Statistik (udkast)
MODELGRUPPEN Arbejdspapir

Dan Knudsen

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Comparing ADAM to the textbook

Abstract:

A large macro model of the Danish economy is compared to a textbook AS-AD model for a small open economy with fixed exchange rates. The basic mechanism looks rather similar, but the dynamics differ. In order to mimic the hump-shaped output response of the large macro model, the stylised output gap equation of the textbook is formulated as an error-correcting equation.

Nøgleord: model

1. Introduction

Large macro-econometric models like ADAM (Annual Danish Aggregate Model) with 12 industries, 5 institutional sectors, 2,500 endogenous and 1,000 exogenous variables seem quite different from the handy models you meet in economic textbooks. But the similarities can be greater than the differences.

This paper compares the two models and shows what it takes before the textbook model reproduces the response of ADAM to a demand shock. The textbook model is an Aggregate Supply-Aggregate Demand model taken from the textbook *Introducing Advanced Macroeconomics* by Sørensen and Whitta-Jacobsen (2010). Denmark has a fixed exchange rate policy vis-á-vis the euro, and the AS-AD model found in chapter 24 is like ADAM designed to describe a small open economy with fixed exchange rates. Besides, both the textbook model and ADAM reflect the traditional synthesis between Keynesian and neo-classical theory by being demand-driven in the short term and supply-driven in the long term.

The model parameters suggested in the textbook are not that different from the parameters that ADAM would imply. The crucial difference reflects the time lag in ADAM's demand equations. There is no lag in the aggregate demand equation of the textbook AS-AD model, but the AS-AD model is able to mimic ADAM's demand side if we give the stylised output gap equation of the textbook into a more general error-correcting form.

In the following four sections, we explain how the AS-AD model relates to ADAM, evaluate key parameters of the AS-AD model on the basis of ADAM, estimate a dynamic output gap equation mimicking ADAM, and conclude.

2. A stylized AS-AD model and a macro-econometric model

The AS-AD model for a small open economy with fixed exchange rate is presented in Sørensen and Whitta-Jacobsen (2010) chapter 24 p. 730 by the following three equations:

AD: $\pi = e^r_{-1} + \pi^f - (1/\beta_1) \cdot (y - \bar{y} - z)$ (1)

SRAS: $\pi = \pi^f + \gamma \cdot (y - \bar{y})$ (2)

Real exchange rate: $e^r = e^r_{-1} + \pi^f - \pi$ (3)

π is domestic inflation, π^f is foreign inflation, e^r is real exchange rate written as foreign price over domestic price in a common currency, y is output, and z is a demand shock. The textbook includes a supply shock in the short-run aggregate supply (SRAS) equation (2), but we shall only work with the demand shock z. A bar indicates long-run equilibrium value, so $\bar{y}$ is long-run equilibrium output and $y - \bar{y}$ is the output gap.

There are two parameters in the textbook AS-AD model: β_1 is a price elasticity determining the impact of the real exchange rate on the output gap, and γ determines the impact of the output gap on inflation.

All three equations can be related to ADAM. It is straightforward to relate the definitional equation in (3) to ADAM, because the real exchange rate of ADAM is defined by a similar equation.

It is also easy to compare the short-run supply equation in (2) to ADAM, because the Phillips curve in ADAM looks much like equation (2). The main difference is that the Phillips curve of ADAM uses the unemployment gap instead of the out gap. Besides, foreign inflation does not enter ADAM's Phillips curve with a coefficient of 1, but that is only a formal difference as long as we are not changing foreign inflation.

We also note that ADAM's Phillips curve determines the wage inflation, while the textbook equation (2) determines price inflation. However, the textbook interprets its prices as GDP deflators that are taken to be proportional to the wage rate in the absence of productivity shock, and we are not shocking productivity. Consequently, it makes sense to compare the price determination of the AS-AD-model directly to the wage determination of ADAM.

Now, we have related equation (2) and (3) of the AS-AD model to ADAM. The aggregate demand equation in (1) does not look that comparable to anything in ADAM, but it helps to put the output gap on the left-hand side and use the definitional relation in (3) to replace $e^r_{-1} + \pi^f - \pi$ by the contemporary exchange rate e^r:

AD: $y - \bar{y} = \beta_1 \cdot e^r + z$ (1')

This formulation reflects that the demand for output in an open economy depends on competitiveness, which is represented by the real exchange rate. ADAM contains the same mechanism. The output gap in (1') is also influenced by the shock variable z representing shocks to domestic demand or to the cyclical part of foreign demand, i.e. to the part of foreign demand not governed by real exchange rate. Again, this is comparable to ADAM, where output can be influenced by a long range of demand shocks.

Basically, equation (1') is representing all the rest of ADAM, i.e. all equations beside the Phillips curve equation and the exchange rate definition. In other words, equation (1') is representing the demand side of ADAM, because without Phillips curve, there is no mechanism to clear the labour market and make the output gap zero in ADAM. Without Phillips curve, the wage rate is exogenous and ADAM will work like a Keynesian demand-driven model, also in the long term.

As previously mentioned, there are about 2500 equations in the ADAM model, and we have to admit that the aggregate AD equation in (1') cannot represent 2498 ADAM equations in the same way that (2) and (3) can represent the Phillips curve and exchange rate definition. A general difference is made by the lags in the demand equations of ADAM. There is no lag in the textbook AD equation (1') implying that we get the full negative output impact of a price

increase in the year of the price increase. In ADAM, there is a lag between wage and price increase plus a lag between the price increase and its negative impact on foreign trade and output. Moreover, the output in ADAM drives the capital formation that has repercussions on output. The lags and the capital formation make the output response of ADAM more slow and hump-shaped. That is the most obvious difference between ADAM's demand side and the aggregate AD equation of the AS-AD model.

The three equations: (1'), (2) and (3), constitute an AS-AD model determining output gap, inflation and real exchange, and they can be combined to an equation that makes output gap a function of itself and the exogenous demand shock z.

