active in period n is denoted K_{n-1}, so we have:

$$K_{-1} = \sum_{j=1}^{J} \sum_{a \in A} \sum_{d \in D} \gamma_{j-1} (a, d) \; a_j^* (a, d) \tag{16}$$

Notice that (without loss of generality) it is assumed that physical capital does not depreciate.

2.3 The producers

The production side is identical to Auerbach and Kotlikoff (1987). There is a single good, that is produced using capital and labor subject to a constant-returns-to-scale technology. Labor across ages and productivity categories differ in efficiency, and we calculate the total labor supply by individuals using equation (15) and the size of the aggregate capital stock using equation (16). For the labor supply this implies that labor across productivity categories are perfect substitutes; what matters is how many efficiency units are supplied.

Production takes place using a CES production function:

$$Y(K, L) = \Lambda \left[\epsilon K^{(1-1/\sigma)} + (1-\epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} \tag{17}$$

where K and L are capital and labor in the period, Y is output, Λ is a scaling constant, ϵ is a capital-intensity parameter and σ is the elasticity of substitution between K and L.

2.3.1 Factor demand

With no depreciation of capital, we have the standard result that the gross wages must equal the marginal revenue product of labor (measured in efficiency units):

$$w = (1-\epsilon) \Lambda \left[\epsilon K^{(1-1/\sigma)} + (1-\epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} L^{-1/\sigma} \tag{18}$$

and the interest rate equals the marginal revenue product of capital:

$$r = \epsilon \Lambda \left[\epsilon K^{(1-1/\sigma)} + (1-\epsilon) L^{(1-1/\sigma)} \right]^{1/(1-1/\sigma)} K^{-1/\sigma} \tag{19}$$

Notice that the output price is numeraire: $p = 1$.

2.4 The government sector

The government sector is kept very simple. Government revenue is raised by taxation of labor income, interest income and a consumption tax:

$$REV\_TAX = \sum_{j=1}^{J} \sum_{a \in A} \sum_{d \in D} \gamma_{j-1}(a,d) \, TAX_L \left[\left(1 - l_j^*(a,d)\right) e_j(d_{j-1}) w \right] +$$

$$\sum_{j=1}^{J} \sum_{a \in A} \sum_{d \in D} \gamma_{j-1}(a,d) \, TAX_A \left(a_j^*(a,d) r \right) +$$

$$\sum_{j=1}^{J} \sum_{a \in A} \sum_{d \in D} \gamma_{j-1}(a,d) \, TAX_C \left[pc_j^*(a,d) \right]$$

where j is an index over generations, d an index of the labor productivity categories, and $\gamma_j(a,d)$ is the share of individuals in generation j with assets a and productivity d.

The revenue from taxation is not transferred back to the consumers, but is consumed (similarly to Auerbach and Kotlikoff (1987)).

2.5 Equilibrium

Finally we need to define what we understand what is understood by an equilibrium in the model:

Definition 1 *A stationary equilibrium for a given set of policy arrangements $\{\kappa_a, \mathcal{I}_a, \ \kappa_c, \mathcal{I}_c, \ \kappa_l, \mathcal{I}_l\}$ (progressivity and tax level) is a collection of value functions $V_j(a,d)$, individual policy rules l_j^*, c_j^* and a_j^*, age-dependent measures of agent types $\gamma_j(a,d)$, relative prices of labor and capital $\{w,r\}$ such that*

    (a) the relative prices $\{w,r\}$ solve the firm's maximization problem (satisfy equation (18) and (19))

    (b) given the relative prices $\{w,r\}$ and government policies $\{\kappa_a, \mathcal{I}_a, \kappa_c, \mathcal{I}_c, \kappa_l, \mathcal{I}_l\}$ the individual policy rules l_j^*, c_j^* and a_j^* solve the consumer's problem (10)

    (c) individual and aggregate behavior are consistent, i.e. that K and L satisfies equations (15) and (16)

(d) the population follows the law of motion given by equations (12), (13) and (14)

(e) commodity markets clear, i.e. that production - given by equation (17) - equals consumption:

$$Y(K,L) = \sum_{j=1}^{J} \sum_{a \in A} \sum_{d \in B} \gamma_{j-1}(a,d) \, c_j^*(a,d) \qquad (20)$$

3 Calibration

This section describes how the model is calibrated, and how the stationary state is calculated.

3.1 The Consumer side

For **the household's intertemporal elasticity of substitution, γ**, we use the estimate used in Auerbach and Kotlikoff (1987) and set $\gamma = 0.25$.

For **the one-period discount factor**, β, we again use Auerbach and Kotlikoff (1987). They use a rate of time preference of 0.015, which is equivalent to $\beta = \frac{1}{1.015} \doteq 0.98522$.

For the **taste parameter reflecting the joy of leisure**, α, we use Auerbach and Kotlikoff's value of $\alpha = 1.5$.

The **elasticity of substitution between leisure and consumption, ρ**, is set to 0.8 (same values as in Auerbach and Kotlikoff (1987)).

3.1.1 The age-dependent (deterministic) productivity term

As mentioned previously $e_j(d_{j-1})$ is separated in the two terms $\widehat{e}_j$ and $\overline{e}_d$. $\widehat{e}_j$ is the (deterministic) age-dependent part and here we use the same equation for productivity over the life-cycle as Auerbach and Kotlikoff (1987), which in turn originate from a cross-sectional regression study by Welch (1979). This hump-shaped profile gives an earnings profile that peaks at age 30, (corresponding to an actual age of 50) at wages that are 45 percent higher than at age 1 (corresponding to 21 years). This hourly productivity profile over the life-cycle is illustrated in the figure below:
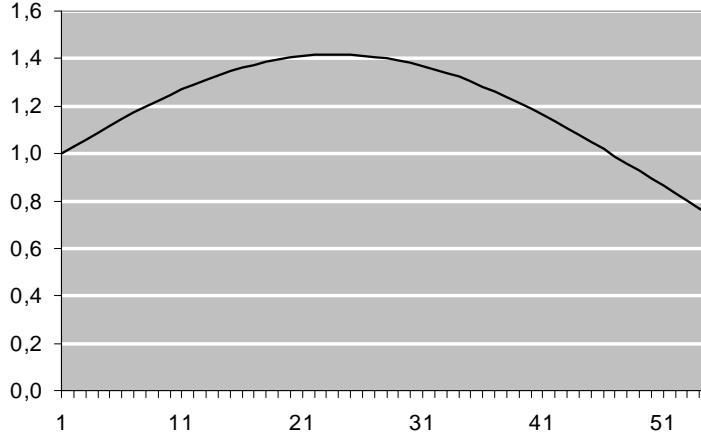
Figure 1. Deterministic age-dependent productivity term, $\widehat{e}_j$.

An agent's total productivity, $e_j\left(d_{j-1}\right)$, is uncertain, and varies around this hump-shaped trend due to the stochastic term, $\overline{e}_d$. The stochastic part, $\overline{e}_d$, is modelled as a random walk, and each year the consumer's $\overline{e}_d$-term goes either up or down (or stays the same). Even with a relatively low probability for a change in wages, of say 10 percent from one year to the next, this can amount to large differences between agents with the lowest, and the agents with the highest productivities (i.e. high variance) over the life cycle. Agents are relatively similar when young (low variance), but a small annual spread in productivity can - after 55 years on the labor market - add up to large differences between agents.