To see that, we use (2) and (3) to replace the real exchange rate in (1') by the cumulated output gap, i.e. replace e^r by the term $-\gamma \cdot \sum \hat{y}$, where the output gap expression $y - \bar{y}$ has been replaced by $\hat{y}$ to shorten the notation. Isolating the contemporaneous output gap on the left-hand side gives us equation (4):

$$\hat{y} = (\beta - 1) \cdot \sum \hat{y}_{-1} + \beta \cdot z\,, \qquad \beta = 1/(1 + \gamma \cdot \beta_1) \qquad (4)$$

In this output gap equation, parameter β describes both the speed of adjustment in the output gap and the first-year reaction in the output gap to a demand shock. The chosen formulation in (4) emphasizes that the cumulated output gap is a function of shock z in the long run, while the output gap returns to zero in the long run, if z is constant. The textbook presents the same output gap equation without cumulating term:

$$\hat{y}_{+1} = \beta \cdot \hat{y} + \beta \cdot (z_{+1} - z), \qquad \beta = 1/(1 + \gamma \cdot \beta_1) \qquad (4')$$

cf. p. 732 in Sørensen and Whitta-Jacobsen (2010). The textbook formulation follows from (4) and you come from (4) to (4') by leading variables 1 period, taking first order differences, and collecting terms.

When comparing the output gap reaction of equation (4) to ADAM it will be crucial to determine the demand shock variable z. In the textbook, the demand shock variable is described by the following equation (5):

$$z = -\beta_2 \cdot (r^f - \bar{r}^f) + \beta_3 \cdot (g - \bar{g}) + \beta_4 \cdot (y^f - \bar{y}^f) + \beta_5 \cdot (\varepsilon - \bar{\varepsilon}) \qquad (5)$$

Equation (5) is an auxiliary equation that formulates the demand shock as a linear function of exogenous variables deviating from their long-run values. Demand shocks can come from, e.g. foreign interest rate r^f, foreign output y^f, government demand g, and breaks in the residual ε of any demand relation that the AD-equation can be said to represent. Possible breaks comprise a confidence-based shift in the consumption function or a technology-based shift in the investment function.

The textbook equation in (5) clarifies how one should interpret shocks to the textbook model, but without estimates for the β parameters it is difficult to calculate the shock variable z. In order to calculate the shock variable, we shall not suggest values for the parameters in (5) but simply use ADAM with exogenous wage rate to calculate the demand shock. If ADAM with exogenous wage is called ADAM_K with K as in Keynes, our auxiliary equation for the demand shock z can be written:

$$z = ADAM\_K\ (r^f - \bar{r}^f, g - \bar{g}, y^f - \bar{y}^f, \varepsilon - \bar{\varepsilon}, etc.) \qquad (5')$$

In this equation (5'), the ADAM_K function makes the output gap reaction a function of shocks to the ADAM_K model. Variables with a bar represent the base line of ADAM_K, so that shocks on the right-hand side of equation (5') are changes to exogenous ADAM variables, while the resulting demand shock z is the effect on the output gap calculated by ADAM_K.

With ADAM_K to calculate shocks for the AS-AD model, we can shock any exogenous variable in ADAM implying that there are many potential arguments for the right-hand side of (5'). Using ADAM_K to formulate demand shocks for the textbook model can be just as interesting as using the textbook auxiliary equation in (5).

Our approach of using ADAM_K to feed the textbook model can also be seen as using the textbook model to crowd out the impact of demand shocks on the output gap instead of leaving it to the normal wage clearing of the labour market in ADAM. In other words, we are introducing a hybrid of ADAM and the textbook model.

As already said, ADAM_K is a short-run model with exogenous wage rate implying that permanent demand changes have a permanent impact on output and output gap in ADAM_K. However, ADAM_K does contain the production functions and capital formation of ADAM, so ADAM_K is more than a traditional short-run model because you need a higher capital stock to produce a higher output. ADAM_K resembles old ADAM versions from the era of incomes policy when the wage rate was assumed to be determined by policy makers.

The output gap equation in (4) implies that a constant increase of 1 in the demand shock z will change the cumulated output gap $\sum \hat{y}$ by $\beta/(1-\beta)$. This is equal to $1/(\gamma \cdot \beta_1)$ illustrating that the two AS-AD parameters, β_1 and γ, affect the output gap similarly. The larger the price elasticity β_1 of demand, the smaller is the domestic price rise needed to neutralize a higher z. And the larger the output gap impact γ on price formation, the smaller is the needed output gap reaction measured by the cumulated output gap $\sum \hat{y}$.

3. Evaluating the two AS-AD parameters

We start by evaluating the real exchange rate elasticity β_1 of output demand.

In the textbook, the real exchange rate is referring to GDP deflators that are assumed to be proportional to the hourly wage rate. Wages and prices are more different in ADAM, where the use of imported capital makes the wage elasticity of value added deflators less than one. Moreover, the ADAM price of exports has its wage elasticity reduced by the content of imported inputs in exports, while the textbook price of exports is represented by a GDP deflator.

The price formation in ADAM is more complicated than the price formation of the textbook AS-AD model, but the wage formation in ADAM resembles the price formation in the textbook. Consequently, we choose to compare a wage-based real exchange rate from ADAM to the price-based real exchange rate of the textbook.

With nominal exchange rate and foreign wage exogenous in ADAM, we can calculate the long-run demand elasticity of output with respect to the real exchange rate as the long-run output effect of reducing the hourly wage rate by 1 per cent in ADAM with exogenous wage, i.e. in ADAM_K. This model calculation is easy but it is not obvious which ADAM variable we should use for measuring the output effect.

The textbook focuses on GDP, which makes it natural to use the ADAM-calculated impact on GDP, or perhaps on GDP at factor cost to avoid the effect from indirect taxes in fixed prices. However, we have no measure of long-run GDP in ADAM, which makes it difficult to describe the GDP-related output gap.

It conforms better with the structure of ADAM to focus on unemployment. ADAM has an explicit measure of long-run unemployment and the gap between actual and long-run unemployment determines the wage change in ADAM. This makes it straightforward to let the unemployment gap illustrate the output gap. More specifically, a positive unemployment gap means that unemployment is above its long-run value, so the output gap could be illustrated by the unemployment gap with a minus.

Subtracting ADAM's long-run unemployment from its labour force determines long-run employment, which allows us to base the output gap measure on either ADAM's employment or ADAM's "desired employment". The latter option is interesting because desired employment reacts like output.

Desired employment N^d is derived from the production functions of ADAM by cost minimization at given output Y, given working hours per employee, and given the relative factor price: user cost/wage rate or u/w. In the first year, there is little response in factor prices and working hours are exogenous. This makes the immediate change in desired employment proportional to the immediate output change. In a longer run, the change in relative factor prices will affect labour productivity and introduce a wedge between desired employment and output.