For the $\overline{e}_d$ productivity term we use a random walk on a logarithmic scale[15], where each productivity category is associated with a 10 percent higher productivity compared to the previous category. In total there are 49 categories where the middle category (number 24) has a unity productivity index, the category above has a productivity index of 1.1^1, the category below has a productivity index of $1.1^{-1}\left(=0.909\right)$, and so forth[16]. The lowest and highest productivity categories are therefore respectively $1.1^{-24}\left(=0.102\right)$ and

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[15] This is the discrete time equivalent of a geometric Brownian motion.

[16] Notice that the first category is called 0 - this is done since the first index C++ vectors per definition is 0 (and not 1 as usual in mathematics). Thus categories where the productivity term $\overline{e}_d$ is below unity are 0-23, and categories where $\overline{e}_d$ is above unity are 25-48. Thus category 24 is status quo, and $\overline{e}_{24} = 1$. In general the productivity category of the d'th category is $= (1.1)^{d-24}$. In the sensivity analysis experiments are carried out where the base number is 1.05 and 1.15 instead of 1.10.

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$1.1^{24} (= 9.849)$, which is quite a wide range. However, either extreme is something that only a very small fraction of the agents will ever experience, and most agents' productivities will fluctuate around the hump-shaped trend shown in Figure 1.

3.1.2 Specification of uncertainty: the Markov process

For the **Markov transition probabilities** $\pi_b(d)$ we model the productivity process as a random walk with no drift.
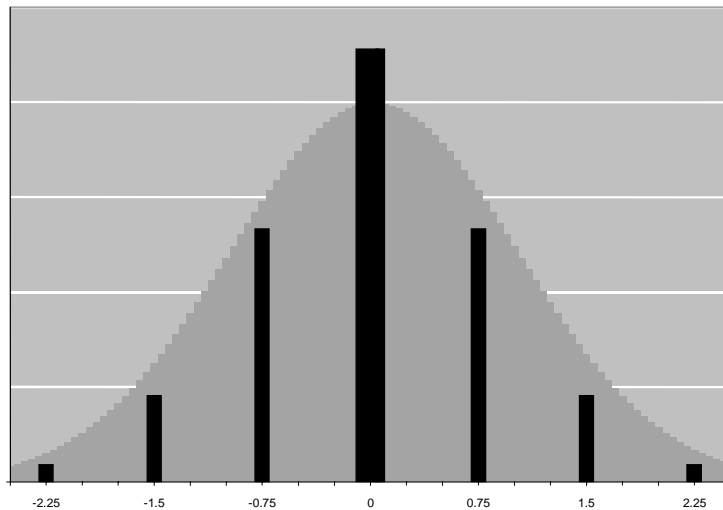

Figure 2. Discrete approximation to the Normal distribution.

This is modelled as Markov chain with 7 possibilities of transition in each period (this discrete approximation to the Normal distribution is illustrated in Figure 2). This means it is only possible to move ± 3 states in each direction (plus the possibility of remaining in the same productivity category from one period to the next). The number of states represents, as pointed out by Deaton (1992, page 185), a trade-off between making a reasonable discrete approximation to the Normal distribution and at the same time not introducing too many states, since this causes a quadratic increase in the computational burden. The table below shows the 7 states, as well as their associated transition probabilities, and their impact on productivity:

transition	-3	-2	-1	0	+1	+2	+3
std dev	-2.25	-1.50	-0.75	0	+0.75	+1.50	+2.25
probability	0.0109	0.0545	0.1598	0.5467	0.1598	0.0545	0.0109
Δpr'tivity ($\overline{e}_d$)	-24.9%	-17.4%	-9.1%	unch.	+10%	+21%	+33.1%

Table 1. Transition probabilities in the Markov process.

As mentioned each of the nodes are $\pm10\%$ apart measured in productivity terms. This number is in the same neighborhood as the value used in Huggett (1996), although smaller (this somewhat ad hoc specification makes this value a good candidate for later sensitivity analysis). An individual who from one period to the next moves 2 steps up the "productivity ladder" has 1.1x1.1=1.21 times (21 percent) higher productivity in next period (and the associated probability that he moves "two steps" up is 5.45 percent). The changes in productivity happens from one year to the next, and with this in mind the numbers in the table above do not seem unreasonable. But even with these small annual changes in productivity, the fact that the consumers live 55 periods means that there over time will be large differences between the least productive and the most productive agents. This results in a distribution of productivities for the population, that over time will become Log Normal as illustrated in the Figure 3 below. The figure shows the distribution of productivities for agents that have lived 55 periods (where the variance in the distribution of productivities within the generation peaks).
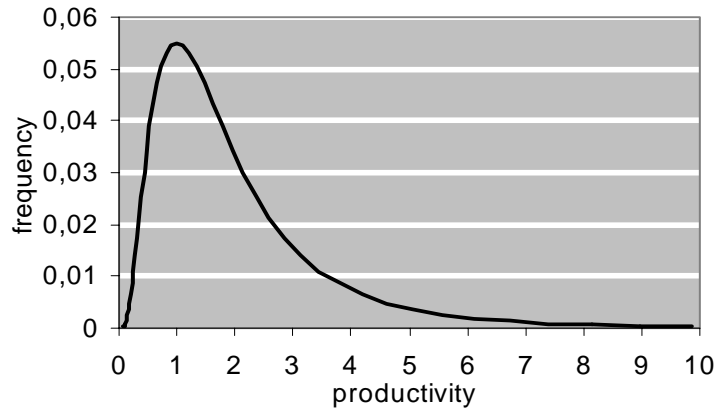

Figure 3. Distribution of productivities for the oldest agents in the population.

In order to keep the number of states low, the Markov chain is bounded below and above to 24 states on each side of the central node. This means