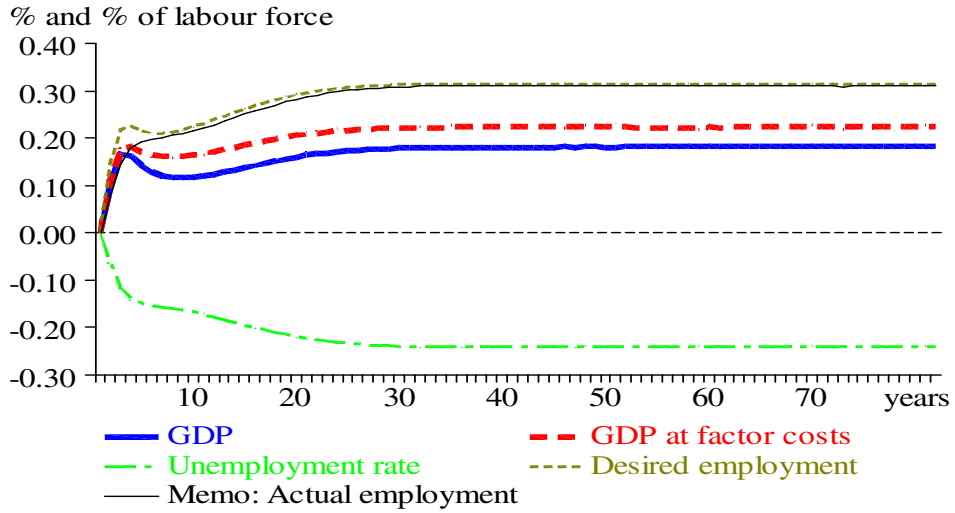
ADAM's employment converges to desired employment in the long run, so there is no difference between long-run actual and long-run desired employment. Using the gap in desired employment as the output gap implies that the output gap $\hat{y}$ of ADAM is set equal to $N^d(Y, u/w) - N^*$, where long-run employment N^* is labour force minus long-run unemployment.

One advantage of this output gap measure is that total desired employment N^d constitutes a simple sum of desired employment in the 12 industries of ADAM, and that N^d can be compared directly to the long-run potential employment N^*

when calculating the output gap. Thus, we avoid deriving a potential GDP on the basis of 12 production functions.

Now, we calculate the impact of 1 per cent lower hourly wage rate using ADAM _K where the wage rate is exogenous. The resulting impact on normal GDP at fixed market prices, on GDP at fixed factor costs, on unemployment and on desired employment are illustrated in figure 1. For comparison, the impact on actual employment is also shown.

Figure 1: Impact of hourly wage rate minus 1% (β_1 estimate), ADAM_K



The long-run impact on ADAM's endogenous variables is reached after some 30 years with housing investments as the last major component to reach its new equilibrium. Long-run impact is 0.18 per cent on standard GDP, 0.22 per cent on GDP at factor cost, minus 0.24 percentage points on unemployment rate, and 0.31 per cent on desired employment.

The 1 per cent lower price of labour decreases labour productivity, which helps to explain that the effect on employment is larger than the effect on GDP. Besides, the lower domestic wage decreases real income and private consumption, and a lower share of private consumption will tend to reduce GDP in fixed market prices by reducing the content of indirect taxes. The lower content of indirect taxes does not affect GDP at factor costs, which increases by more than standard GDP in market prices. Finally, the unemployment rate drops by only 0.24 percentage point because the 0.31 per cent increase in employment is partly neutralized by a higher labour force that partly adapts to higher employment in ADAM.

The output impact of 1 per cent lower exogenous wage rate in ADAM is comparable to the output impact of 1 per cent higher real exchange rate in the AS-AD model, so now we have four β_1 estimates between 0.18 and 0.31 for ADAM's AD equation. All four are clearly lower than the β_1 of 0.72 suggested in the textbook. However, the difference is to a large extent formal reflecting that the textbook calibrates β_1 to reflect the price elasticity of output, while our ADAM calculation describes the wage elasticity of output.

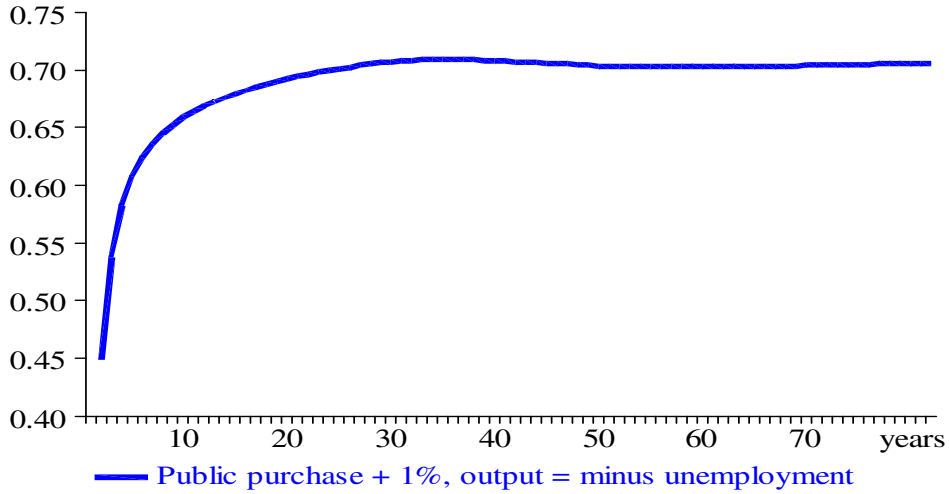
The difference between our ADAM calculation and the textbook calibration of β_1 is further scrutinized in appendix.

To quantify the stylised textbook model, we also need to determine the parameter γ for the output gap in the short-run supply equation.

For this assessment, we choose to represent ADAM's output gap by minus the unemployment gap that enters the wage relation of ADAM, which comes close to the short-run supply equation of the textbook. And in order to estimate γ, ADAM is used to calculate the effect of a permanent 1 per cent increase in public purchase. More specifically, γ is derived as the long-run effect on real exchange rate, ∇e^r, divided by the long-run effect on ADAM's cumulated output gap, $\nabla \sum \hat{y}$, where the effect-describing operator ∇ indicates that the baseline variable is subtracted from the calculated variable.

This gives us a γ-estimate close to 0.70, cf. figure 2. The 0.70 reflects the coefficient for the unemployment gap in ADAM's wage relation supplemented by a contribution from the inflation term of the wage relation.