that the transition probability matrix is a 49x49 band diagonal matrix where this period's state only can communicate with next period's ±3 neighboring states - a total of 7 states (the 49 states is quite a large number - it can be compared to the 18 states in Huggett (1996)). This 49-state Markov transition matrix is illustrated in Figure 4 on the next page.
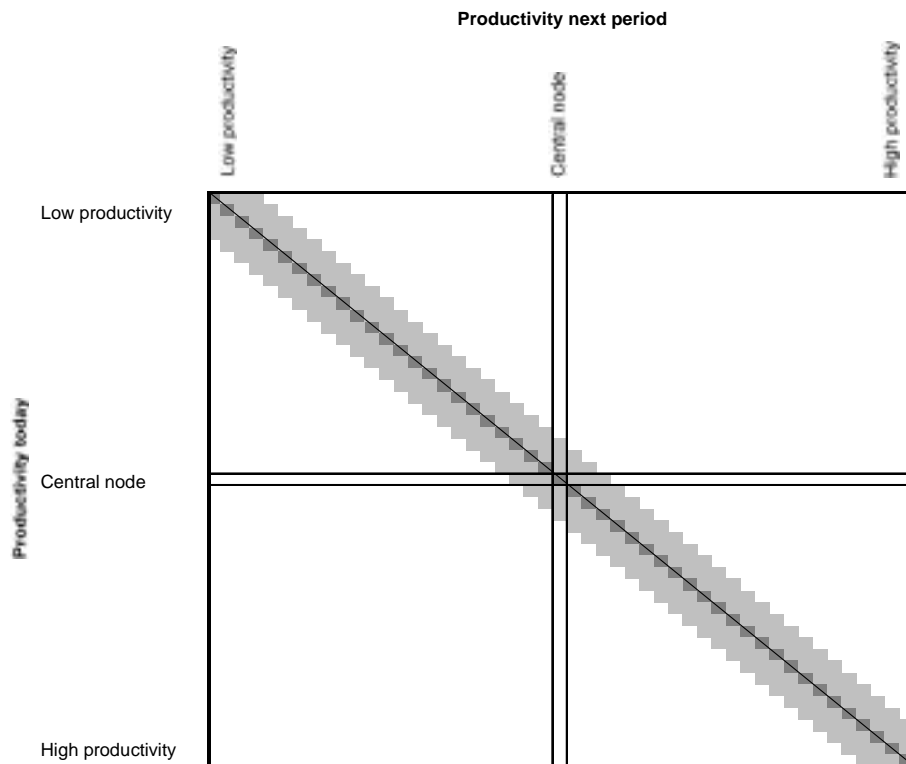

Figure 4. The Markov transition matrix.

In the figure the light grey fields indicate the states to which transition is possible from the present state (given by the row). Status quo, i.e. the individual's productivity term $\overline{e}_d$ does not change from one period to the next, is illustrated by dark grey (but note that the individual's total productivity, e_j, is likely to change from one year to the next because of the age-dependent term: $\widehat{e}_j$). The probability that this happens is shown in Table 1 above and is 54.67%. The probability that the individual's productivity goes either one "step" up or down is 15.98%. Notice that it is not possible to move from category 24 (the central node) to category 40 from one year to the next - it is only possible to transit ±3 steps in each direction.

Bounding the stochastic process above and below[17] introduces a small truncation error since some agents (although not many) will eventually reach either the upper or lower boundary (where the probabilities must be adjusted to make each row sum to unity[18] - since no agents can leave the system). Individuals that for instance "draw" a +3 productivity increase in 9 periods in a row, will reach the upper productivity boundary (with the associated productivity level that is almost 10 times larger than initially, and similarly some very unlucky individuals will reach the lower bound that is around one 10th of their initial productivity. However the number of agents that reach the upper or lower bound are 0.13 percent of all agents in the final period of their lives[19], and this is a sufficiently small fraction that (given the other sources of error that are present in these computations) it is reasonably safe to assume that this truncation does not introduce any significant errors on the results.

3.1.3   Initial distribution of productivities

The initial distribution on productivity categories is centered around the central node such that 54.67 percent have the productivity level 1.000 (node 24), 15.98 percent have the productivity associated with nodes 25 and 23, 5.45 percent have the productivity associated with nodes 26 and 22, and finally 1.09 percent begin with productivity levels associated with nodes 27 and 21. In other words $\eta_{24} = 0.5467$, $\eta_{23} = \eta_{25} = 0.1598$, $\eta_{22} = \eta_{26} = 0.0545$, and $\eta_{21} = \eta_{27} = 0.0109$.

Of cause other choices of initial distribution could be made. However the underlying idea that individuals' productivity varies over the life cycle, from less variability when young to span a larger spectrum later in life, seems quite plausible.

3.2   The producers

Since the production side is identical to Auerbach and Kotlikoff and we use the same parameters as them. The elasticity of substitution $\sigma = 1.0$, i.e.

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[17]In this case the upper and lower bounds are 1.1^{24} (= 9.849) and 1.1^{-24} (= 0.102).

[18]This is done by adding the residual probability mass at the boundary value (state 0 and 48). This implies that agents who reach the upper (lower) boundary do not get stuck, but can experience a decrease (increase) in productivity the next period.

[19]This number is found by introducing an absorbing state at the upper and the lower bound and computing the number of agents that end up in these states after 55 periods.

a Cobb-Douglas production function, the capital intensity parameter, ϵ, is 0.25, and the production function constant: $\Lambda = 0.8927$.

3.3 The public sector

The initial tax is a 15 percent income tax. In all subsequent experiments is the government revenue held constant at this level (in the literature this is known as differential incidence experiments). Clearly the size of the government sector is important, and is subject to sensitivity analysis in section 5.2.

3.4 Solving the model in Steady State

The model is solved in a manner similar to Auerbach and Kotlikoff (1987) or İmrohoroğlu, İmrohoroğlu and Joines (1993). This means performing the following procedure until convergence:

1. Guess aggregate L and K (and tax rates if they are endogenous)

2. Use the factor-demand equations ((18) and (19)) to calculate guesses for w and r.

3. Solve the Dynamic Program and obtain the decision rules c_j^*, a_j^* and l_j^*.

4. Compute the new aggregate capital stock (using equation 16) and the new labor-supply (using equation 15) (and the tax revenue if it is endogenous).

5. Check if K and L are converging - if not go to step 1, and use a convex combination of the old and the new estimates for K and L as initial guesses (with endogenous tax rates, adjust the tax rates up(down) if the revenue is too low (high)).

3.4.1 Solving the dynamic program

The general equilibrium model is solved in stationary state using the iterative techniques discussed in Auerbach and Kotlikoff (1987). The consumer's problem is solved using dynamic programming as described in Petersen (2001), as well as in Bertsekas (1995) and Ljungqvist and Sargent

(2000). Several of the techniques discussed in İmrohoroğlu, İmrohoroğlu and Joines (1998) and Judd (1998) are used to speed up the computations.

The discrete grid for assets is successively refined in each iteration, and when convergence is achieved the grid-points in the mesh are 0.0005 units apart which corresponds to 0.02 percent of the average asset holdings. The labor supply is also constrained to lie on a discrete grid: in this case the grid-points are 0.00005 units apart corresponding to 0.015 percent of the average labor supply. Refining these grids further does not alter the results significantly but influences the computation time dramatically. The practical computations are carried out in MS Visual C++ 5.0.

4 Results

This section presents the main results from the comparison between progressive and proportional labor income taxation. Before looking at the results in the present model with idiosyncratic earnings uncertainty, it is worthwhile to review the main findings in chapter 8 in Auerbach and Kotlikoff (1987) who perform a similar comparison in a deterministic framework:

- Progressive taxation induces intertemporal speculation in labor supply. With progressive taxation the marginal taxes are higher in the highly productive years in the life-cycle. To avoid these high marginal taxes, agents choose to work less in the middle ages, and more when old.

- Removing progressivity causes a welfare increase in the new steady-state of 0.69%. This amount is sensitive to the revenue-requirements: with an approx. 66% higher revenue requirement, the gain in welfare is 1.62%.