Figure 2: Wage impact/cumulated output gap (γ estimate), ADAM



This γ-estimate of 0.70 for the AS-AD model representing ADAM is clearly higher than the γ of 0.3 in the 2010 edition of the textbook. And, it is only natural that our ADAM-calculated γ is the highest, because wages react more than prices.

In an ADAM calculation, it takes almost 3 per cent wage change to produce a 1 per cent price change of manufactured exports suggesting that the wage elasticity of this price is one third. Thus, the wage-related γ of 0.70 corresponds to a price-related γ of around 0.23 or somewhat lower than the textbook γ of 0.3. Incidentally, the 0.23 is closer to the γ of 0.2 suggested in the 2005 edition of the textbook. Note also, that a price-related demand elasticity β_1 of 0.72 for the textbook model corresponds to a wage-related β_1 of 0.24 for ADAM, if the wage elasticity of prices is one third in ADAM.

The speed of output gap adjustment in the stylised AS-AD model is described by the β parameter combining β_1 and γ, cf. the output gap equation (4) in the previous section 2. Equation (4) implies that, without shocks, the output gap will adjust towards zero according to a simple autoregressive equation:
$\hat{y}_{+1} = \beta \cdot \hat{y}$
The textbook parameters, $\beta_1 = 0.72$ and $\gamma = 0.3$, imply a β of 0.82 and a half-life of 3.5 years ($0.82^{3.5}$=0.5), while the ADAM-calculated parameters , $\beta_1 = 0.24$ and $\gamma = 0.70$ (output gap represented by minus unemployment gap), imply a β of 0.86 and a half-life of 4.5 years.

This discussion of parameters has summarized the perhaps moderate difference between the stylized AS-AD model parametrized in the textbook and the same stylized AS-AD model parametrized on the basis of ADAM. However, we have not yet covered the full difference between the textbook and ADAM. Notwithstanding the similarity of the crucial β parameter, it is necessary to enhance the dynamics of a stylized AS-AD model before it can mimic ADAM.

4. An estimated ADAM output gap equation

In the textbook AS-AD model, the dynamic adjustment of the output gap is created solely by the short-run supply equation in (2) determining the adjustment path for the real exchange rate. On the demand side, output adjusts immediately to the exchange rate according to equation (1).

In ADAM, the Phillips curve produces similar dynamics in the nominal wage rate and thereby also in the wage-based real exchange rate. However, in ADAM it takes time for prices to react to wages and time for foreign trade to react to prices. Moreover, an increase in the Danish wage level raises the real income of consumers. This Harberger-Laursen-Metzler effect implies that a positive demand shock to ADAM will be accompanied by higher private consumption enhancing the need for crowding out via higher wage rates. Moreover, the higher demand triggers a hump-shaped reaction in investments in ADAM reflecting the accelerator mechanism. All in all, adjustment in ADAM is more complicated than adjustment in the textbook AS-AD model.

In order to mimic the richer dynamics of ADAM, we shall put the output gap equation in (4) on a less restricted error-correcting form:

$$\Delta\hat{y} = \alpha_1 \cdot \Delta z - \theta \cdot \hat{y}_{-1} + \alpha_2 \cdot \sum \hat{y}_{-1} + \alpha_3 \cdot z_{-1} \qquad \text{(4ADAM)}$$

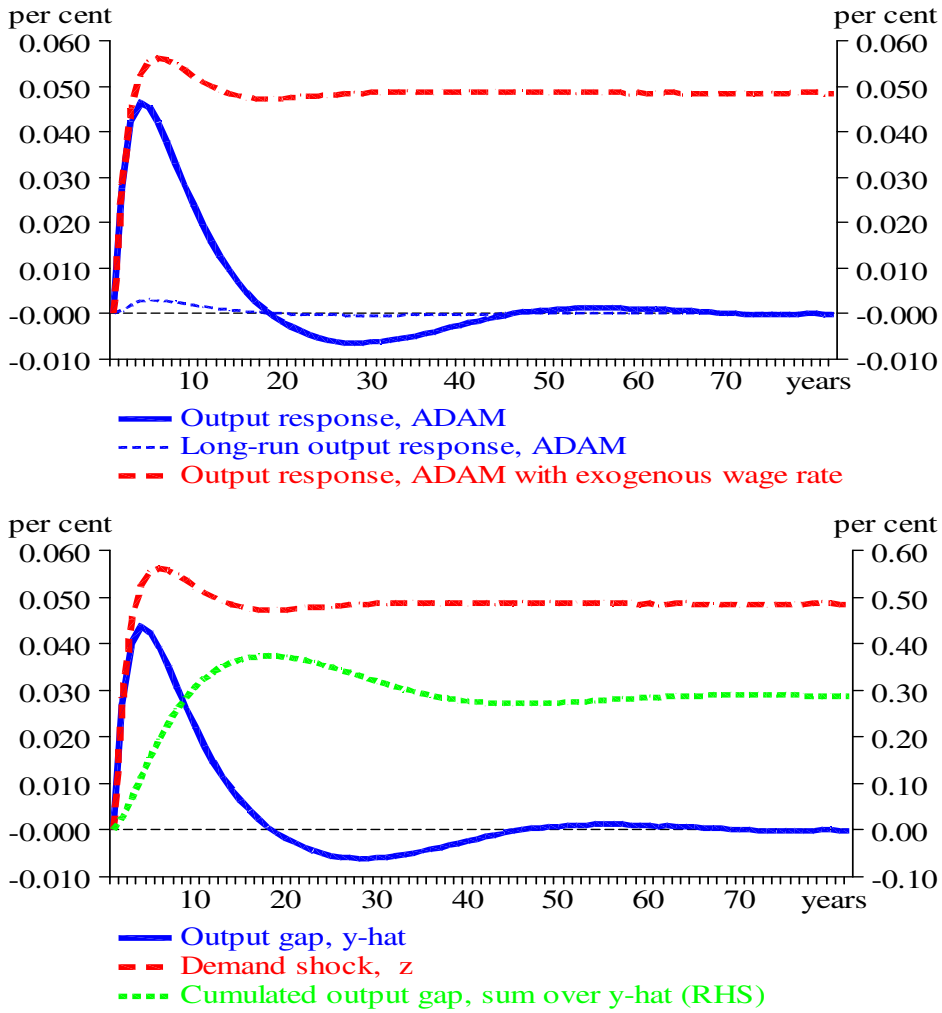
In this equation, we have introduced the lagged output gap on the right-hand side with parameter θ, and there are no restrictions on the other three parameters. Consequently, there are four parameters to estimate in the error-correcting equation (4ADAM): θ, α_1, α_2 and α_3. In (4), there was only one parameter, β, to estimate. The error-correcting form in (4ADAM) is encompassing the parsimonious textbook equation (4) that will emerge if θ is estimated to 1, $\alpha_1$and α_3 to β, and α_2 to $\beta - 1$.