- There is an increase in labor supply of 3.9%, an increase in the capital stock of 5.1%, and an increase in production of 4.5%. With a higher revenue requirement the corresponding numbers are 6.3%, 10.7% and a 7.0%.

4.1 Aggregate results

As mentioned previously experiments in this paper are of a differential incidence kind, i.e. that the government revenue is constant (at the level that a

15 percent income tax generates under a proportional income tax). In the experiments reported below it means that the progression, κ, is increased (decreased) at the same time as the intercept, $\underline{\tau}$, is decreased (increased) (see equation (8)). The progression of labor earnings, κ, is exogenously chosen while the intercept (of labor earnings), $\underline{\tau}$, is endogenously computed such that the overall revenue is constant.[20] Notice that the capital income tax in all cases below stays at a 15 percent proportional rate.[21]

Progressivity	$\kappa=0$	$\kappa=0.125$	$\kappa=0.250$	$\kappa=0.375$	$\kappa=0.500$	$\kappa=0.625$
Production	26.832	26.648	26.489	26.345	26.219	26.104
Capital stock	120.637	119.181	117.940	116.838	115.895	115.029
Consumption	21.650	21.485	21.340	21.211	21.095	20.989
L suppl (eff)	18.915	18.818	18.733	18.656	18.588	18.525
L suppl (hours)	14.444	14.388	14.340	14.297	14.258	14.223
Avg. Lsuppl eff	1.3096	1.3079	1.3064	1.3049	1.3036	1.3024
L-inc tax (%)	15.000	13.977	13.106	12.354	11.697	11.117
Avg. Linc tax %	15.000	15.134	15.253	15.360	15.458	15.550
Util ($\times 10^{-6}$)	-5.4200	-5.4144	-5.4111	-5.4095	-5.4091	-5.4098

Table 2. Labor income tax reform.

The next table contains the same values in index (where the proportional case, $\kappa = 0$, is 100). Compared to the results from the deterministic model - the "Auerbach and Kotlikoff case" - we see an expected negative impact from progressivity on production, consumption and the capital stock. Labor supply also goes down, and compared to the case with proportional taxation ($\kappa = 0$) the decrease in the number of efficiency units is larger than the decrease in number of hours worked under progressive taxation. This means that the average efficiency per hour worked (the row labelled *Avg. L-supply eff.* in the tables above) decreases with progressivity. This is the same intertemporal labor supply effect found by Auerbach and Kotlikoff: to avoid the high progressive taxes agents choose to work less when highly productive (middle aged), and work more when less productive (young or old). This

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[20]Auerbach and Kotlikoff (1987) chose to endogenize κ, which is the other way around. Both methods give the same results, but for the purpose of the simulations in this paper, it is convenient to be able to fix the level of progression exogenously.

[21]Ideally the optimal progressivity should be determined jointly for both available tax instruments. However, this would add a (computationally challenging) dimension to the problem: from a one dimensional search (for the optimal level of progression in the labor earnings), to a two dimensional search with respect to the optimal level of progression in labor earnings *and* the optimal level of progression of capital earnings. The run-time of the present calculations makes this practically infeasible.

in turn affects savings: agents who retire later will tend to accumulate less assets at a given age, than would an agent who plan to retire earlier.

Progressivity	κ=0	κ=0.125	κ=0.250	κ=0.375	κ=0.500	κ=0.625
Production	100	99.31	98.72	98.19	97.72	97.29
Capital stock	100	98.79	97.76	96.85	96.07	95.35
Consumption	100	99.24	98.57	97.97	97.44	96.95
L suppl (eff)	100	99.49	99.04	99.63	98.27	97.94
L suppl (hours)	100	99.61	99.28	99.98	98.71	98.47
Avg. Lsuppl eff	100	99.85	99.74	99.63	99.53	99.44
L-inc tax (%)	100	93.18	87.37	82.36	77.98	74.11
Avg. Linc tax %	100	100.89	101.69	102.40	103.05	103.67
Util	100	100.105	100.165	100.192	100.202	100.189

Table 3. Labor income tax reform (*index*: proportional=100).

But whereas these negative effects were captured by Auerbach and Kotlikoff, the present model also captures the positive risk-mitigating effects from progressive taxation. This after-tax smoothing property of the progressive tax system has value for the consumers, as can be seen from the row, *Utility newborn*, in the table: *ex ante* expected utility for an newborn[22] is actually larger under the progressive scheme. This overall result stands in contrast with the deterministic case: the average agent, embodied by a newborn consumer, actually *prefers* progressive taxation. However, there is a limit to this preference for progressivity: at some level of progressivity the negative distortionary effects from increased taxation is larger than the positive after-tax income smoothing effects, and the overall effect will fall and eventually become negative. That such an optimal level in fact exists can be seen from the table: for the values of κ shown in the table, the highest utility is achieved when κ=0.5. Where exactly the optimal level of progressivity is located cannot be determined precisely from the selected values of κ above: but it appears to be located somewhere in the interval $\kappa \in [0.375, 0.625]$.

4.1.1 Optimal progressivity

In principle the location of the optimal level of progression can be determined using the model: the level of progression could be made endogenous and the

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[22] By newborn is meant an individual who has no history, i.e. does not yet know to what productivity category he initally is assigned. Formally it is calculated as $U_{newborn} = \sum_j \eta_j V_1(0,j)$ where η_j is the initial distribution when the first period begins.

expected utility of a newborn could be maximized (instead of maximizing utilities for each of the 7 initial groups of consumers given by η_i). However, more interesting than the exact optimal level given the specifications used here, is how the welfare varies with the degree of progressivity, κ. Figure 5 below shows the welfare improvement (relative to the proportional case, $\kappa = 0$) of progressive taxation for values of κ in the $[0, 1]$ range:
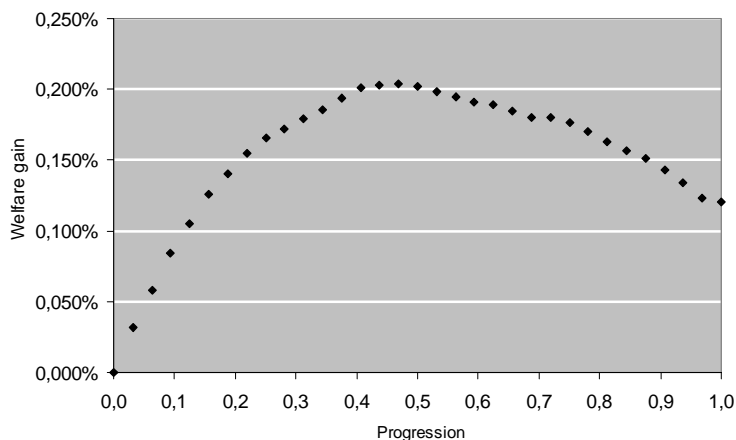

Figure 5. Welfare gains from progressivity.

The figure shows the benefits from progressivity, and at some level (the optimal level of progression, $\kappa = 0.46875$ in the figure) the welfare gain is the highest. For higher levels of progressivity the welfare gain decreases (but still remains positive). Notice that the figure is not symmetric: welfare goes up relatively fast for low levels of progression, whereas it decreases relatively more slowly for levels of progression higher than the optimal level. Secondly it should be noted that the welfare gains in all cases are positive (for the interval under consideration). There are two effects working in opposite directions: the beneficial income-smoothing effect (increasing in κ) and the negative distortionary effect from progression for a constant revenue requirement (increasing in κ). In addition there is an effect from the size of the revenue that must be raised (since higher revenue means higher distortions); in the figure this effect is kept constant, since the revenue collected in all cases is the same. In the sensitivity analysis experiments are carried out with a higher revenue requirement, which gives a different relationship between progressivity and welfare gains.

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4.2   Life-cycle behavior

The results reported in the tables above were on the aggregate level. However, it is also interesting to take a look at the underlying life-cycle effects. Let us first look at what happened on average for each age-group. The figure below shows the average end-of-period assets (a_j^*) per generation:
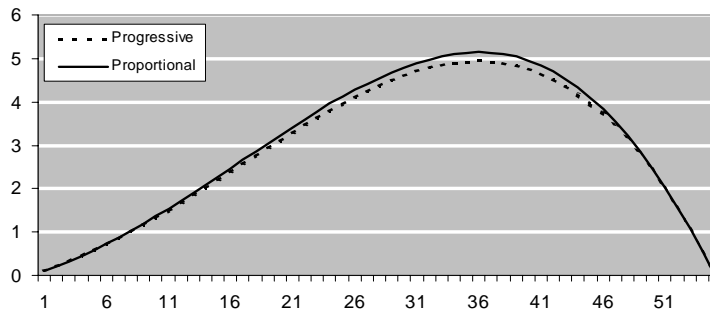

Figure 6. Average wealth over the life cycle.

In the figure above, and the following figures, *progressive* will be taken to mean the previously found optimal level of progression, i.e. $\kappa = 0.46875$. The results clearly show that the decrease in wealth from Table 3 is distributed evenly over the life-cycle: for every age is the average asset holdings larger under proportional labor income taxation. This effect was largely expected, and can be explained by the intertemporal speculation in labor supply that the next figure shows.

The results shown in this figure are in accordance with Auerbach and Kotlikoff (1987): under a progressive labor income tax individuals choose to work less when highly productive (middle aged), and more when less productive (i.e. old). Here the switch occurs after 38 years in the labor market (corresponding to a real age of 58): under progressive taxation individuals younger than 58, work less hours than they would under proportional taxation, and reversely individuals older than 58 years work more hours under progressive taxation. Thus there are two effects that have negative influence of savings: a) there is less income to save out of when younger, and b) there is a smaller need for a large nest-egg when old, since labor supply is higher.
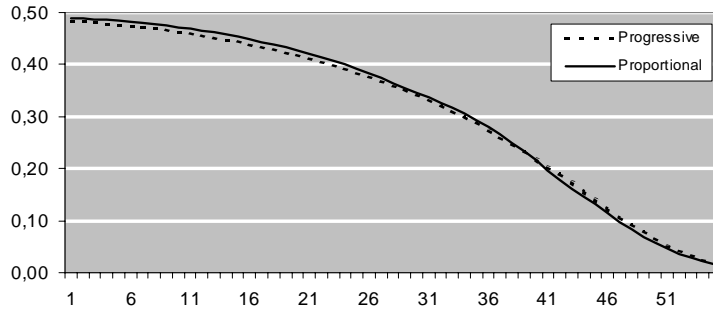
Figure 7. Average labor supply in hours over the life cycle.

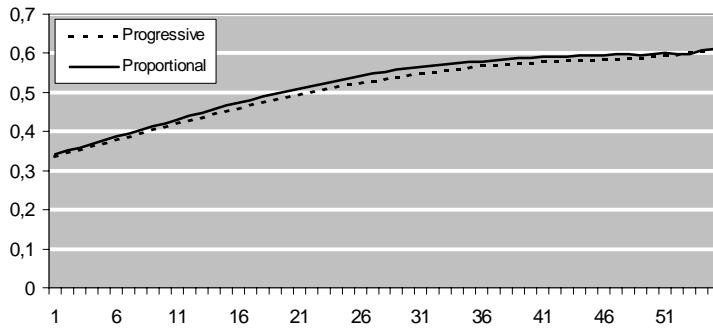Finally Figure 8 shows what happens to consumption over the life-cycle:

Figure 8. Average consumption over the life cycle.

Since life-cycle income is lower under progressive taxation so is the consumption. Notice that consumption is not smooth: the consumer's Keynes-Ramsey rule says to smooth annual utility, $u(c, l)$, from equation (9) and not consumption alone. Therefore the kink in consumption around retirement is in accordance with consumption smoothing: since the consumer cannot decrease labor supply further, he starts purchasing more consumption goods to smooth the annual utility.

4.3   Distributional effects

The life-cycle figures above showed what happened with the average agent. However behind these averages are large distributional effects. Since these

effects are not present in a deterministic model such as Auerbach and Kotlikoff (1987) these are of particular interest.

Figure 9 below shows how wealth is distributed in the case with proportional taxation. The discs in the figure represent the size of the population (i.e. the number of people) with the given age and assets - this is a way of showing 3-dimensional data. The full line in Figure 6 represented an average over these values.
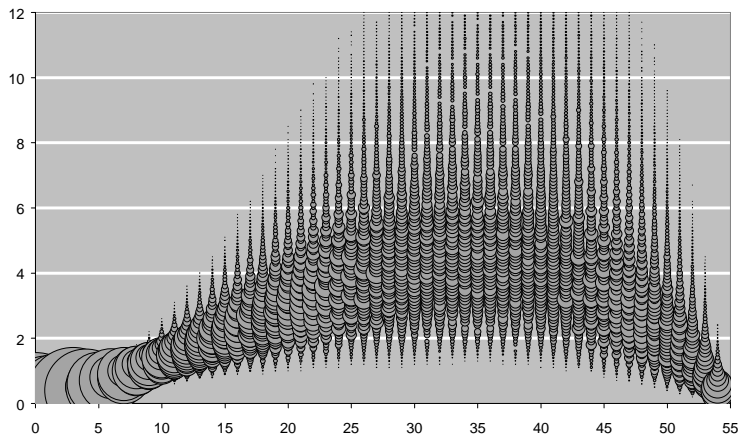

Figure 9. Wealth distribution under proportional taxation.