The parameters in (4ADAM) will be estimated on the basis of ADAM calculations. More specifically, we shall re-use the public purchase shock

without budget restriction from the previous section and represent the impact on output by the impact on unemployment with a minus. The effect of the shock is calculated by the full ADAM model and by ADAM without wage relation, i.e. by ADAM and ADAM_K.

The results of 1 per cent higher public purchase are shown in figure 3 where the upper panel shows the impact on output in both ADAM and ADAM_K plus the impact on long-run output in ADAM. The lower panel shows the impact on the three variables, $\hat{y}$, $\sum\hat{y}$, and z, that enter (4ADAM). The output gap $\hat{y}$ is represented by minus the unemployment gap in ADAM, $\sum\hat{y}$ is cumulated $\hat{y}$ starting in year 1, and z is the fall of unemployment in ADAM_K. The impact on the output gap $\hat{y}$ shown in the lower panel is equal to the output response of ADAM minus the long-run out output response of ADAM shown in the upper panel, and the demand shock z is the output response in ADAM with exogenous wage.

Figure 3: Public purchase + 1%, ADAM-calculated responses, effect on output is minus effect on unemployment



In the first couple of years, it is difficult to distinguish the calculation on ADAM from the calculation on ADAM_K, but thereafter the difference

widens. In the long run, the impact on unemployment is zero in ADAM while unemployment remains its baseline in ADAM_K.

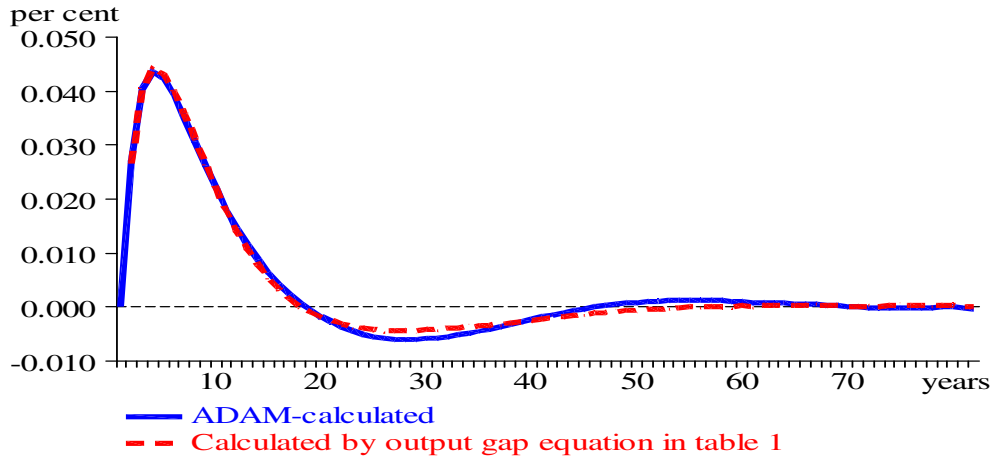
The institutional time lag in the wage indexation of unemployment benefits creates a slight temporary drop in the replacement ratio, and this drives the slight temporary increase in the ADAM-calculated long-run output that is visible in the upper panel of figure 3. In the long run, the wage rate attains its new level and grows in parallel to its original base line. This makes the lag in the wage indexation irrelevant, and higher public purchase has no long-run effect on unemployment in ADAM.

The lower panel of figure 3 illustrates that the cumulated output gap is overshooting its long-run target for a number of years. This overshooting is reflected in the wage rate of in ADAM, and it is the overshooting of the wage rate and its impact on competitiveness that forces the output gap to undershoot its target of zero for a number of years. In the long run, when the output gap has attained its steady-state value of zero, the wage rate will have attained its new steady-state value, so that also the wage gap is closed in the long run.

The result of estimating output equation (4ADAM) on the effects of the public purchase shock is shown in table 1. There is no stochastics in the ADAM calculation, so we have no standard errors for the four parameters. However, the high fit of (4ADAM) is illustrated by the high R square and by figure 4 that compares the ADAM-calculated output gap to the output gap calculated by inserting the ADAM_K-calculated shock z in output gap equation (4ADAM).

Table 1: Estimate of output gap equation (4ADAM)
$\Delta\hat{y} = \alpha_1 \cdot \Delta z - \theta \cdot \hat{y}_{-1} + \alpha_2 \cdot \sum \hat{y}_{-1} + \alpha_3 \cdot z_{-1}$
$\Delta\hat{y} = 0.938 \cdot \Delta z - 0.136 \cdot \hat{y}_{-1} - 0.0120 \cdot \sum \hat{y}_{-1} + 0.070 \cdot z_{-1}$, R2=0.998
OLS estimate based on public purchase experiment. Effect on output gap represented by minus effect on unemployment gap.

Figure 4: Approximating ADAM's output gap (minus unemployment gap), public purchase + 1%

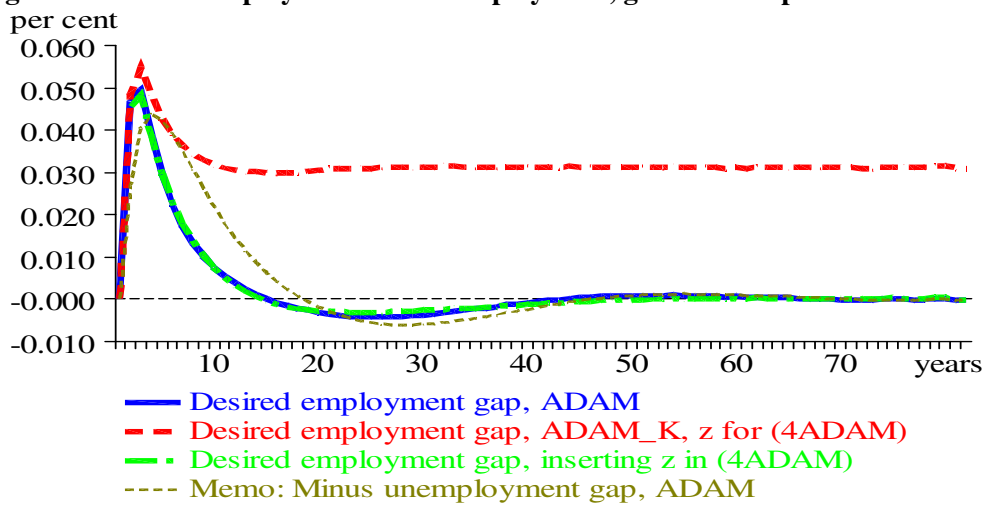


ADAM is a large dynamic model and it is not evident that we would achieve a high fit when estimating the rather simple equation (4ADAM). However, the fit in table 1 and figure 4 is hardly a coincidence because (4ADAM) has a