Recall that current productivity in the first period can be in one of 7 categories, with the majority starting with a productivity level of unity (54.67% start in category 24: $\eta_{24} = 0.5467$). This group is represented by the largest disc in the figure, since this group chooses end-of-period assets $a_1^* = 0.1037$. This particular group is then next period split into 7 smaller groups (according to the transition probabilities shown in Table 1). Another way of showing this distribution is the figure below that shows selected percentiles in the distribution of assets:
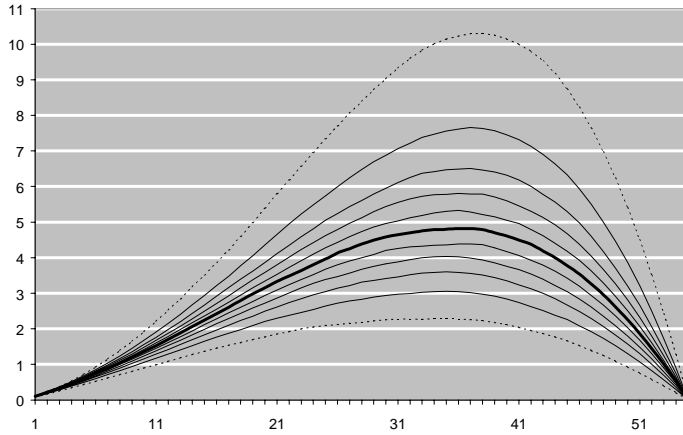
Figure 10a. Percentiles in the wealth distribution under proportional taxation.

In the figure the median (the 50th percentile) is shown with the bold line, and each of the other full lines represent the 10 percent percentiles (from the bottom, the 10th, 20th, 30th, 40th percentile, and above the median the 60th, 70th, 80th and 90th percentiles), in addition to the 2 and 98 percentile that is represented by a dotted line. This figure should be compared with Figure 10b that shows a similar figure under progressive labor income taxation.

Notice how a move to progressive labor income taxation (not surprisingly) compresses the wealth distribution: the second percentile moves marginally up under progressivity, and the 98'th percentile goes down with around 10 percent. For the richest of the rich - a *very* small group not shown in the figure - the effect is even more dramatic: the richest agent in the economy moves from a peak wealth of around 45 to 28. Similarly there are agents with zero assets in both cases, but they are *very* few.

The model can also be used to show aggregate distributional effects. Figure 11 below shows a Lorenz-curve of the pre-tax income. The horizontal axis shows the total population, and the vertical axis shows the total pre-tax income.
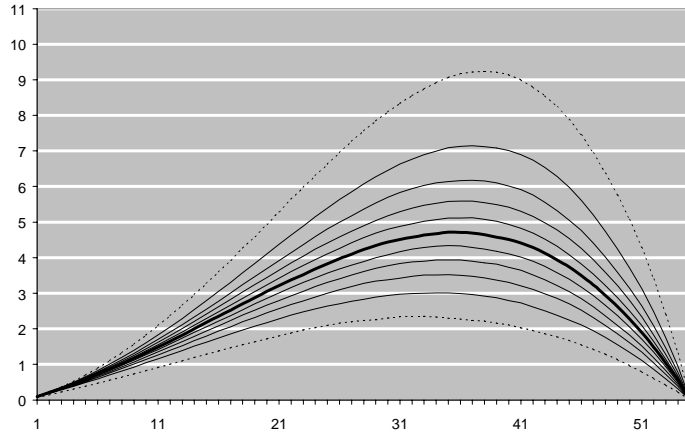
30

Figure 10b. Percentiles in the wealth distribution under progressive taxation.
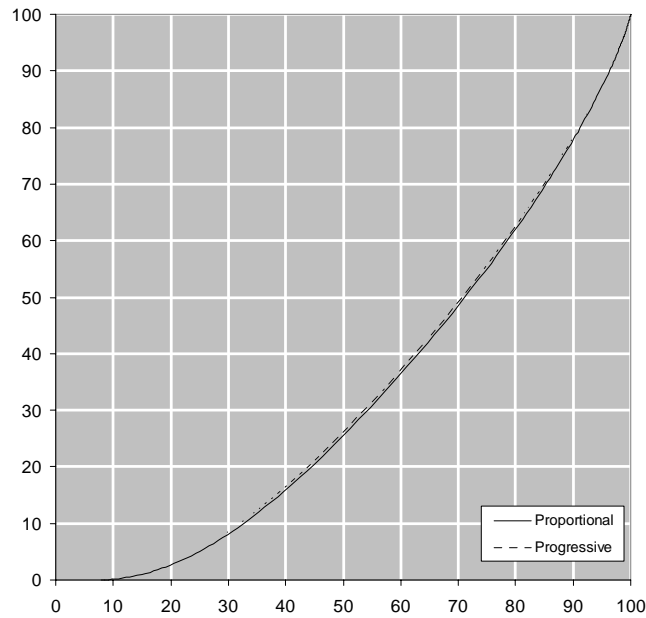

Figure 11: Lorenz curve for pre-tax income.

As expected the figure shows that the pre-tax income distribution becomes more even under progressive taxation of labor earnings; this is because individuals with high productivity, $e_j(d)$, lower their labor supply (and vice-

versa for individuals with lower productivity). This effect can also be seen from Figure 12 below, that shows what happens to pre-tax and post-tax income under proportional and progressive labor income taxation:
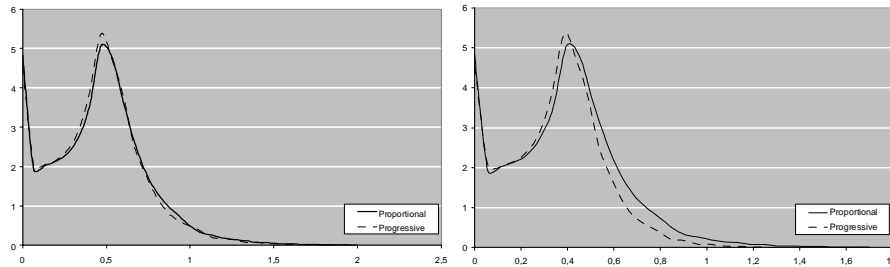

Figure 12. Distribution of labor earnings before taxes (left) and after taxes (right).

In the figures the earnings are along the horizontal axis, and the frequency along the vertical axis. The change in tax regime affects both before and after tax earnings. For the part of the population with an income below 0.6 their before-tax income is higher under progressive taxation - this effect is due to an increase in the number of hours worked. For the part of the population with a higher income (around 23% of the population) the effect is the other way around: they choose to work less, and therefore receive a lower before-tax income. As can be seen from the right figure (showing the after-tax earnings) the progressive tax system magnifies this effect: the distribution of after-tax labor earnings moves quite a lot.

Clearly the overall wealth distribution is also affected, and Figure 13 below shows the Lorenz-curves for asset-holdings. Again this change in the Lorenz-curve does not look like much, but the fact is that it takes very large changes in the distribution to make the Lorenz-curve move substantially (this is true for any Lorenz-curve). There are large changes going on in the wealth distribution, which was well illustrated by the Figures 10a and 10b above.
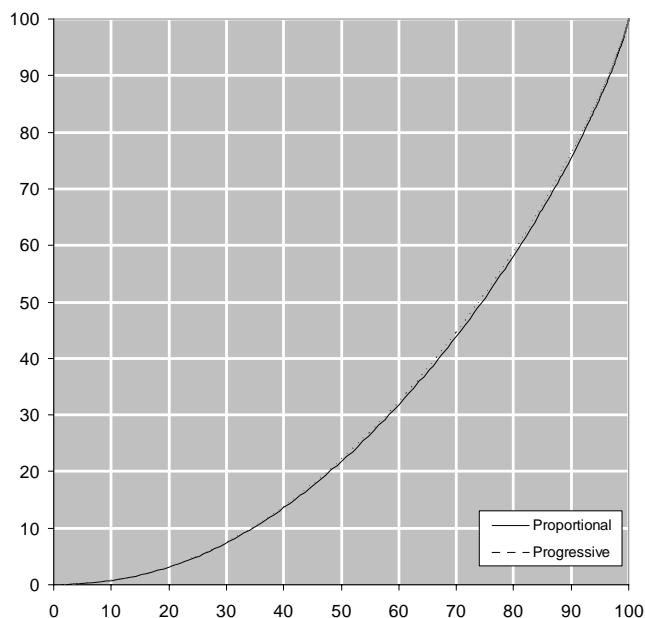
Figure 13: Lorenz curve for assets.

5 Sensitivity analysis

This section examines the sensitivity of the results with respect to central parameter values and specifications. As with any other sensitivity analysis in CGE models, this sensitivity analysis can never be complete and exhaustive. It is impossible to try every combination of parameters and assumptions; it is necessary to restrict the attention to a smaller subset of assumptions.

Compared to Auerbach and Kotlikoff (1987) the main change in the set-up used above was the introduction of uncertainty in earnings. For this reason it is important to examine the result's dependence on the (somewhat ad-hoc) specification of uncertainty. The first part of the sensitivity analysis looks at different specifications of uncertainty. The second objection to the A-K framework is that the public sector is too small, and that the 15 percent initial income tax is too low (at least with a European context in mind). Therefore the second sensitivity experiment is to increase the revenue requirement to 25 percent - a similar experiment is carried out in Auerbach and Kotlikoff (1987). The third interesting experiment, is to see

how different values of the important elasticities of substitution affect the results - again this type of experiment is also carried out in Auerbach and Kotlikoff (1987)

5.1 Alternative specifications of uncertainty

As mentioned above it is not possible to make an exhaustive examination of how the specification of uncertainty affects the results. Uncertainty was modelled as a random walk, and one may argue that some alternative specification is better. For instance earnings could be a mean-reverting process, and hence should be modelled as a so-called Ornstein-Uhlenbeck process, as in Huggett (1996). On the other hand one could argue, that productivity should be modelled as a jump-diffusion process, or by some other Ito-process.[23] Ultimately this choice between alternative formulations is an empirical matter. That is: which of the specifications fits the data best..

Since the purpose of this paper is to examine how uncertainty in earnings affect policy conclusions, and not to achieve the maximum degree of realism *per se*, these alternative specifications of the stochastic process will not be pursued here. Instead it will be examined how more or less variance/volatility in the random walk (i.e. more or less uncertainty) affects the results. Previously the steps on the discrete "productivity-ladder", $\overline{e}_d$, were 10 percent apart: the section below presents the results from the policy experiments when the steps on the "productivity"-ladder are lower (5 percent) and higher (15 percent).

Lower volatility in productivity

With less variability/uncertainty in earnings, the model becomes closer to the A-K model. In fact lowering the productivity stepsize to zero means that the model turns into the A-K model (this would make the model deterministic). Setting the steps on the "productivity-ladder" 5 percent apart (as opposed to 10 percent in the base case), means that income is a lot less variable/volatile. This implies that the lowest productivity category has the productivity index 0.31 ($=1.05^{-24}$) and the highest category has the index 3.23 ($=1.05^{24}$) - a much smaller interval than before, where the lower and upper indices were 0.1 and 9.8 (factor 10 versus the base case's factor 97).[24]

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[23] For an introduction to these stochastic processes see Dixit and Pindyck (1994, ch. 3).

[24] Here factor is used to mean the ratio between the highest and the lowest productivity categories (i.e. $\overline{e}_{48}/\overline{e}_0$).

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Therefore the earnings potential of the most productive agents is smaller, and hence the asset holdings of the richest percentile is lower: the richest agent in the economy now has asset holdings of 16, which must be compared to the richest agent in the base case who held 45 in assets.

Table 4 below shows the effect of increasing the progressivity (increasing κ) on labor income taxation. The table is in index values (relative to the proportional case, $\kappa = 0$) and must be compared to Table 3.

Progressivity	κ=0	κ=0.125	κ=0.250	κ=0.375	κ=0.500	κ=0.625
Production	100	99.500	99.045	98.630	98.253	97.908
Capital stock	100	99.249	98.556	97.927	97.358	96.840
L suppl (eff)	100	99.584	99.208	98.865	98.553	98.266
Utility newb.	100	99.861	99.719	99.579	99.442	99.311

Table 4. Alternative with lower uncertainty (productivity step=5%).

This time the effect of higher progressivity is negative, and the optimal level of progression is zero. The risk-mitigating effects of progression are still present, but are outweighed by the negative distortionary effects.

Higher volatility in productivity

Setting the steps on the "productivity-ladder" 15 percent apart (as opposed to 10 percent in the base case), means that income is a lot more variable/volatile. This implies that the lowest productivity category has the productivity index 0.035 ($=1.15^{-24}$) and the highest category has the index 28.63 ($=1.15^{24}$) - a much wider interval than before: factor 819 versus the base case's factor of 97. This means that the earnings potential of the most productive agent now is much higher relative to the base case, and therefore the asset holdings of the richest percentile is higher: the richest agent in the economy has asset holdings of 124, which must be compared to the richest agent in the base case who held 45 in assets.

Table 5 below shows the effect of increasing the progressivity (increasing κ) in labor income taxation. The table is in index values (relative to the proportional case, $\kappa = 0$) and must be compared to Table 3 (for the base case).

Progressivity	$\kappa=0$	$\kappa=0.125$	$\kappa=0.250$	$\kappa=0.375$	$\kappa=0.500$	$\kappa=0.625$
Production	100	98.982	98.165	97.501	96.936	96.467
Capital stock	100	97.948	96.450	95.339	94.447	93.773
L suppl (eff)	100	99.327	98.741	98.230	97.778	97.380
Utility newb.	100	100.967	101.569	101.947	102.210	102.409

Table 5. Alternative with higher uncertainty (productivity step=15%).

Utility of a newborn goes up with increasing progression - and within the range of progressivity shown in the table (κ below 0.625) it keeps increasing; in this case there is no optimal level of progression (within the range under consideration). But the main result from the base case still holds: progression improves welfare.

5.2 Higher revenue requirement

Obviously the effects on progressivity depend crucially on the size of the government sector. A priori it is not clear how an increased revenue requirement affects the conclusions drawn from the analysis, i.e. how it affects the trade-off between the negative effect on labor supply and the positive risk-mitigating effect. Table 6 below shows the effect of increasing the income tax to 25 percent under proportional taxation (where it was 15 percent in the base case). The model is specified as the base case (e.g. the productivity steps are 10 percent apart). As before the values in the table are index (relative to the proportional case, $\kappa = 0$, under the higher revenue requirement):

Progressivity	$\kappa=0$	$\kappa=0.125$	$\kappa=0.250$	$\kappa=0.375$	$\kappa=0.500$	$\kappa=0.625$
Production	100	98.661	97.457	96.409	95.448	94.142
Capital stock	100	97.379	95.244	93.440	91.862	89.699
L suppl (eff)	100	99.092	98.205	97.420	96.674	95.671
Utility newb.	100	100.027	99.630	99.305	98.866	98.139

Table 6. Alternative with higher revenue requirement.