similar high fit when estimated on the basis of other demand shocks. Besides, the estimated (4ADAM) in table 1 can also mimic the impact of the public purchase shock on the ADAM-calculated gap in desired employment.

The impact on the gap in desired employment is shown in figure 5. Figure 5 illustrates that the desired employment gap responds more quickly than the unemployment gap reflecting that desired employment is proportional to output in the short run. Moreover, figure 5 confirms that the estimated (4ADAM) can track the ADAM-calculated desired employment gap rather well. For this calculation, the shock variable z in (4ADAM) is the impact on desired employment gap calculated by ADAM with exogenous wage rate.

Figure 5: Desired employment and unemployment, government purchase + 1%



5. Comparing the gap response of ADAM and textbook

In order to illustrate the relation between ADAM and the textbook AS-AD model, we can compare the output gap calculated by our estimated (4ADAM) to the output gap calculated by the parametrized output gap equation of the textbook, where the parameter β of equation (4) is set to 0.82.

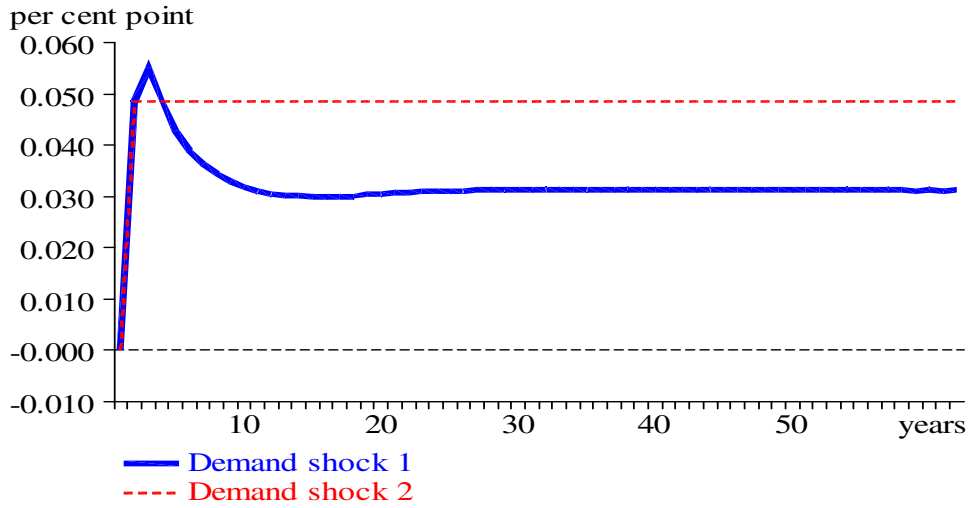
Thus, the two output gap equations to compare are:

$$\Delta\hat{y} = 0.938 \cdot \Delta z - 0.136 \cdot \hat{y}_{-1} - 0.0120 \cdot \sum \hat{y}_{-1} + 0.070 \cdot z_{-1} \qquad \text{(4ADAM)}$$

$$\hat{y} = -0.18 \cdot \sum \hat{y}_{-1} + 0.82 \cdot z \qquad (4)$$

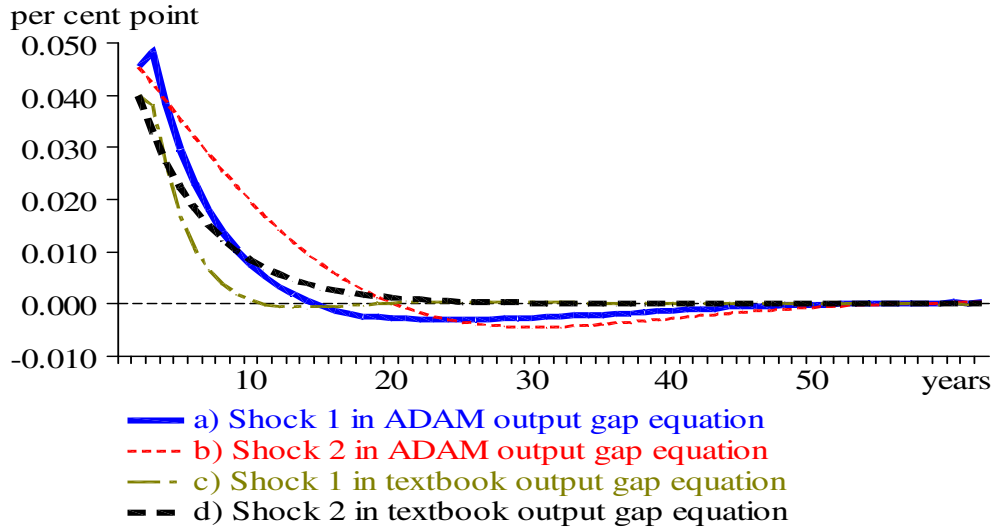
The comparison is based on two demand shocks. For shock number one we reuse the impact on desired employment gap of a constant 1 per cent increase in public purchase calculated by ADAM with exogenous wage rate. It is a shock with a peak, cf. the illustration in figure 6. Shock number two is a uniform shock repeating the first-year value of the first shock throughout the calculation period. A uniform shock is in accordance with the textbook that would not imply a shock with a peak to represent a constant 1 per cent increase of public purchase.