With the higher revenue requirement, lower levels of progression can affect the welfare of a newborn positively, whereas for higher levels of progression it is detrimental to welfare. For the selected levels of progression shown in the table, it is likely that the optimal level of progression lies somewhere in the interval $\kappa \in]0.0; 0.25[$.[25] Notice that this is a lower optimal level of

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[25] As in the previous simulations it would be nice to be able to make a graph of the

progression than in the base case simulations.

5.3 Alternative elasticities of substitution

Apart from the obvious important alternative experiments performed above, it is also relevant to see how the important parameters in the standard A-K set-up affect the result. There are many candidates for the important parameters in the model, but the attention here will be limited to the two elasticities of substitution: (i) the intertemporal elasticity of substitution (γ), and (ii) the intratemporal elasticity of substitution between consumption and leisure (ρ). In the calibration of these parameters the same values were chosen as Auerbach and Kotlikoff (1987), and we will choose the same the alternative specification of these values as these authors.

In the baseline simulations the intertemporal elasticity of substitution (γ) was set equal to 0.25, and below experiments will be carried out with $\gamma = 0.1$ and $\gamma = 0.5$. The intratemporal elasticity of substitution between consumption and leisure (ρ) was 0.8 in the base case, and in the alternative formulations below simulations will be performed with $\rho = 0.3$ and $\rho = 1.5$. Table 7 shows the results of a move from proportional income taxation (at the 15 percent rate as in the base case) to progressive labor income taxation (with $\kappa = 0.25$).

	Base case	intertemporal elasticity of substitution		elasticity between consumption and leisure	
		low (0.1)	high (0.5)	low (0.3)	high (1.5)
Production	98.72	98.71	98.64	99.40	95.48
Capital stock	97.76	97.50	97.57	96.84	95.11
Labor supply	99.04	99.11	98.99	99.90	95.59
Util newborn	100.165	101.864	100.014	100.001	100.642

Table 7. Alternative elasticities of substitution.

When the intertemporal elasticity of substitution is high, this means that

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utility as a function of the level of progression (similar to Figure 5). This would require the model to be solved a large number of times, and since each simulation takes a couple of days to complete, this is unfortunately not practically feasible. Future research with faster computers - as well as better solution strategies for solving the dynamic programming - may make this possible. Inspiration for improving the solution method should probably be found in Judd (1996, 1998, 1999) and Rust (1996, 1997).

the consumer has better opportunities to substitute consumption over time. In this case variability in earnings hurt the consumer less, and therefore the risk-mitigating effects of progression are lower. The table confirms this: the utility gain for a newborn is a lot lower (higher) when the intertemporal elasticity of substitution is high (low). However, the results are reasonably robust to these rather large changes in the elasticities of substitution; progressive taxation of labor income can still be welfare improving (but whether there is an optimal level of progression with the alternative elasticities cannot be determined from the table).

6 Summary

This paper investigated the welfare implications of introducing progressive taxation of labor earnings in a CGE model. The consumers faced uninsurable idiosyncratic earnings uncertainty, borrowing-constraints and an endogenous labor decision - the rest of the model was similar to Auerbach and Kotlikoff (1987). In this set-up the welfare implications of progressive taxation were not only negative, as was the case in Auerbach and Kotlikoff (1987, chapter 8). In the presence of labor earnings uncertainty and borrowing constraints, progressive taxation has a risk-mitigating effect, since it distributes relative tax payments unevenly, such that the consumers with the highest income pay the most, and that those with low incomes pay less.

The analysis showed that indeed it was possible to find positive welfare implications of progressive taxation. Obviously middle-aged consumers with high productivity (and hence high income) dislike progressive taxation (since in isolation it implies a higher average tax rate for them), and middle-aged consumers with a low productivity (and hence low income) prefer progressive taxation (since in isolation it implies a lower average tax rate for them). However for the average new-born agent who does not yet know his future earnings stream, the overall effect is positive: he actually *prefers* progressive taxation.

Nonetheless, there is a limit to this preference for progression: at some level of progressivity the negative distortionary effect from increased taxation is larger than the positive risk-mitigating effect, and the overall effect will fall and eventually become negative. With the specification used, it turned out that there was an optimal level of progression in the model, where welfare was approximately 0.2 percent higher than in the case with proportional labor income taxation. While this number may seem small, it is interesting

that it is positive.

The sensitivity analysis illuminated the robustness of these results. Not surprisingly the results were sensitive to the "degree of uncertainty" for the consumers (i.e. the variability/volatility in the stochastic process governing labor earnings). With low volatility in productivity the negative distortionary effects dominated, and progressivity was found to decrease welfare (in the limit zero variability in earnings would make the model converge to the model used by Auerbach and Kotlikoff (1987)). On the other hand, higher earnings uncertainty made agents prefer progressive taxation of labor earnings. In a situation where the government revenue requirement was higher, the sign of the welfare effect depended on the progressivity in the labor income tax: with a low degree of progressivity in labor earnings welfare is higher than under proportional taxation, but for high degrees of progressivity the consumers prefer proportional taxation. Finally the results' dependence on the elasticities of substitution were tested, and the sign of the overall conclusions held, even though the quantitative effects were different.

6.1 Suggestions for future research

Clearly the model used here is not "the model to end all models" and can be improved. A first import task is on the data side, and would be to get a better estimate of the earnings process. While the simple random walk specification used here presents an improvement over a deterministic model, it is probably not the most realistic description of earnings uncertainty. For accurate policy evaluations - as opposed to the more academic exercises in this paper where calibration is taken lightly - getting this modelled correctly would be very important (as indicated by the sensitivity analysis).

One could also argue that the way the revenue from taxation is spent represents an over-simplification. In reality, only a part of the government sector's revenue is spent on public goods, but a large part is handed back to the consumers as income transfers. Typically these transfers are graduated by income (or for instance by asset holdings). This risk-mitigating feature of these transfers from the government is not captured by the present model. This calls for the introduction of means-tested transfers, or public-assistance programs as in Hubbard, Skinner and Zeldes (1994, 1995), or other forms of transfers that are not given uniformly.[26]

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[26] See also Rust and Phelan (1996) who, in a dynamic programming framework, analyze how institutional details of the U.S. Social Security and Medicare system affect individual

Finally, it would be an advantage to introduce some kind of endogenous human capital formation, for instance along the lines of Heckman, Lochner and Taber (1998). This would allow agents with low productivity to go back to school and increase their productivity, rather than in the present model where they can only sit and wait for their productivity to go up (or down). Some work has been done in this field by Lord and Rangazas (1998), who analyze endogenous human capital formation in a 3-period model with earnings uncertainty; here an extension to the 55-period framework used in this paper would undoubtedly improve the quality of the analysis.

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behavior.

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