Figure 6: Demand shocks for output gap equations, government purchase + 1%



When demand shock 1 is inserted into equation (4ADAM), the resulting output gap response comes close to the result of increasing public purchase in ADAM. Thus, the output gap peaks in year 2, turns negative after 15 years, and returns to the zero axis after 40 years, cf. curve a) in figure 7. When shock 2 is inserted into the textbook equation (4), the first-year response of the output gap is lower because the coefficient for the contemporary shock is lower in (4) than in (4ADAM). After year 1, equation (4) makes the output gap follow the geometrical progression $\hat{y}_{+1} = 0.82 \cdot \hat{y}$. In principle, the progression will never reach zero, but after year 20, it is difficult to see the difference between curve d) and the zero-axis in figure 7.

Figure 7: Output gap in ADAM and textbook, public purchase +1%



The difference between curve a) and d) in figure 7 is caused by the difference between the two demand shocks and by the difference between the two equations, (4ADAM) and (4).

Shock 1 is larger in the short run than in the long run, and the additional short-run boost to wages and prices should shorten the duration of the first round positive impact on output gap. This is confirmed by figure 7, where curve c)

reaches and slightly crosses the zero-axis already after 10 years, which is a much faster approach to the zero-axis, than that of curve d). Thus, it has speeded up the adjustment of the textbook equation to replace the uniform shock 2 by shock 1. Besides, it slows down the adjustment of the ADAM-related equation if shock 1 is replaced by the uniform shock 2, cf. figure 7 where curve b) crosses the zero-axis 5 years after curve a).

7. Conclusion

We have compared the large Danish macro model ADAM to a small stylized AS-AD model describing a small open economy with fixed exchange rates. The raison d'être for ADAM is that it can address a range of issues because of its many equations and variables. A textbook model is not about details but about the big picture, and there are only few variables and equations in a standard textbook model.

However, ADAM and an AS-AD for a small open economy seem to agree a lot on the big picture. If we augment the dynamics of the textbook equation explaining the output gap, the textbook equation also reproduce the response the output gap in ADAM.

Appendix: Price elasticity in the text book and in ADAM

In the textbook, the price elasticity β_1 of output with respect to the real exchange rate is evaluated by means of the following expression derived in chapter appendix 23.8 on p. 715:

$$\beta_1 = \frac{[(M\ /\ Y) \cdot (\ \epsilon_E + \epsilon_M - 1)] + [\epsilon_D \cdot (D\ /\ Y)]}{1 - D_Y}$$

Import ratio $M\ /\ Y$= 0.3, export + import price elasticity $\epsilon_E + \epsilon_M$= 3, demand share $D\ /\ Y$ = 0.8, domestic demand elasticity with respect to real exchange rate, ϵ_D= -0.3, derivative of output with respect to total demand D_Y=0.5.
With the quoted textbook values, the expression sets β_1 to 0.72.

We want to relate this expression to ADAM's AS-AD model: (1ADAM), (2ADAM) and (3ADAM), and we start by rewriting the expression as (A1):

$$\beta_1 = [(M\ /\ Y) \cdot (\ \epsilon_E + \epsilon_M - 1)] + [\epsilon_D \cdot (D\ /\ Y)] + [\beta_1 \cdot D_Y] \qquad \text{(A1)}$$

In ADAM's AS-AD model, the real exchange rate is wage based, so that (A1) is formulating the output impact of 1 per cent lower Danish wage rate. The output impact is a sum of three square parentheses. The first parenthesis contains the output contribution from net export responding positively to the lower wage. The second contains output contribution from domestic demand responding negatively to the lower wage. The third contains output contribution from domestic demand responding positively to the higher output.

All three output contributions can be identified in an ADAM calculation. To do that, consider the central national accounts relation where GDP in market prices reflects domestic demand and net foreign trade:

$$GDP = CO + CP + IH + IXH + E - M$$

CO is public consumption, CP private consumption, IH housing investments, IXH other investments, E exports, and M imports. The simple relation holds in current prices.

We are not interested in current prices, but the percentage volume change of GDP from its baseline can be calculated as a weighted sum of percentage volume changes from baseline in the right-hand-side variables. Weights indicate the shares of the variables in baseline GDP evaluated at lagged prices. Using weights and volume changes from the end of the baseline period where all weights and volume changes have stabilized, the following relation holds.

$$GDP\% = CO\% \cdot 0.270 + CP\% \cdot 0.495 + IBH\% \cdot 0.067 + IXBH\% \cdot 0.148 + E\% \cdot 0.566\text{-}M\% \cdot 0.546$$

Where $X\%$ is the percentage volume change in variable X from X's baseline, and weights sum to 1 (=0.270+0.495+0.067+0.148+0.566-0.546). From the calculation on 1 per cent lower wage rate in ADAM with exogenous wage, we can insert volume changes (per cent) from base line:

$$GDP\% = 0.18 = 0 \cdot 0.270 - 0.31 \cdot 0.495 - 0.28 \cdot 0.067 + 0.29 \cdot 0.148 + 0.69 \cdot 0.566 - 0.14 \cdot 0.546 = -0.31 \cdot 0.495 - 0.28 \cdot 0.067 + 0.29 \cdot 0.148 + 0.69 \cdot 0.566 - 0.14 \cdot 0.546$$

GDP and weights are measured in market prices, i.e. including indirect taxes. Private consumption includes the bulk of indirect taxes, and if we reduce private consumption by 20 percent and change the weights for private consumption and the other items correspondingly, our national account relation calculates the 0.22 per cent change in GDP at factor costs. GDP at factor cost

must be the ADAM output concept that comes closest to the textbook world, and the difference between 0.22 for ADAM's β_1 and the textbook's β_1 of 0.72 is the formal difference to analyse in the following.

To simplify the comparison, we ignore a couple of finer points in ADAM: We use the 0.31 per cent fall in private consumption for housing investments as well instead of the 28 per cent fall calculated by ADAM. Thus, we have a 0.31 per cent fall in households demand consisting of consumption plus housing investments. By doing that, we are ignoring the effect of a small fall in the relative price of housing consumption when the wage rate drops 1 per cent at given import prices. We also use the GDP change of 0.22 per cent for other investments ignoring the 0.29 per cent calculated by ADAM. The ignored difference reflects a compositional change towards foreign-competing industries like manufacturing and agriculture that are both capital intensive.

The simplifications just mentioned disappear in the rounding-off errors, so that the expression for ADAM's β_1 still holds:

$$\beta_1 = 0.22 =$$
$$-0.31 \cdot 0.448 - 0.31 \cdot 0.073 + 0.22 \cdot 0.162 + 0.69 \cdot 0.619 - 0.14 \cdot 0.597 =$$
$$-0.31 \cdot 0.521 + 0.22 \cdot 0.162 + 0.69 \cdot 0.619 - 0.14 \cdot 0.597$$

In order to identify the negative impact on households' purchasing power, we formulate the -0.31 per cent change in household expenditure as the GDP volume increase of 0.22 per cent minus the purchasing-power impact of the lower wage rate. And to identify the price-driven impact on imports, we express the 0.14 per cent increase in imports as the GDP increase minus the fall in imports relative to GDP. This gives us:

$$\beta_1 = 0.22 =$$
$$(0.22 - 0.53) \cdot 0.521 + 0.22 \cdot 0.162 + 0.69 \cdot 0.619 - (0.22\text{-}0.08) \cdot 0.597$$

We can now recollect terms on the right-hand side and get three output contributions that are comparable to the three contributions suggested in (A1) above:

$$\beta_1 = 0.22 =$$
$$[0.619 \cdot 0.69 + 0.597 \cdot 0.08] + [-0.53 \cdot 0.521] + [0.22 \cdot 0.086] =$$
$$[E\ /\ Y \cdot \epsilon_E + M\ /\ Y \cdot \epsilon_M] \quad + [-\epsilon_D \cdot (D\ /\ Y)] + [\beta_1 \cdot D_Y] \qquad \text{(A2)}$$

It follows that the ADAM-based suggestions for parameters and variables are: Export ratio $E\ /\ Y$ = 0.619, import ratio $M\ /\ Y$ = 0.597, export wage elasticity ϵ_E = 0.619 and import wage elasticity ϵ_M = 0.08; domestic demand elasticity with respect to real exchange rate, ϵ_D= -0.53 and domestic demand share $D\ /\ Y$ = 0.521; derivative of output with respect to total demand D_Y=0.086 and β_1 is 0.22. Now, the squared parentheses can be interpreted one by one.

The first parenthesis in (A1) and in (A2) indicates the output effect from net foreign demand driven by higher competitiveness. The value of the parenthesis is 0.6 in (A1) with textbook parameters and 0.475 in (A2) with ADAM-based parameters. The difference reflects that a substantial difference in the response of foreign trade is partly offset by higher trade shares in the ADAM calculation and by a difference in method.

The high import and export shares in the ADAM calculation reflect the present level when GDP is measured at factor costs. The competitiveness-driven 0.69

per cent increase in exports is a simple ADAM-calculated result, and it should be straightforward to discuss the difference to the textbook.

In the textbook, the sum of export and import price elasticity is set to 3. It seems a fair guess that the export price elasticity accounts for 2 of the 3, and 2 is clearly higher than 0.69. However, the 0.69 does not represent ADAM's price elasticity. It represents ADAM's wage elasticity. One per cent lower wage rate produces a moderate 0.26 per cent long-term fall in the total export price, partly because prices of energy, agricultural products and sea freight are exogenous, but also because there is a content of imports in exports.

Thus, the difference between 0.69 and 2 per cent for the impact on exports reflects the difference between wage and price elasticity. The wage rate in ADAM would have to be reduced by 3.85 per cent to reduce the export price by 1 percent and that would produce 2.7 per cent additional export volume. Also the difference between 0.08 and 1 per cent for the impact on imports is related to the difference between wage and price elasticity because 1 per cent lower wage rate reduces Danish output prices by less than 1 per cent. Besides, the residually derived negative price-driven import impact of 0.08 per cent may be too modest, because the volume-driven impact on imports is higher than the GDP increase due to high import content in investments and exports.

Last but not least, the difference between the first parenthesis in (A1) and (A2), respectively, reveals a difference in method between the textbook and our ADAM calculation. The textbook is calculating the nominal impact of net exports when it subtracts the assumed price fall of 1 per cent from the sum of export and import price elasticity. However, if we are interested in the volume contribution of net exports to real GDP we should not subtract the price fall from the volume contribution. If the textbook calculation were revised accordingly, its β_1 estimate would ceteris paribus increase from 0.72 to 1.32. The textbook approach may reflect that the terms of trade is assumed to have a technical impact on imports in fixed prices, cf. p. 701 of the textbook. In the ADAM calculation, the negative impact from the terms of trade is carried by the second parenthesis.

The second parenthesis represents the GDP contribution from the Harberger-Laursen-Metzler effect, i.e. from the fall in domestic purchasing power driven by 1 per cent change in the real exchange rate. The textbook sets this contribution to -0.24 per cent ($= -0.3 \cdot 0.8$), which is close to the -0.28 per cent ($= -0.53 \cdot 0.521$) in the ADAM calculation. The higher negative impact on the purchasing power, -0.53 against -0.3, reflects the higher import content in the ADAM calculation. While the lower ADAM weight for private demand in GDP, 0.521 against 0.8, reflects that the business investments of ADAM follow GDP and not household demand. Besides, the GDP share of exogenous public demand is higher and there is a small positive net export in ADAM's baseline.

The third parenthesis can be seen as a multiplier contribution where higher GDP increases the demand for GDP. In the textbook, the multiplier

contribution is 0.36 ($= 0.72 \cdot 0.5$) reflecting a D_Y coefficient of 0.5. In the ADAM calculation, the multiplier contribution is only:
$0.019 = 0.22 \cdot (0.521 + 0.162 - 0.597) = 0.22 \cdot 0.086$
ADAM's D_Y coefficient is only 0.086 because the import content of 0.597 is deducted. The import content reduces the multiplier contribution in an open economy. A lower D_Y coefficient would reduce the textbook estimate of β_1.

Summing up, we may revise the formal textbook calculation, e.g. replace the nominal change in foreign trade: $(M\ /\ Y) \cdot (\epsilon_E + \epsilon_M - 1)$ by the real change: $(M/_Y) \cdot (\epsilon_E + \epsilon_M)$ and reduce the D_Y coefficient because of the import content. However, the basic issue is the price concept as mentioned in the main text. The textbook is referring to price changes of 1 per cent and it would take wage rate changes of 2 to 4 per cent in ADAM to produce these price changes. Consequently, it is as expected that the price elasticity of GDP is higher than the wage elasticity.

Litterature:

Statistics Denmark, 2013. ADAM – a model of the Danish economy.

Sørensen, P.B. and H.J. Whitta-Jacobsen, 2005. Introducing Advanced Macroeconomics. Mc Graw Hill.

Sørensen, P.B. and H.J. Whitta-Jacobsen, 2010. Introducing Advanced Macroeconomics. Mc Graw Hill.